

# The Role of Utterances in Bradwardine's Theory of Truth\*

Elia Zardini

*Instituto de Investigaciones Filosóficas*  
*Universidad Nacional Autónoma de México*  
*Northern Institute of Philosophy*  
*University of Aberdeen*  
elia.zardini@abdn.ac.uk

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## Abstract

Thomas Bradwardine’s theory of truth has recently received much attention, due to its seeming ability to offer a solution to a wide range of semantic paradoxes which preserves classical logic and semantics without jettisoning a unified truth property and predicate. It does so by focussing on truth as a property of utterances, and by appealing to general principles about utterance truth and utterance saying to justify unobvious restrictions on the principle that, if  $u$  is an utterance of the sentence ‘ $P$ ’,  $P$  only if  $u$  is true. The paper analyses the crucial role that the focus on utterances as truth bearers plays in Bradwardine’s theory, arguing that this feature of the theory makes it unable to deal with semantic paradoxes targeted at kinds of truth bearers differing from utterances in the relevant respects.

So suppose that Socrates says only this: Socrates says something false, which we’ll call  $a$ . And let this be proposed: Socrates says something false. Then, this must be conceded. [7.1.1] Objection: But then this is true: Socrates says something false, and Socrates says this, therefore Socrates says something true. [ad 7.1.1] Reply: One should say that the minor premise is false, since he doesn’t say this proposition—the one proposed by you and conceded by me—but another one similar to it, namely  $a$ . (Thomas Bradwardine, *Insolubilia*, 7.1.)<sup>1</sup>

## 1 Introduction and Overview

Philitas of Cos, an ancient Greek poet and scholar of the early Hellenistic age (born ca. 340 BC, died ca. 285 BC), is (somewhat dubiously) reported to have died owing to the distress incurred by thinking hard about the sentence that says of itself that it is false (see Bocheński [1970], p. 131). Many great ancient philosophers seem not to have taken the philosophical and logical puzzle presented by that sentence so seriously as Philitas did according to that report—indeed, in their extant writings they either ignore it completely

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<sup>1</sup>“*Dicat ergo Sortes tantum istam: Sortes dicit falsum, que sit a. Et proponatur ista: Sortes dicit falsum, tunc ista est concedenda. [7.1.1] Contra: tunc hec est vera: Sortes dicit falsum, et Sortes dicit istam, ergo Sortes dicit verum. [ad 7.1.1] Dicendum quod minor est falsa, quia non dicit istam propositionem propositam a te et concessam a me, sed aliam sibi similem, scilicet a*”. Throughout, I’ll avail myself of the new edition of the tract due to Stephen Read and recently appeared as Bradwardine [2010] (although my translation will diverge on minor points from Read’s).

or relegate it to the status of a fallacy receiving an obvious and straightforward treatment.<sup>2</sup> There are for sure some that seem to have granted the paradox a greater philosophical and logical interest (such as Chrysippus, who wrote perhaps as many as 28 books on the topic, see again Bocheński [1970], p. 131), but, alas, their writings on the matter have almost entirely gone lost. It is only from the Middle Ages onwards that we possess an extensive literature showing both a clear appreciation of the force of the paradox and a considerable acumen in coming up with solutions to it, many of which have not only historical but also theoretical interest to the contemporary eye. Prominent among such solutions is the one offered by Thomas Bradwardine (member of the Oxford *calculatores* and later Archbishop of Canterbury) in his tract *Insolubilia*, written in the early 1320s.<sup>3</sup>

Bradwardine's solution is developed within the more general framework of his theory of truth. The theory shares with many other medieval theories of truth an exclusive focus on *utterance* truth (in a sense of 'utterance' to be clarified presently). Indeed, the solution cleverly exploits features of utterances in such a way as to reach a delicate equilibrium among different, potentially contrasting elements that are often considered to be desiderata on a theory of truth aiming to deal with the semantic paradoxes: preserving classical logical and semantic principles, avoiding introducing a hierarchy of truth properties and predicates, upholding the idea that there are paradoxical things that say of themselves that they are false, vindicating the thought that any such thing is false while offering a principled and reasonable explanation of why that does not imply that any such thing is also true (since what it says—i.e. that it is false—is the case). The first part of the paper will study in some detail how such a surprising squaring of the circle is achieved. What will be presented is a way in which the shift to utterances in the theory of truth can allow for an otherwise very unlikely combination of elements in one's solution to the semantic paradoxes—a way which differs in crucial respects from a certain application of an utterance-based theory of

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<sup>2</sup>An example of the first kind is Plato, an example of the second kind is Aristotle (see *De sophisticis elenchis*, 25, 180a27–b7). In a sense, it is of course possible that in that text Aristotle is offering an adequate solution whose real force we contemporaries are failing to appreciate, but I think it's much more likely that it's the Stagirite who's failing to appreciate the real force of the paradox. Unsurprisingly, most medievals thought the opposite, so much so that they regarded as a condition of adequacy on a solution to the paradox that it vindicate Aristotle's contention that the paradox arises from a fallacy *secundum quid et simpliciter* (see Spade [1987], p. 33).

<sup>3</sup>See Read [2002] for some more biographical information, and for a sympathetic exposition of Bradwardine's theory (see also Read [2006]; [2007]; [2008a]; [2008b]; [2009]; [2010a]; [2010b], which together constitute a powerful attempt at establishing Bradwardine's approach as a worthy contender in the contemporary debate on the semantic paradoxes).

truth which has gained some popularity in the recent literature on the semantic paradoxes (see the references in fn 20) and which Bradwardine himself vigorously opposed.

Exactly because of this reliance on utterances, the worry arises however that the solution will not be extensible to semantic paradoxes involving a notion of truth applying to things of a different kind. The second part of the paper will substantiate this worry by considering semantic paradoxes involving notions of truth applying to *propositions* and *sentences*. It will be argued that an extension of Bradwardine's solution to these cases fails to engage with the peculiar coarse-grainedness of individuation characterising the relevant domains (the domain of propositions and that of sentences) and that such a failure generates consequences of the uttermost implausibility.

The rest of the paper is organised as follows. Section 2 introduces the idea of taking utterances as the carriers of truth and falsity and delves a little into some important issues concerning their ontology. Section 3 discusses what is arguably the foundation of Bradwardine's theory of truth—the relation of saying between utterances and propositions—articulating Bradwardine's claim that saying is closed under consequence. On this basis, section 4 briefly presents Bradwardine's theory of truth and falsity proper, and its bearing on some traditional semantic principles. Section 5 sets forth versions of the most common semantic paradoxes in an utterance-theoretic framework, demonstrating how they create trouble for a naive theory of utterance truth. Section 6 explains how, on the contrary, Bradwardine's own theory of truth is able to deal with such paradoxes. Section 7 sets forth versions of the most common semantic paradoxes in a proposition-theoretic and sentence-theoretic framework, demonstrating how they create trouble for a naive theory of propositional and sentential truth. By way of illustration, section 8 proposes an extension of Bradwardine's theory to sentential truth, arguing that its application to the semantic paradoxes for sentential truth breaks down at a crucial place where it would require a distinction that simply cannot be drawn once one has ascended to the level of sentences. Given this failure of Bradwardine's approach, section 9 draws the conclusion that the theory cannot be applied so as to offer a fully general solution to the semantic paradoxes and hence is unlikely to have uncovered the real root of the problem even in the case of the semantic paradoxes for utterance truth.

## 2 Utterances

One can certainly come to know what *truth* is without knowing—at least in full generality—what is *true*: knowledge of what truth is doesn't require knowledge of how things stand with respect to any possible factual question (for example, it doesn't require knowledge of whether it is true that there are more than 1,963 books in the departmental library). Still, it would seem that one can hardly come to know what truth is without knowing what *kinds of things* are capable of being true, just as it would seem that one can hardly come to know what moral goodness is without knowing what *kinds of things* are capable of being morally good. Outcomes? Actions? Intentions? Persons?<sup>4</sup>

The question of what kinds of things are capable of being true is known in the contemporary debate on truth as the question of what are the kinds of things that are *truth bearers*, a truth bearer being a thing which, by its own nature, is capable of being true (see Kirkham [1992], pp. 41–72 for an in-depth discussion of the problem of truth bearers).<sup>5</sup> Sentences? Utterances?

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<sup>4</sup>Notice that, for moral goodness just as for truth and many other natural, not gerymandered properties, one can certainly come to know what *moral goodness* is without knowing—at least in full generality—what is *morally good*: knowledge of what moral goodness is doesn't require knowledge of how things stand with respect to any possible moral question (for example, it doesn't require knowledge of whether it is morally good to become a vegetarian).

<sup>5</sup>Only 'capable of being true'. The sentence 'Snow is black' is by many theorists considered to be a truth bearer, yet it is not true. Granted, it is *false* and not just simply *untrue* (i.e. not true). (Contrast with the Mont Blanc, which is untrue without being false.) So can 'capable of being true' be replaced by 'either true or false'? No, because that would build into the very basics of the theory of truth an unwanted commitment to some form or other of the principle of *bivalence* (which we can provisionally state as the principle saying that every truth bearer is either true or false). The principle of bivalence is rejected by some theorists of truth on grounds relating to the semantic paradoxes (see e.g. Kripke [1975]), which will concern us in the following (it is also rejected by some theorists of truth on other grounds, which need not concern us here). Notice also that the *modality* in 'capable of being true' is somewhat *sui generis* and cannot easily be reduced to *metaphysical* modality (the modality pertaining to what is necessary and to what is possible according to the metaphysical laws of being), by dint of something like 'being such that it is metaphysically possible that it is true'. The sentence ' $2 + 2 = 5$ ' is by many theorists considered to be a truth bearer, yet it is not metaphysically possible that it is true. The modality would seem rather to relate to the *kind* that a thing is a sample of: being capable of being true is to be explained, very roughly, as being a sample of a kind which *by itself* does not prevent its samples from the (metaphysical) possibility of truth. Thus, the sentence ' $2 + 2 = 5$ ' is capable of being true because of its being a sample or other of sentencehood (and because, contrary to mountainhood, sentencehood by itself does not prevent its samples from the (metaphysical) possibility of truth), not because of

Propositions? Beliefs? The question presupposes, plausibly enough, that the truth bearer/not truth bearer distinction carves up things in *kinds*, so that, for every kind, either every sample of that kind is a truth bearer or none of its samples is. The question also quickly leads to the question concerning the *dependency relations* between different kinds of truth bearers: can the truth of a truth bearer of a certain kind be explained in terms of the more fundamental truth of a truth bearer of another kind? For example, can the truth of the *sentence* ‘Snow is white’ be explained in terms of the more fundamental truth of the *proposition* that the sentence expresses, namely of the proposition that snow is white?

Many a theory of truth has to make substantial assumptions about truth bearers. On approaching Bradwardine’s theory of truth, what immediately strikes the eye of the contemporary reader is the quick admission of *utterances* (*propositiones*) as truth bearers and indeed the theory’s firm and resolute restriction to *utterance truth*. In the following, it will be my main critical contention that such a restriction hampers Bradwardine’s theory in very significant respects, in the sense that his solution to the Liar paradox (to be introduced in due time) crucially relies on features of utterance truth that are not present for the truth of other plausible truth bearers, like e.g. propositions or sentences. However, I propose to focus for the time being (sections from 2 to 6) on the presentation of Bradwardine’s theory of utterance truth, leaving for later (sections 7 and 8) an assessment of what its merits and demerits may be when understood more broadly as a general theory of truth.

We first need a better understanding of the *ontology* of utterances. Relying more on what Bradwardine and other medieval authors he seems to follow suit merely imply rather than explicitly say and subject to an important qualification to be entered shortly, the technical term ‘utterance’, as used in contemporary philosophy of language, seems to match fairly well what *propositiones* are understood to be by such authors. They are crucially a distinct kind of thing from what *sentences* are nowadays understood to be. In contemporary philosophy of language, sentences are understood to be *abstract* linguistic entities,<sup>6</sup> indeed *types*, capable of multiple tokenings in

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its being the specific sample of sentencehood it is.

<sup>6</sup>The abstract/concrete distinction is notoriously hard to be made precise (see e.g. Lewis [1986], pp. 81–86). An intuitive understanding of it will suffice here. Also, notice that, given that for most natural languages there is no one-to-one correspondence between basic *letters* and basic *sounds*, it does make a substantial difference whether we understand such abstract entities as composed of atoms that are letters or as composed of atoms that are sounds (both understandings are of course legitimate, even though I suspect that the latter is much closer than the former to our pre-theoretical conception of a sentence). Finally, notice that such abstract linguistic entities can be individuated either coarse-grainedly,

the concrete world.<sup>7</sup> Utterances are then typically understood to be concrete *tokens* of these types, what speakers of a language produce—with sounds, ink, gestures and whatnot—when they use the language.

Thus far reaches the convergence between Bradwardine’s *propositiones* and utterances on the contemporary understanding. Their divergence consists in the fact that, while Bradwardine’s *propositiones* are conceived of as *products* of speech, *continuing objects* whose existence usually stretches beyond that of a speaking or writing (especially so in the case of a writing) and which can be differently used by different speakers, utterances on the contemporary understanding are rather conceived of as *acts* of speech, *occurring events* which usually don’t last for longer than the time of a speaking or writing and which can be used (to stretch a little the ordinary meaning of ‘use’ so as to cover acts as its objects) in only one way by only a single speaker (see García-Carpintero [1998], pp. 534–535 for a useful contrast between these two conceptions).

For example, suppose that Yīn writes on a big board the sentence ‘This site is dangerous’ placing it next to a perfectly safe playground, and that after some time the same board with the same inscription is taken over by Yáng to be placed next to an unsafe construction site. Then, in this situation Bradwardine’s *propositio* is the unique inscription on the board, which persists at least from the time of writing until and including the board’s stay next to the construction site, and which is used on a first occasion by Yīn to mislead children into thinking that the playground is dangerous and on a second occasion by Yáng to warn people that the construction

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solely in terms of their *syntactic* properties, or more fine-grainedly, in terms of their *syntactic and semantic* properties. For example, on the former understanding—and assuming sounds to be the atoms—there is just one sentence that sounds exactly like ‘Empedocles leaped’ and has the syntactic structure that this sentence has in English, while, on the latter understanding, there are at least two such sentences, namely the (English) one meaning that Empedocles leaped and the (German) one meaning that Empedocles loves (again, both understandings are of course legitimate, even though I suspect that the latter is much closer than the former to our pre-theoretical conception of a sentence). Throughout, I’m assuming a fine-grained, semantic individuation of sentences (see also fn 12 for a further clarification). Throughout, I’m also employing quotations marks and display to pick out such entities (and to pick out sub-sentential entities at the same level of fine-grainedness of individuation), occasionally understanding both devices as creating a *quasi-quotation* environment in the sense of Quine [1951], pp. 33–37 (context will determine whether quotation or quasi-quotation is intended).

<sup>7</sup>The type/token distinction, and the nature of its peculiarity with respect to other distinctions in the vicinity (like the property/exemplifier distinction, or the kind/sample distinction, or the set/member distinction etc.) is a subject of much controversy in contemporary metaphysics (see Wetzel [2008] for a critical survey). Again, an intuitive understanding of it will suffice here.

site is dangerous. In the same situation, however, there are at least two utterances on the contemporary understanding: the speech act performed by Yīn, existing at most between the time of writing and the board's removal from next to the playground<sup>8</sup> and misleading children into thinking that the playground is dangerous, and the speech act performed by Yáng, existing at most throughout the board's stay next to the construction site and warning people that the construction site is dangerous. This cautionary note being made, I will henceforth stick to using 'utterance' and its like, screening off its contemporary connotations and stipulatively understanding it rather simply as a translation of '*propositio*' and its like.<sup>9</sup> Utterances will thus understood

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<sup>8</sup>It is a matter of some delicacy what exactly the *temporal location* (and, with it, the precise *ontology*) of Yīn's speech act is best understood to be. On some views, it occurs just at the time of Yīn's writing on the board (or maybe of her placing the board next to the playground). On some other views, it also occurs throughout the board's stay next to the playground. On yet other views, there are in fact several speech acts, each of which occurs exactly at one of the times at which the board is read (or maybe at one of the times during the board's stay next to the playground). And more views are possible. Fortunately, we don't need to adjudicate among them here: the important point is that, on *all* such views, the temporal location of Yīn's speech act is going to be substantially different from the temporal location of her inscription. Analogous considerations apply of course to Yáng's speech act.

<sup>9</sup>It is instructive to ask oneself why contemporary theorists have shifted away from the traditional product-oriented conception of utterances to the new act-oriented conception. I believe that one main reason for this theoretical departure has been the (mostly unacknowledged) tendency in favour of having an *absolute* notion of truth as applying to material objects. For, on the product-oriented conception, it is most natural to think that, say, the utterance (i.e. the inscription) on the board is first false (when used by Yīn) and then true (when used by Yáng): this is plausibly taken to entail that the utterance is false at some time and true at some other (later) time, from which it would seem to follow that its truth must in some sense be relative to times and cannot thus be absolute. The need for such a relativisation, which might be thought to be problematic on independent grounds, disappears if one adopts a more fine-grained individuation of utterances, such as that afforded by the act-oriented conception. For, on such a conception, uses of the same product at different times are distinct acts, and so will count as different utterances, each of which possesses its own truth value (either truth or falsity) absolutely (at least at the times at which it exists). For what's worth, I think that a medieval author would have rejected the step signalled by 'from which it would seem to follow' in the previous reasoning, presupposing as it does that, as opposed to being such-and-such, exemplifying the property of being such-and-such cannot itself be a temporal affair (and so that the fact that things are, say, red at some times and not red at other times implies that things do not exemplify the property of being red absolutely, but bear instead a binary relation of redness to times, or a ternary relation of exemplification to the property of being red and to times). And I believe that, even granting that step, a medieval author would still have insisted that a particular time—namely, the present—is privileged in such a way that, although being true is generally relative to times, this is compatible with there being something recognisable as being true absolutely, and that such a thing coincides with (and



to be vocal sounds, inscriptions on paper, projections on a screen etc.

With particular regard to the abstract/concrete distinction, I should like to stress that this interpretation of Bradwardine’s understanding of the ontology of utterances is, as far as I can judge from the text of the *Insolubilia*, (highly plausible but) not indisputable. As I’ve already indicated, the text is not explicit about what utterances really are, and most predicates applied to them allow both for a reading under which they apply to abstract objects of some kind or other and for a reading under which they apply to concrete objects of some kind or other (see e.g. “[...] Socrates concedes something false [...] Socrates denies something false, Socrates hears something false, Socrates sees something false, Socrates writes something false, Socrates reads something false [...]”, Bradwardine [2010], 8.0).<sup>10</sup> Still, much of what Bradwardine says makes most straightforwardly sense if utterances are interpreted to be concrete objects, and some passages offer indeed considerable resistance to any alternative interpretation (see e.g. “[...] let this be written: a is not true, which Socrates sees indistinctly from afar, in such a way that he perceives well that there is a material object and in such a way that he indeed perceives distinctly the subject a. And then he can stipulate that that a, which he distinctly sees, signify the entire material object which he indistinctly sees, and he understands by a that material object”, Bradwardine [2010], 3.1.6).<sup>11</sup>

As may be gathered from the epigraph of this paper and as I’ll explain in detail in section 6, the theory espoused by Bradwardine does certainly require that utterances be more fine-grainedly individuated than sentences are, but, again, that is after all compatible with their being a *sui generis* kind of (very fine-grainedly individuated) abstract objects. Yet, one would have expected from Bradwardine some discussion of such unusual objects if those were to provide the domain of truth bearers for his theory of truth (not so if such

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possibly consists in) being true relative to the present time.

<sup>10</sup> “[...] *Sortes concedit falsum* [...] *Sortes negat falsum*, *Sortes audit falsum*, *Sortes videt falsum*, *Sortes scribit falsum*, *Sortes legit falsum*”. If it is not immediately obvious, for at least some of the relevant predicates like ‘ $\tau_0$  sees  $\tau_1$ ’, that they allow for a reading under which they apply to abstract objects of some kind or other in their second place, one may wish to consider sentences like ‘Socrates sees beauty everywhere’ (this point may need a notion of abstractness consistent with spatiotemporal location; if it does, that seems anyways the correct kind of notion under which linguistic types—which can be written down on paper, sand, walls etc.—can be said to be abstract).

<sup>11</sup> “[...] *scribatur ista: a non est verum, quam videat Sortes a remotis indistincte, ita quod bene percipiat ibi esse unum materiale, et subiectum scilicet a distincte percipiat. Et tunc ipse potest imponere illud a, quod distincte videt, ad significandum illud totum materiale quod videt indistincte et intelligit illud materiale per a*”.

a domain is provided by common-or-garden concrete tokens of sentences). Be that as it may, fortunately we don't have to settle this issue here: to fix ideas, I have in effect been assuming and I'll indeed continue to assume with the majority of the commentators that utterances are concrete tokens of sentences (although see Read [2008b], p. 211 for a dissenting voice),<sup>12</sup> but what I'll say will apply just as well, *mutatis mutandis*, under an interpretation according to which utterances are abstract objects of a suitable kind.

### 3 Saying and Closure

A prominent property utterances are supposed to have in Bradwardine's theory is that of *saying things* (*significare*). I believe that the notion should be regarded as (the fundamental) primitive of the theory, intended to track important aspects of the pre-theoretic notion of saying, and that it is best introduced by intuitive examples.<sup>13</sup> We can for example call ' $\mathfrak{S}$ ' a particular self-standing utterance of the English sentence 'Snow is white', like this:

$\mathfrak{S}$  : Snow is white.<sup>14</sup>

What does  $\mathfrak{S}$  say? Well, intuitively it at least says that snow is white. Note that, in spite of its seeming obviousness, this is actually non-trivial. For the principle of *utterance-saying disquotation*:

(USD) For every utterance  $u$  of the sentence ' $P$ ',  $u$  says that  $P$ <sup>15</sup>

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<sup>12</sup>Somewhat confusingly, that paper uses 'sentence' and its like to mean the envisaged *sui generis* kind of very fine-grainedly individuated abstract objects. Throughout, I'm using 'sentence' and its like in a more standard way, to mean what linguists and logicians mean by it (at least roughly: there might be variations of detail within this common understanding). For example, if  $\tau$  is an English determiner phrase (like 'this sentence') and  $\Phi$  a congruent English verb phrase (like 'is false'), there is exactly one English sentence  $[[\tau]_{\text{DP}}[\Phi]_{\text{VP}}]_{\text{S}}$  (like 'This sentence is false').

<sup>13</sup>There is arguably a sense of 'say' (usually labelled in contemporary philosophy of language as '*pragmatic*' rather than '*semantic*') such that, among other things, a typical utterance of 'Most students passed the exam' "says" (or "conveys", or "communicates", or "implies" etc.) that not every student passed the exam. In the following, we'd do better to screen off this pragmatic sense and focus on the semantic one, even though we'll see that Bradwardine endorses very strong closure principles for saying which do not perfectly match with the contemporary notion of what an utterance "semantically expresses" either.

<sup>14</sup>Of course, for each different token of this paper ' $\mathfrak{S}$ ' will denote a different utterance of the sentence 'Snow is white', but the differences between such utterances will not matter for our purposes. Analogous considerations apply to the utterances displayed below.

<sup>15</sup>' $P$ ' as it occurs (twice) in (USD) is to be understood *schematically* (see fn 17 on my

is demonstrably incorrect in its full generality. The trouble comes from a certain *context dependence* of some English expressions,<sup>16</sup> context dependence which makes it the case that different tokens of the same expression produced in different contexts refer to different objects, and so that the utterances of which these tokens are part<sup>17</sup> talk about different objects, which in turn presumably entails a divergence in saying between such utterances. This specific form of context dependence is usually referred to nowadays as ‘*indexicality*’ (although the term by no means has a fully specific and agreed on meaning in the contemporary debate). Let’s consider an example. Take an ordinary utterance  $u_0$  of the English sentence ‘I am British’ produced by Tony Blair. The expression ‘I’ is indexical, since a token of it typically refers<sup>18</sup> to its producer,<sup>19</sup> and different tokens can be produced by different speakers. In our situation, ‘I’’s token in  $u_0$  refers to Blair. By universal instantiation plus the empirical fact that  $u_0$  is an utterance of ‘I am British’, the relevant instance of (USD):

(USD<sup>I am British</sup>) For every utterance  $u$  of the sentence ‘I am British’,  $u$  says that I am British

yields:

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use of ‘occur’ and its like). Hence, (USD) itself is to be so understood, namely as equivalent to the set of sentences that are obtainable by replacing the schematic ‘ $P$ ’ with any declarative English sentence (or, more liberally, with any declarative sentence belonging to any possible extension of English), like e.g. ‘For every utterance  $u$  of the sentence ‘Grass is green’,  $u$  says that grass is green’ (one may wish to put additional restrictions on the admissible instances of such schemas, for example that the sentence does not suffer from certain kinds of reference failure). Later on, I will sometimes use ‘ $P$ ’, ‘ $Q$ ’ and ‘ $S$ ’ (possibly with subscripts) not so much as schematic for sentences but as (bindable) sentential variables. I will remain neutral about the proper interpretation of quantification using such variables (context will disambiguate between the schematic use and the quantifying use).

<sup>16</sup>Throughout, I’ll use ‘expression’ and its like to refer to (possibly sub-sentential) linguistic types as semantically individuated (see fn 6).

<sup>17</sup>Throughout, somewhat oversimplifying I’ll assume *parthood* to be the relation holding between *tokens* of component expressions and *utterances* of sentences in which these expressions occur. I reserve ‘occur’ and its like to denote the relation between *component expressions* and *compound expressions* they help to compose.

<sup>18</sup>Note that, in this explanation of the context dependence of ‘I’, I’m following Bradwardine and the medievals in assigning semantic properties to *tokens* rather than to *expressions relative to contexts* (as is usual—even though not universal—practice in contemporary formal semantics; see Kaplan [1989], pp. 522–523, 546 for a *locus classicus* where this distinction is drawn and the second option favoured). A contemporary semanticist would usually give a slightly different explanation in terms of the latter (see Reichenbach [1947], pp. 284–287 for a deviation from contemporary orthodoxy).

<sup>19</sup>Typically, but arguably not always (see Predelli [2005], pp. 40–75).

(USD $_{u_0}^{\text{I am British}}$ )  $u_0$  says that I am British.

Consider next my utterance  $u_1$  of (USD $_{u_0}^{\text{I am British}}$ ).  $u_1$  is arguably false, since it says that  $u_0$  says that I (Elia Zardini, the author of this paper, certainly different from Tony Blair) am British, but  $u_0$  presumably doesn't say any such thing, since it talks about Blair rather than Elia Zardini, and so presumably only says of Blair, rather than of Elia Zardini, that he is British. Hence, some utterance (i.e.  $u_1$ ) of a sentence (i.e. (USD $_{u_0}^{\text{I am British}}$ )) which is a consequence of an instance of (USD) (i.e. (USD $^{\text{I am British}}$ )) in conjunction with an uncontroversial empirical fact (i.e. the fact that  $u_0$  is an utterance of 'I am British') is false, and it is in this specific (and very natural) sense that (USD) is incorrect in its full generality.

In this connection, it is interesting to observe that some philosophers and logicians anterior to Bradwardine held that the failures of (USD) extended well beyond those determined by context dependence. They thought that the best solution to the paradox constituted by the following *Liar utterance*:

$\mathfrak{L}_0$  :  $\mathfrak{L}_0$  is false

was to maintain that, despite appearances,  $\mathfrak{L}_0$  does not say that  $\mathfrak{L}_0$  is false—indeed, does not say anything at all (in the conceptual space of the medieval solutions to the semantic paradoxes, the proponents of this view were known as '*cassantes*').<sup>20</sup> We'll discuss in section 5 what the paradox is exactly supposed to be, but, in the meanwhile, we can record Bradwardine's uncompromising and harsh opposition to *cassatio*:

But because these nullifiers of the first and second kind appear too asinine, one should not argue with them any further, as Aristotle says in *Metaphysics* B 4: "One should not consider seriously the speculations of myth". Hence the view of the nullifiers has been sufficiently nullified. (Bradwardine [2010], 5.6.)<sup>21</sup>

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<sup>20</sup>As far as I know, there are only two attested anonymous medieval sources endorsing the view: two tracts both entitled *Insolubilia* and dating respectively from the late twelfth/early thirteenth century and from around 1225 (see Spade [1987], pp. 29, 43–45 for details). Some prominent contemporary *cassantes* are Ryle [1950]; Bar-Hillel [1957]; Prior [1961]; Kneale [1971]; Mackie [1973], pp. 237–295; Goldstein [1992]; Smiley [1993]; Williamson [1998]; Sorensen [2001], pp. 165–184. The view seems to go back to Chrysippus (see Bocheński [1970], pp. 132–133). I criticise the view, at some length and in such respects as to affect also Bradwardine's own solution to the paradox, in Zardini [2008].

<sup>21</sup>"*Sed quia isti cassantes primi et secundi nimis asinini existunt, non est cum eis amplius disputandum, dicente Aristotele 3<sup>o</sup> Metaphysice: "de fabulose sophisticantibus non est dignum cum studio intendere". Opinio ergo cassantium satis est cassata*".

So much for what  $\mathfrak{S}$  intuitively does say. What does it intuitively *not* say? Well, intuitively, it does not say, at least, that snow is black, or that  $2 + 2 = 4$ , or that snow is not white etc. Does  $\mathfrak{S}$  also say anything stronger than (or weaker than, or independent from) that snow is white?

Before presenting Bradwardine’s (partial) answer to this question, I’d like first to say a bit more about what are the things that are said by utterances. Precious little is said by Bradwardine in this regard. Indeed, the main constructions used by Bradwardine when talking about what is said by a certain utterance (*‘significare’* or one of its like followed by an infinitive clause, *‘quod’*-clause or adverbial phrase, as in *‘significat se esse veram’* (see e.g. Bradwardine [2010], 6.4), *‘significat quod a est verum’* (see e.g. Bradwardine [2010], 6.6.1) and *‘significans aliter quam est’* (see e.g. Bradwardine [2010], 6.2) respectively) do not involve any overt ontological commitment. Even when *‘significare’* or one of its like is followed by a determiner or noun phrase as its direct object (as in *‘significat aliquod aliud’* (see e.g. Bradwardine [2010], 6.6.1) and *‘significat affirmationem’* (see e.g. Bradwardine [2010], 6.5.1) respectively), this is most naturally interpreted as a *higher-order* construction carrying as little ontological commitment as the determiner phrase ‘everything I want to be’ in ‘He is everything I want to be’.<sup>22</sup> I believe that Bradwardine’s theory could be stated and developed by sticking to such higher-order phrases, but it will ease our discussion considerably if we allow ourselves to the typical contemporary first-order talk of certain *objects* being said by utterances. I’ll follow contemporary usage in calling such objects *‘propositions’*. Such talk will always have to be understood as being (possibly) non-committing and as being at least in principle eliminable in favour of the corresponding (clumsier) higher-order talk (in fact, I’ll still use higher-order talk in some cases where it actually allows for snappier formulations). Consequently, I’ll also understand saying as a *first-order relation* between utterances and propositions.

Once these expository steps are taken, a further step will also prove extremely convenient: namely, to think of *consequence* (or *entailment*) as a *first-order relation among propositions* and hence say that the proposi-

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<sup>22</sup>For one, Read [2008b], pp. 209–213 insists, rightly in my view, on the higher-order interpretation of such constructions. Notice that I am not saying that these, when interpreted as being higher-order, are not ontologically committing. I am only pointing out that it is not at all clear that they are—in contrast to the determiner phrase ‘some apples’ in ‘I ate some apples’—and consequently that one should not be too hasty in attributing to Bradwardine’s theory a commitment to a realm of entities constituting the objects that are said by utterances (see Rayo and Yablo [2001] for an illuminating contemporary discussion of the problem of the ontological commitment of higher-order phrases).

tions  $\langle P_0 \rangle$ ,  $\langle P_1 \rangle$ ,  $\langle P_2 \rangle \dots$  entail the proposition  $\langle Q \rangle$ <sup>23</sup> iff the argument ‘ $P_0$ ,  $P_1$ ,  $P_2 \dots$  Therefore,  $Q$ ’ is valid. Again, I would like to stress that nothing in Bradwardine’s text evidences a commitment on his part to a realm of objects among which the relation of consequence holds: the most typical ways in which Bradwardine expresses the claim that the argument ‘ $P_0$ ,  $P_1$ ,  $P_2 \dots$  Therefore,  $Q$ ’ is valid are ‘This consequence is valid:  $P_0$ ,  $P_1$ ,  $P_2 \dots$  therefore,  $Q$ ’ (see e.g. ‘*ista consequentia est bona: homo est sedens, ergo homo est asinus*’, Bradwardine [2010], 6.5.4), ‘And it follows:  $P_0$ ,  $P_1$ ,  $P_2 \dots$  therefore  $Q$ ’ (see e.g. ‘*et sequitur: a non est verum, ergo non est ita totaliter sicut significatur per a*’, Bradwardine [2010], 6.6.1), ‘From its being the case that  $P_0$ , that  $P_1$ , that  $P_2 \dots$  it follows that  $Q$ ’ (see e.g. ‘*ex a esse falsum sequitur a esse verum*’, Bradwardine [2010], ad 7.2.5) and ‘Its being the case that  $Q$  follows from its being the case that  $P_0$ , that  $P_1$ , that  $P_2 \dots$ ’ (see e.g. ‘*a esse verum sequitur ad a esse falsum*’, Bradwardine [2010], 7.2.5). None of these constructions clearly commits to certain objects (such as  $\langle P_0 \rangle$ ,  $\langle P_1 \rangle$ ,  $\langle P_2 \rangle \dots$ ) standing in the relation of consequence to a certain object (such as  $\langle Q \rangle$ ). Still, the first-order talk just introduced will be helpful in simplifying the exposition and, as far as I can see, does not distort Bradwardine’s theory in any way which may be relevant to our concerns.

With these expository arrangements in place, we can easily state the only principle which Bradwardine uses (in conjunction with acceptable instances of (USD)) to determine whether, given a certain utterance  $u$  and a certain proposition  $\langle P \rangle$ ,  $u$  says that  $P$  (cf Bradwardine [2010], 6.3; I’m by and large following the reconstruction of what the principle should really amount to given by Read in the introduction to his edition, see Bradwardine [2010], pp. ???). The principle is a very strong principle of *closure of saying under consequence*:

(C) If  $u$  says that  $P_0$ , that  $P_1$ , that  $P_2 \dots$  and  $\langle P_0 \rangle$ ,  $\langle P_1 \rangle$ ,  $\langle P_2 \rangle \dots$  entail  $\langle Q \rangle$ , then  $u$  says that  $Q$ .

Thus, to go back to the example of  $\mathfrak{S}$ , since e.g.  $\langle \text{Snow is white} \rangle$  entails  $\langle \text{Either snow is white or grass is green} \rangle$ , (C), together with the empirical

<sup>23</sup>I’m using ‘ $\langle \varphi \rangle$ ’ to denote the proposition denoted by the phrase ‘the proposition that  $\varphi$ ’ (where ‘ $\varphi$ ’ is a *metalinguistic* variable over declarative English sentences). In those cases covered by acceptable instances of (USD), such a proposition will be among the propositions said by an utterance of  $\varphi$ . Of course, in those cases where, on the contrary, (USD) fails—such as indexicality—for some utterance  $u$  of  $\varphi$ , ‘ $\langle \varphi \rangle$ ’ will not denote a proposition said by  $u$  (for instance, in the previous example about Blair,  $\langle \text{I am British} \rangle$  is the proposition that Elia Zardini is British, and this proposition is not among the propositions said by Blair’s utterance  $u_0$  of ‘I am British’).

fact that  $\mathfrak{S}$  is an utterance of ‘Snow is white’ and the acceptable instance of (USD):<sup>24</sup>

(USD<sup>Snow is white</sup>) For every utterance  $u$  of the sentence ‘Snow is white’,  $u$  says that snow is white,

yields that  $\mathfrak{S}$  not only says that snow is white, but also says that either snow is white or grass is green. Ditto for the other infinitely many consequences of  $\langle$ Snow is white $\rangle$ .

Finally, we should note that, as with other medieval logicians, Bradwardine distinguished and accepted as legitimate at least two different notions of consequence (see e.g. Bradwardine [2010], ad 6.5.4). On the one hand, *absolute* consequence (*consequentia simpliciter*) corresponds roughly to the contemporary notion of *necessary truth preservation* from the premises to the conclusion: absolute consequence guarantees that it is necessary that, if all the premises are true, so is the conclusion.<sup>25</sup> On the other hand, *as-a-matter-of-fact* consequence (*consequentia ut nunc*) can be glossed roughly as *contingent truth preservation* from the premises to the conclusion: as-a-matter-of-fact consequence only guarantees that, if all the premises are true, so is the conclusion, but it does not guarantee that this is necessarily so.

As-a-matter-of-fact consequence is a complex and quite problematic notion, whose proper treatment lies outside the scope of this paper. Here, we will rest content with one straightforward way of introducing it, which exploits a previously understood notion of absolute consequence and goes as follows:

(AS)  $\langle P_0 \rangle, \langle P_1 \rangle, \langle P_2 \rangle \dots$  as-a-matter-of-fact entail  $\langle Q \rangle$  iff, for some  $S_0$ ,

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<sup>24</sup>Along with many medieval authors, I’m presupposing that ‘Snow is white’ is not indexical (and in particular that tense is not indexical).

<sup>25</sup>Notoriously, this is arguably going to be a *more inclusive* notion than the one defined by classical first-order logic or by any of its formal extensions. This is due to the fact that absolute consequence itself is a kind of *material* rather than *formal* consequence (see Read [1994] for a useful discussion of the formal/material distinction). For reflect that the intuitive idea of necessary truth preservation outruns the strictures of formal notions (such as implication, existence, identity etc.), allowing for example  $\langle$ A man is sitting $\rangle$  absolutely to entail  $\langle$ An animal is sitting $\rangle$ , even though not formally so, but in virtue of the occurrence of the non-logical notions of manhood and animalhood. It is still an important question to ask what the formal component of absolute consequence is. Throughout, I will assume it to be some form of classical higher-order logic. I think there is clear textual evidence that this choice approximates very well Bradwardine’s implicit one (see Zardini [2011a] for an exploration of what happens to Bradwardine’s theory if a certain non-classical logic is chosen instead as the formal component of absolute consequence; cf fn 31).

$S_1, S_2 \dots$  such that  $S_0, S_1, S_2 \dots, \langle P_0 \rangle, \langle P_1 \rangle, \langle P_2 \rangle \dots, \langle S_0 \rangle, \langle S_1 \rangle, \langle S_2 \rangle \dots$  absolutely entail  $\langle Q \rangle$ .

Thus, supposing that everyone who is sitting is talking, even though  $\langle \text{Socrates is sitting} \rangle$  does not absolutely entail  $\langle \text{Socrates is talking} \rangle$  (for it is possible that the former is true while the latter is false), it does entail it as-a-matter-of-fact, for  $\langle \text{Socrates is sitting} \rangle$  and  $\langle \text{Everyone who is sitting is talking} \rangle$  do absolutely entail  $\langle \text{Socrates is talking} \rangle$ , and, we're supposing,  $\langle \text{Everyone who is sitting is talking} \rangle$  is in fact the case. We won't pursue further the investigation of the notion of as-a-matter-of-fact consequence, save for noting that it is crucial to the workings of Bradwardine's theory that saying be closed not only under absolute consequence but also under as as-a-matter-of-fact consequence. We will understand (C) accordingly.

## 4 Truth

With so much by the way of background, the presentation of the core of Bradwardine's theory can be very brief. The core of the theory consists of two (non-stipulative, real) definitions of *truth* and *falsity* and goes as follows (cf Bradwardine [2010], 6.2; I'm slightly regimenting Bradwardine's definitions in ways that needn't concern us here):

(BT) For every utterance  $u$ ,  $u$  is *true* iff:

- (i) For some  $P$ ,  $u$  says that  $P$ ;
- (ii) For every  $P$ , if  $u$  says that  $P$ , then  $P$ ;

(BF) For every utterance  $u$ ,  $u$  is *false* iff, for some  $P$ ,  $u$  says that  $P$  and it is not the case that  $P$ ,

where, in conformity with what we have seen in section 3, the notion of saying is understood as being governed by (C).

Next, observe that, even though not explicitly endorsed by Bradwardine, the following principle *connecting utterancehood with saying* is very much enforced by his other theoretical commitments:

(US) For every utterance  $u$ , for some  $P$ ,  $u$  says that  $P$ .



The following semantic principles are then easily derivable from (BT), (BF) and (US):

**Theorem 1.** *The semantic principles of bivalence:*

(BIV) *For every utterance  $u$ , either  $u$  is true or  $u$  is false*

*and contravalance:*

(CONTRAV) *For every utterance  $u$ , it is not the case that [ $u$  is true and  $u$  is false]<sup>26</sup>*

*hold.*

*Proof.*

- (BIV): assume that  $u$  is an utterance. Then, by (US), for some  $P$ ,  $u$  says that  $P$ . Therefore, either, for some  $P$ ,  $u$  says that  $P$  and, for every  $P$ , if  $u$  says that  $P$ , then  $P$ , or, for some  $P$ ,  $u$  says that  $P$  and it is not the case that  $P$ —that is, by (BT) and (BF),  $u$  is either true or false.
- (CONTRAV): assume for *reductio* that  $u$  is an utterance which is both true and false. Then, by (BT) and (BF), for some  $P$ , both  $P$  and it is not the case that  $P$ . Contradiction. By *reductio*,  $u$  is not both true and false.

□

I should stress here that, while I think that putting things this way may be illuminating for the contemporary reader, I don't mean to suggest that in Bradwardine's own thinking there is a direction of argumentation from (US) (plus (BT) and (BF)) to (BIV) (or, for what's worth, to (CONTRAV)). Indeed, I'd rather be inclined to think that he regarded (BIV) as a self-evident truth (it is in effect set forth as an assumption in Bradwardine [2010], 6.3), and would have accepted (US) simply as a consequence of (BIV), (BT) and (BF): both truth and falsity require saying something, so that, if being an utterance requires being either true or false, then being an utterance requires saying something. Incidentally, it is the thought that truth requires saying something that motivates clause (i) in (BT) and belies the alternative, more

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<sup>26</sup>Throughout, I'll use square brackets to disambiguate constituent structure in English.

simple definition that would result by leaving out clause (i) (as for matters of exegesis, Bradwardine’s official definition says: “[...] a true utterance is a speech signifying things only as they are” (Bradwardine [2010], 6.2),<sup>27</sup> where I take it as implicitly understood that such an utterance does say something). For, at least on the usual contemporary understanding of universal quantification and implication, ‘For every  $P$ , if  $u$  says that  $P$ , then  $P$ ’ is vacuously true if, for no  $P$ ,  $u$  says that  $P$ . Hence, ‘For every  $P$ , if the Mont Blanc says that  $P$ , then  $P$ ’ is vacuously true since, for no  $P$ , the Mont Blanc says that  $P$ . However, it seems to make little sense to count the Mont Blanc as being true, and so the alternative, more simple definition of truth resulting from (BT) by leaving out clause (i) is exposed as being too weak.

## 5 Liar Utterances

As is well-known, an utterance like  $\mathfrak{L}_0$  poses a problem for many naive theories of truth. Let’s take for example the very simple theory based on the principle of *utterance-truth disquotation*:

(UTD) For every utterance  $u$  of the sentence ‘ $P$ ’,  $u$  is true iff  $P$ .

(UTD) has certainly flaws which are independent from considerations arising from the semantic paradoxes. Prominent among such considerations are those analogous to the ones we have seen (in section 3) to affect (USD). Take again an ordinary utterance  $u_0$  of the English sentence ‘I am British’ produced by Tony Blair. By universal instantiation plus the empirical fact that  $u_0$  is an utterance of ‘I am British’, the relevant instance of (UTD):

(UTD<sup>I am British</sup>) For every utterance  $u$  of the sentence ‘I am British’,  $u$  is true iff I am British

yields:

(UTD <sub>$u_0$</sub> <sup>I am British</sup>)  $u_0$  is true iff I am British.

Consider next my utterance  $u_1$  of (UTD <sub>$u_0$</sub> <sup>I am British</sup>).  $u_1$  is arguably false, since it says that  $u_0$  is true iff I (Elia Zardini, the author of this paper, certainly different from Tony Blair) am British, but, while  $u_0$  is presumably true (since

<sup>27</sup> “[...] *propositio vera est oratio significans tantum sicut est*”.

it talks about Blair rather than Elia Zardini, and so presumably only says of Blair, rather than of Elia Zardini, that he is British, and Blair is indeed British), I (Elia Zardini) am not British. Hence, some utterance (i.e.  $u_1$ ) of a sentence (i.e.  $(\text{UTD}_{u_0}^{\text{I am British}})$ ) which is a consequence of an instance of (UTD) (i.e.  $(\text{UTD}^{\text{I am British}})$ ) in conjunction with an uncontroversial empirical fact (i.e. the fact that  $u_0$  is an utterance of ‘I am British’) is false, and it is in this specific (and very natural) sense that (UTD)—just as (USD)—is incorrect in its full generality.<sup>28</sup>

What matters now is that, even setting aside such problems arising from context dependence, the semantic paradoxes seem to be sufficient for providing a conclusive refutation of the unrestricted validity of (UTD). This can be seen by reflecting that, by universal instantiation plus the empirical fact that  $\mathfrak{L}_0$  is an utterance of ‘ $\mathfrak{L}_0$  is false’, the relevant instance of (UTD):

$(\text{UTD}_{\mathfrak{L}_0}^{\mathfrak{L}_0 \text{ is false}})$  For every utterance  $u$  of the sentence ‘ $\mathfrak{L}_0$  is false’,  $u$  is true iff  $\mathfrak{L}_0$  is false

yields:

$(\text{UTD}_{\mathfrak{L}_0}^{\mathfrak{L}_0 \text{ is false}})$   $\mathfrak{L}_0$  is true iff  $\mathfrak{L}_0$  is false,

from which it follows that either [ $\mathfrak{L}_0$  is true and  $\mathfrak{L}_0$  is false] or [ $\mathfrak{L}_0$  is not true and  $\mathfrak{L}_0$  is not false], which contradict (CONTRAV) and (BIV) respectively. In order better to grasp the inner workings of Bradwardine’s solution to the semantic paradoxes, it will also be instructive to go through a natural version of the paradoxical reasoning which does not explicitly rely on (UTD). Assume for *reductio* that  $\mathfrak{L}_0$  is true. Then what it says must be the case, and what it says is that  $\mathfrak{L}_0$  is false. Thus, if  $\mathfrak{L}_0$  is true, then  $\mathfrak{L}_0$  is both true and false. Contradiction with (CONTRAV). By *reductio*,  $\mathfrak{L}_0$  is not true, and so, by (BIV),  $\mathfrak{L}_0$  is false (under no assumption). But then what it says is the case, since what it says is that  $\mathfrak{L}_0$  is false. Thus,  $\mathfrak{L}_0$  must be true (under no assumption). But we have previously shown that  $\mathfrak{L}_0$  is false (under no assumption), and so we are landed in a contradiction with (CONTRAV).

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<sup>28</sup>In Zardini [2008], pp. 545–561 I argue that problems from context dependence (even if not from that particular form of context dependence which consists in indexicality) are likely to beset a theory very similar to the core of Bradwardine’s theory. The extension of those criticisms to Bradwardine’s own theory is however not straightforward, since the criticisms rely on additional assumptions that, while highly plausible for utterances conceived of as acts of speech (as is the case in contemporary philosophy of language), are more problematic for utterances conceived of as products of speech (as is the case in Bradwardine, see section 2).

## 6 Applying Bradwardine’s Theory of Truth

Any theory of truth aspiring to completeness must offer a diagnosis and solution to the semantic paradoxes. This need was no doubt acutely perceived by Bradwardine: indeed, the tract *Insolubilia* containing the exposition of his theory really consists in a detailed treatment of the paradoxes (together with an interesting survey and criticisms of the other main approaches of the time).  $\mathfrak{L}_0$  and its like hardly represent more than the tip of the iceberg of the semantic paradoxes, and Bradwardine’s careful consideration and discussion of a wide range of paradoxes involving semantic and epistemic notions shows a clear awareness of this fact on his part. Still, in the rest of this paper, I propose to focus attention exclusively on his solution to the Liar paradox (which, in our utterance-theoretic framework, is represented by an utterance like  $\mathfrak{L}_0$ ): such solution contains the key idea that is also at play in the solutions offered by Bradwardine for many other paradoxes, and the worry about Bradwardine’s general treatment of the paradoxes that I want to raise here can be sharply and fruitfully specified with respect to his specific solution to the Liar paradox.

The key idea is to block the second leg of the paradoxical reasoning, the one going from ‘ $\mathfrak{L}_0$  is false’ to ‘ $\mathfrak{L}_0$  is true’. The possibility of blocking that part of the reasoning becomes especially salient under the aegis of (BT), since this requires that in order for  $\mathfrak{L}_0$  to be true *everything*  $\mathfrak{L}_0$  says must be the case. Now, the first leg of the argument may be taken to establish that  $\mathfrak{L}_0$  is false—indeed, given Bradwardine’s theory, the result would seem to be readily available:

**Theorem 2.** (BT), (CONTRAV), (BIV) and an acceptable instance of (USD) entail that  $\mathfrak{L}_0$  is false.

*Proof.* Assume for *reductio* that  $\mathfrak{L}_0$  is true. By universal instantiation plus the fact that  $\mathfrak{L}_0$  is an utterance of ‘ $\mathfrak{L}_0$  is false’, the relevant acceptable instance of (USD):

(USD <sup>$\mathfrak{L}_0$  is false</sup>) For every utterance  $u$  of the sentence ‘ $\mathfrak{L}_0$  is false’,  $u$  says that  $\mathfrak{L}_0$  is false

yields:

(USD <sub>$\mathfrak{L}_0$</sub>  <sup>$\mathfrak{L}_0$  is false</sup>)  $\mathfrak{L}_0$  says that  $\mathfrak{L}_0$  is false,

which, together with (BT) and the assumption that  $\mathfrak{L}_0$  is true, entails that  $\mathfrak{L}_0$  is false. Contradiction with (CONTRAV). Hence, by *reductio*,  $\mathfrak{L}_0$  is not true, and so, by (BIV),  $\mathfrak{L}_0$  is false.

□

Thus, given again (USD $_{\mathfrak{L}_0}^{\mathfrak{L}_0 \text{ is false}}$ ), we have that at least one of the things said by  $\mathfrak{L}_0$ —namely, that  $\mathfrak{L}_0$  is false—is the case. In the context of Bradwardine’s theory, this is however still compatible with the possibility that  $\mathfrak{L}_0$  is false, since it might be that *some other things* said by  $\mathfrak{L}_0$  are not the case. Given (C), we have that  $\mathfrak{L}_0$  not only says that  $\mathfrak{L}_0$  is false, but also says everything which absolutely or as-a-matter-of-fact follows from<sup>29</sup>  $\langle \mathfrak{L}_0 \text{ is false} \rangle$ . Yet, if we want to uphold the verdict of theorem 2 (as Bradwardine does), it’s clear that the proposition said by  $\mathfrak{L}_0$  which is not the case cannot be any proposition which absolutely or as-a-matter-of-fact follows from  $\langle \mathfrak{L}_0 \text{ is false} \rangle$ , for it is in the very nature of absolute and as-a-matter-of-fact consequence that everything which absolutely or as-a-matter-of-fact follows from something that is the case is also the case (see section 3). But what else could be said by  $\mathfrak{L}_0$  which is neither absolutely nor as-a-matter-of-fact entailed by  $\langle \mathfrak{L}_0 \text{ is false} \rangle$ , and which in addition fails to be the case?

It is probably the greatest achievement of Bradwardine’s logical acumen in this area to have made a compelling case that there is indeed a proposition said by  $\mathfrak{L}_0$  which is neither absolutely nor as-a-matter-of-fact entailed by  $\langle \mathfrak{L}_0 \text{ is false} \rangle$ , and which in addition fails to be the case. The case in question is the one adduced by Bradwardine as a proof of the following crucial theorem (cf Bradwardine [2010], 6.6.1, 6.6.2; my reconstruction of Bradwardine’s proof owes much to Read [2002], pp. 191–193):

**Theorem 3.**  $\mathfrak{L}_0$  says that  $\mathfrak{L}_0$  is true.

*Proof.* By (USD $_{\mathfrak{L}_0}^{\mathfrak{L}_0 \text{ is false}}$ ),  $\mathfrak{L}_0$  says that  $\mathfrak{L}_0$  is false, and, by (C), everything which is absolutely or as-a-matter-of-fact entailed by it. Take the weakest proposition  $\langle P \rangle$ , said by  $\mathfrak{L}_0$ , which as-a-matter-of-fact entails everything that  $\mathfrak{L}_0$  says that is not already as-a-matter-of-fact entailed by  $\langle \mathfrak{L}_0 \text{ is false} \rangle$ .<sup>30</sup> Then the weakest proposition, said by  $\mathfrak{L}_0$ , that as-a-matter-of-fact entails

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<sup>29</sup>Throughout, I’m using ‘following from’ and its like to denote the converse relation of the relation denoted by ‘entail’ and its like (where the *converse* relation of a relation  $R$  is the relation  $R'$  such that, for every  $x$  and  $y$ ,  $x R y$  iff  $y R' x$ ).

<sup>30</sup>The assumption, here and in the next step of the proof, that there is such a proposition is non-trivial, since it requires the space of propositions to have certain properties. Discussion of the matters arising in this connection lies outside the scope of this paper.

everything that  $\mathfrak{L}_0$  says is  $\langle P \text{ and } \mathfrak{L}_0 \text{ is false} \rangle$ . By (BF),  $\langle \mathfrak{L}_0 \text{ is false} \rangle$  as-a-matter-of-fact entails that it is not the case that  $[P \text{ and } \mathfrak{L}_0 \text{ is false}]$ , and this together with  $\langle P \rangle$  absolutely entails that it is not the case that  $\mathfrak{L}_0$  is false,<sup>31</sup> and so, by (BIV),<sup>32</sup> that  $\mathfrak{L}_0$  is true. But  $\mathfrak{L}_0$  says that  $\mathfrak{L}_0$  is false and says that  $P$ , and so, by (C),  $\mathfrak{L}_0$  says that  $\mathfrak{L}_0$  is true.

□

But now we can argue with Bradwardine as follows.  $\mathfrak{L}_0$  both says that  $\mathfrak{L}_0$  is false (by  $(\text{USD}_{\mathfrak{L}_0}^{\mathfrak{L}_0 \text{ is false}})$ ) and that  $\mathfrak{L}_0$  is true (by theorem 3). By (CONTRAV), however, it is not the case that  $\mathfrak{L}_0$  is both true and false, and so either it is not the case that  $\mathfrak{L}_0$  is true or it is not the case that  $\mathfrak{L}_0$  is false. Either way, something  $\mathfrak{L}_0$  says is not going to be the case (namely, either  $\langle \mathfrak{L}_0 \text{ is true} \rangle$  or  $\langle \mathfrak{L}_0 \text{ is false} \rangle$ ). Therefore, by (BF),  $\mathfrak{L}_0$  is false. It is false, even if, on the face of it, it seems to say only what is the case (namely,  $\langle \mathfrak{L}_0 \text{ is false} \rangle$  and whatever absolutely or as-a-matter-of-fact follows from it), for it also says something more, as revealed in theorem 3.

An interesting awkwardness seems however to remain. For Bradwardine says that  $\mathfrak{L}_0$  is false, and this attitude of his can be described as an endorsement of the proposition  $\langle \mathfrak{L}_0 \text{ is false} \rangle$ , or of the sentence ‘ $\mathfrak{L}_0$  is false’. Indeed, the endorsement of such a proposition or sentence is plausibly performed by uttering, in thought or talk, an utterance that expresses such a proposition or tokens such a sentence. But expressing  $\langle \mathfrak{L}_0 \text{ is false} \rangle$  or tokening ‘ $\mathfrak{L}_0$  is false’ is precisely what  $\mathfrak{L}_0$  also does. Yet, Bradwardine wishes to maintain that his own utterance (let’s call it ‘ $\mathfrak{T}_0$ ’) is true while  $\mathfrak{L}_0$  is false. Both  $\mathfrak{T}_0$  and  $\mathfrak{L}_0$  token the same sentence, namely ‘ $\mathfrak{L}_0$  is false’. This is itself not particularly problematic, since it is simply not the case that utterances of the same sentence must have the same truth value, as should be clear from our discussion of  $(\text{UTD}^{\text{I am British}})$ . Still, the straightforward reason for why

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<sup>31</sup>In its use of *modus ponendo tollens* (‘It is not the case that  $[P_0 \text{ and } P_1]$ ;  $P_0$ . Therefore, it is not the case that  $P_1$ ’), this proof is relevantly suspicious—that is, suspicious from the perspective of a prominent family of logics (so-called ‘*relevant logics*’) that require a genuine connection between premises and conclusion of a valid argument (see Anderson and Belnap [1975] for a classical presentation). This can be seen by reflecting that, in any logic that (very plausibly) accepts ‘It is not the case that  $[[P \text{ and it is not the case that } P] \text{ and } Q]$ ’, the premise ‘ $P$  and it is not the case that  $P$ ’ would entail, by *modus ponendo tollens*, the conclusion ‘It is not the case that  $Q$ ’, where no genuine connection seems to exist between premise and conclusion. In Zardini [2011a], I study some semantic conditions under which a proof of theorem 3 can be expected to be relevantly valid.

<sup>32</sup>Throughout, I’ll assume that such principles as (BIV), (CONTRAV), (BT), (BF) etc. bear the hallmark of necessary truth relevant to absolute consequence, and so that, if, together with  $\langle P_0 \rangle, \langle P_1 \rangle, \langle P_2 \rangle \dots$  they absolutely entail  $\langle Q \rangle$ , then  $\langle P_0 \rangle, \langle P_1 \rangle, \langle P_2 \rangle \dots$  already absolutely entail  $\langle Q \rangle$ .

this is not the case is that two utterances of the same sentence may express different propositions (as we envisaged in our discussion of (UTD<sup>I am British</sup>)). However,  $\mathfrak{T}_0$  and  $\mathfrak{L}_0$  not only token the same sentence, but, in so doing, also express the same proposition, namely  $\langle \mathfrak{L}_0 \text{ is false} \rangle$ . How can it then be that one is true while the other is false?<sup>33</sup>

The key lies in seeing that, even though the proposition that the two utterances “directly” express by dint of tokening the particular sentence they token is the same, the particular identities of the utterances conspire to determine that  $\mathfrak{L}_0$  says also some other proposition that is not said by  $\mathfrak{T}_0$ . In our presentation (see theorem 3), this other proposition is  $\langle P \rangle$ , and it is because  $\mathfrak{L}_0$ , but not  $\mathfrak{T}_0$ , says that  $P$  that  $\mathfrak{L}_0$ , but not  $\mathfrak{T}_0$ , says that  $\mathfrak{L}_0$  is true (given (C)). From such a perspective,  $\mathfrak{L}_0$  can be truthfully declared to be false by  $\mathfrak{T}_0$ : the similarities between them in terms of what sentence they token and of what proposition they “directly” express by tokening such a sentence are more than compensated by the substantial differences in what propositions they ultimately say.

## 7 Liar Propositions and Liar Sentences

By now, we’ve been using the truth-like predicate for propositions ‘ $\tau$  is the case’ rather extensively (the predicate is “truth-like” in the sense that it denotes a property<sup>34</sup> of *objective representational correctness*). We might just as well have followed many other philosophers and used once again the predicate ‘ $\tau$  is true’ to apply also to propositions. I’ve decided to use a different predicate since in this context it’s clearly important to keep distinct the two notions of utterance truth on one side and of propositional truth on the other side, however closely they may be related (see section 2). Let’s from now on use ‘ $\tau$  is true<sup>P</sup>’ as our truth-like predicate for propositions and ‘ $\tau$  is true<sup>S</sup>’ as our truth-like predicate for sentences (a property of objective representational correctness does seem to be defined over sentences as well—

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<sup>33</sup>In Zardini [2008], pp. 545–561 I discuss a quite common contemporary combination of views under which even this circumstance, rather than being surprising, should generally be expected to be realised. For reasons already alluded to in fn 28, such a combination of views sits rather uncomfortably with Bradwardine’s conception of utterances as products of speech. In any event, the case in question badly fits the conditions under which the aforementioned combination of views predicts the divergence in truth value of two utterances expressing the same proposition.

<sup>34</sup>Here and in the following, I’m availing myself to ‘property’-talk merely for convenience: this is not essential to the point being made, but it greatly facilitates its verbal presentation.

at least over those that are not context dependent). We also adopt analogous stipulations regarding falsity.

Now, note that we've been assuming that a truth-like predicate such as ' $\tau$  is true<sup>P</sup>' obeys what are in effect rather suspicious principles, treating ' $\langle\varphi\rangle$  is true<sup>P</sup>' (or any other sentence ' $\tau$  is true<sup>P</sup>' with  $\tau$  denoting  $\langle\varphi\rangle$ ) as *intersubstitutable* with  $\varphi$ . Such principles require the existence of a truth-like property (the one to be assigned to ' $\tau$  is true<sup>P</sup>') which, under a certain assumption, a reasoning familiar from the semantic paradoxes shows to be of dubious coherence. The assumption is that there is some self-referential device ensuring the existence of a proposition—expressed by, say, ' $L_0$ '—which is such that [ $L_0$  iff  $\langle L_0 \rangle$  is not true<sup>P</sup>] (in an intuitive sense,  $\langle L_0 \rangle$  is a proposition that “says” of—or at least “implies” about—itsself that it is not true<sup>P</sup>). Given ' $L_0$  iff  $\langle L_0 \rangle$  is not true<sup>P</sup>', intersubstitutability of ' $\langle L_0 \rangle$  is true<sup>P</sup>' with ' $L_0$ ' yields ' $\langle L_0 \rangle$  is true<sup>P</sup> iff  $\langle L_0 \rangle$  is not true<sup>P</sup>'—a classical contradiction.<sup>35</sup>

Clearly, a fully analogous paradox can be run by talking about sentences rather than propositions (and by using the ordinary sentence-referring device ' $\dots$ ' rather the proposition-referring device ' $\langle \dots \rangle$ '). Again, we assume that there is some self-referential device ensuring the existence of a sentence—say, ' $L'_0$ '—which is such that [ $L'_0$  iff ' $L'_0$ ' is not true<sup>S</sup>] (in an intuitive sense, ' $L'_0$ ' is a sentence that “says” of—or at least “implies” about—itsself that it is not true<sup>S</sup>). Given ' $L'_0$  iff ' $L'_0$ ' is not true<sup>S</sup>', intersubstitutability of ' $L'_0$ ' is true<sup>S</sup>' with ' $L'_0$ ' yields ' $L'_0$ ' is true<sup>S</sup> iff ' $L'_0$ ' is not true<sup>S</sup>'—a classical contradiction.

Outside of concrete mathematical contexts, where the self-reference exploited in these paradoxes is well-understood under the heading of diagonalisation theorems, such self-reference may look in its abstractness very artificial and of dubious intelligibility (What does ' $L_0$ ' mean? How could we establish that [ $L_0$  iff  $\langle L_0 \rangle$  is not true<sup>P</sup>]?). In the absence of a more concrete and plausible example, this is certainly a justified worry, and explains the popularity of an alternative way of setting up semantic paradoxes for propositional and sentential truth. We now assume the availability of a stronger self-referential device ensuring the existence of a proposition,  $\mathcal{L}_1$ , which is identical to the

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<sup>35</sup>I have stated and will state these paradoxes using first-order talk of propositions and talk of the property (of some propositions) of being true<sup>P</sup> (for a similar presentation, see Horwich [1998], pp. 40–42). I should therefore mention that essentially the same paradoxes can be set up in a (possibly) non-committing higher-order framework (see Zardini [2008], pp. 563–566, which also exploits a more natural device of self-reference that I'll employ later in this paper as well). I'll give an example of this at the end of section 8. Moreover, since the higher-order framework of Zardini [2008] is compatible with a substitutional interpretation of the higher-order quantifiers, an analogous point applies in relation to the use (forthcoming in the text) of first-order talk of sentences and of talk of the property (of some sentences) of being true<sup>S</sup>.



proposition  $\langle \mathcal{L}_1 \text{ is not true}^P \rangle$  (again, in an even more intuitive sense,  $\mathcal{L}_1$  is a proposition that “says” of itself that it is not true<sup>P</sup>). Given the instance of reflexivity ‘ $\mathcal{L}_1$  is not true<sup>P</sup> iff  $\mathcal{L}_1$  is not true<sup>P</sup>’, intersubstitutability of ‘ $\mathcal{L}_1$  is true<sup>P</sup>’ with ‘ $\mathcal{L}_1$  is not true<sup>P</sup>’ yields ‘ $\mathcal{L}_1$  is true<sup>P</sup> iff  $\mathcal{L}_1$  is not true<sup>P</sup>’—a classical contradiction.

Again, clearly, a fully analogous paradox can be run by talking about sentences rather than propositions. Again, we assume that there is some self-referential device ensuring the existence of a sentence,  $\lambda_1$ , which is identical to the sentence ‘ $\lambda_1$  is not true<sup>S</sup>’ (again, in an even more intuitive sense,  $\lambda_1$  is a sentence that “says” of itself that it is not true<sup>S</sup>). This can happen e.g. if, taking ‘ $\lambda_1$ ’ to be a hitherto uninterpreted sign, we stipulate that it is to refer to the sentence ‘ $\lambda_1$  is not true<sup>S</sup>’ (see Kripke [1975], p. 693, whose idea here seems to have been anticipated by Bradwardine [2010], 3.1.6: “[...] a can signify this: a is not true, since a is an expression that signifies by convention, and hence can be stipulated to signify anything”;<sup>36</sup> see also the other passage from the same paragraph quoted in section 2). Given the instance of reflexivity ‘ $\lambda_1$  is not true<sup>S</sup> iff  $\lambda_1$  is not true<sup>S</sup>’, intersubstitutability of ‘ $\lambda_1$  is true<sup>S</sup>’ with ‘ $\lambda_1$  is not true<sup>S</sup>’ yields ‘ $\lambda_1$  is true<sup>S</sup> iff  $\lambda_1$  is not true<sup>S</sup>’—a classical contradiction.

To clinch matters about the intelligibility of the self-referential mechanism needed to generate the paradoxes at hand, let’s consider the proposition  $\langle \text{The proposition mentioned in the first sentence of the sixth paragraph of section 7 of such-and-such paper is not true}^P \rangle$  (where ‘such-and-such’ stands in for a definite description that in the actual world picks out this paper). Given the instance of reflexivity ‘The proposition mentioned in the first sentence of the sixth paragraph of section 7 of such-and-such paper is not true<sup>P</sup> iff the proposition mentioned in the first sentence of the sixth paragraph of section 7 of such-and-such paper is not true<sup>P</sup>’, intersubstitutability of ‘ $\langle \text{The proposition mentioned in the first sentence of the sixth paragraph of section 7 of such-and-such paper is not true}^P \rangle$  is true<sup>P</sup>’ with ‘The proposition mentioned in the first sentence of the sixth paragraph of section 7 of such-and-such paper is not true<sup>P</sup>’ yields ‘ $\langle \text{The proposition mentioned in the first sentence of the sixth paragraph of section 7 of such-and-such paper is not true}^P \rangle$  is true<sup>P</sup> iff the proposition mentioned in the first sentence of the sixth paragraph of section 7 of such-and-such paper is not true<sup>P</sup>’, which, together with the identity ‘ $\langle \text{The proposition mentioned in the first sentence of the sixth paragraph of section 7 of such-and-such paper is not true}^P \rangle$  is the proposition mentioned in the first sentence of the sixth paragraph of section 7 of such-and-such paper’

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<sup>36</sup> “[...] *a potest significare istam: a non est verum, quia a est vox significativa ad placitum, ergo potest imponi ad significandum quodcumque*”.

(guaranteed by empirical facts), yields ‘The proposition mentioned in the first sentence of the sixth paragraph of section 7 of such-and-such paper is true<sup>P</sup> iff the proposition mentioned in the first sentence of the sixth paragraph of section 7 of such-and-such paper is not true<sup>P</sup>’—a classical contradiction.

Again, clearly, a fully analogous paradox can be run by talking about sentences rather than propositions. Let’s consider the sentence ‘The sentence mentioned in the second sentence of the seventh paragraph of section 7 of such-and-such paper is not true<sup>S</sup>’ (with ‘such-and-such’ as in the previous paragraph). Given the instance of reflexivity ‘The sentence mentioned in the second sentence of the seventh paragraph of section 7 of such-and-such paper is not true<sup>S</sup> iff the sentence mentioned in the second sentence of the seventh paragraph of section 7 of such-and-such paper is not true<sup>S</sup>’, intersubstitutability of ‘‘The sentence mentioned in the second sentence of the seventh paragraph of section 7 of such-and-such paper is not true<sup>S</sup>’ is true<sup>S</sup>’ with ‘The sentence mentioned in the second sentence of the seventh paragraph of section 7 of such-and-such paper is not true<sup>S</sup>’ yields ‘‘The sentence mentioned in the second sentence of the seventh paragraph of section 7 of such-and-such paper is not true<sup>S</sup>’ is true<sup>S</sup> iff the sentence mentioned in the second sentence of the seventh paragraph of section 7 of such-and-such paper is not true<sup>S</sup>’, which, together with the identity ‘‘The sentence mentioned in the second sentence of the seventh paragraph of section 7 of such-and-such paper is not true<sup>S</sup>’ is the sentence mentioned in the second sentence of the seventh paragraph of section 7 of such-and-such paper’ (guaranteed by empirical facts), yields ‘The sentence mentioned in the second sentence of the seventh paragraph of section 7 of such-and-such paper is true<sup>S</sup> iff the sentence mentioned in the second sentence of the seventh paragraph of section 7 of such-and-such paper is not true<sup>S</sup>’—a classical contradiction.

Finally, to end this all too brief illustration of the varieties of Liar paradoxes for propositional and sentential truth, it’s interesting to observe that Bradwardine’s own theoretical machinery makes it possible to construct some such paradoxes which achieve propositional or sentential self-reference *via* utterance self-reference. To see this, reflect first that, even though, given (C), the notion of *the* proposition expressed by an utterance is not well-defined, there should be no bar to introducing in the following way a somehow stricter notion that does require uniqueness (assuming very plausibly that, for every utterance  $u$ , there is one and only one sentence  $u$  is an utterance of):

**Definition 1.** For every utterance  $u$ ,  $\langle P \rangle$  is the *proposition disquotationally associated with  $u$*  iff  $u$  is an utterance of ‘ $P$ ’.<sup>37</sup>

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<sup>37</sup>The claim that the notion of being disquotationally associated with an utterance  $u$

Consider then an utterance  $\mathfrak{L}_3$  of the sentence ‘The proposition disquotationally associated with  $\mathfrak{L}_3$  is not true<sup>P</sup>’. Given the instance of reflexivity ‘The proposition disquotationally associated with  $\mathfrak{L}_3$  is not true<sup>P</sup> iff the proposition disquotationally associated with  $\mathfrak{L}_3$  is not true<sup>P</sup>’, intersubstitutability of ‘⟨The proposition disquotationally associated with  $\mathfrak{L}_3$  is not true<sup>P</sup>⟩ is true<sup>P</sup>’ with ‘The proposition disquotationally associated with  $\mathfrak{L}_3$  is not true<sup>P</sup>’ yields ‘⟨The proposition disquotationally associated with  $\mathfrak{L}_3$  is not true<sup>P</sup>⟩ is true<sup>P</sup> iff the proposition disquotationally associated with  $\mathfrak{L}_3$  is not true<sup>P</sup>’, which, together with the identity ‘⟨The proposition disquotationally associated with  $\mathfrak{L}_3$  is not true<sup>P</sup>⟩ is the proposition disquotationally associated with  $\mathfrak{L}_3$ ’ (guaranteed by definition 1 and empirical facts), yields ‘The proposition disquotationally associated with  $\mathfrak{L}_3$  is true<sup>P</sup> iff the proposition disquotationally associated with  $\mathfrak{L}_3$  is not true<sup>P</sup>’—a classical contradiction.

Again, clearly, a fully analogous paradox can be run by talking about sentences rather than propositions, using the analogue of the notion of the proposition disquotationally associated with an utterance—namely, the notion of *the sentence an utterance is an utterance of* (assuming very plausibly, again, that, for every utterance  $u$ , there is one and only one sentence  $u$  is an utterance of). Let’s consider then an utterance  $\mathfrak{L}'_3$  of the sentence ‘The sentence  $\mathfrak{L}'_3$  is an utterance of is not true<sup>S</sup>’. Given the instance of reflexivity ‘The sentence  $\mathfrak{L}'_3$  is an utterance of is not true<sup>S</sup> iff the sentence  $\mathfrak{L}'_3$  is an utterance of is not true<sup>S</sup>’, intersubstitutability of ‘‘The sentence  $\mathfrak{L}'_3$  is an utterance of is not true<sup>S</sup>’ is true<sup>S</sup>’ with ‘The sentence  $\mathfrak{L}'_3$  is an utterance of is not true<sup>S</sup>’ yields ‘‘The sentence  $\mathfrak{L}'_3$  is an utterance of is not true<sup>S</sup>’ is true<sup>S</sup> iff the sentence  $\mathfrak{L}'_3$  is an utterance of is not true<sup>S</sup>’, which, together with the identity ‘‘The sentence  $\mathfrak{L}'_3$  is an utterance of is not true<sup>S</sup>’ is the sentence  $\mathfrak{L}'_3$  is an utterance of’ (guaranteed by empirical facts), yields ‘The sentence  $\mathfrak{L}'_3$  is an utterance of is true<sup>S</sup> iff the sentence  $\mathfrak{L}'_3$  is an utterance of is not true<sup>S</sup>’—a classical contradiction.

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is stricter than the notion of being said by  $u$  is subject to an important qualification. For notice that, given what we’ve seen in section 3, we cannot always assume that the proposition disquotationally associated with an utterance  $u$  is in effect said by  $u$ : for example, given definition 1, the proposition disquotationally associated with an ordinary utterance  $u$  of the English sentence ‘I am British’ produced by Tony Blair is ⟨I am British⟩ (which is true<sup>P</sup> iff Elia Zardini is British), but  $u$  does not say that I am British (rather, it says that Blair is British).

## 8 Extending Bradwardine's Theory of Truth?

Could we apply to these cases an analogue of Bradwardine's solution to the Liar paradox for utterance truth? Without loss of generality, let's see how things would pan out in the case of  $\lambda_1$ . To do so, we have in effect to extend Bradwardine's theory, originally conceived as a theory of utterance truth, to a theory of truth<sup>S</sup>. Given that the pivotal notion of the theory of utterance truth is the notion of utterance saying, this in turn requires that there be available an analogous notion of sentence saying (let's use 'say<sup>S</sup>' and its like to express such a notion). We seem to have just as good a grasp of the notion of saying<sup>S</sup> as we have of the notion of saying that is needed in the theory of utterance truth: what a sentence says<sup>S</sup> is *whatever information it is that the sentence is determined to carry by the semantic rules of the language*. Again, understood in this way, the envisaged extension of Bradwardine's theory could be stated and developed by sticking to higher-order phrases when talking about the things said<sup>S</sup> by sentences, but it will ease our discussion considerably if we allow ourselves to the typical contemporary first-order talk of certain objects being said<sup>S</sup> by sentences. I will call such objects '*contents*'. Such talk will always have to be understood as being (possibly) non-committing and as being at least in principle eliminable in favour of the corresponding (clumsier) higher-order talk (in fact, I'll still use higher-order talk in some cases where it actually allows for snappier formulations). Consequently, I'll also understand saying<sup>S</sup> as a first-order relation between sentences and propositions.

Moreover, there is every reason to think that the contents said<sup>S</sup> by sentences are the same objects as the propositions said by utterances. Indeed, not only is such coincidence extremely plausible even if one introduces contents independently from propositions (as we've done), but it is actually forced if one introduces contents in a more reductive fashion, for example by defining what contents are said<sup>S</sup> by a sentence in the following way:

**Definition 2.** A sentence  $\varphi$  says<sup>S</sup> a content  $c$  iff  $c$  is a proposition that would be said by every utterance of  $\varphi$ .

In view of this, I think it's safe to identify contents with propositions and will do so in the following (but I will not assume definition 2).

Of course, given again what we have seen in section 3, we cannot always assume that a sentence says<sup>S</sup> something: for example, for all that has been said so far, the sentence 'I am British' may well say<sup>S</sup> nothing—indeed, there

does not seem to be any intuitive information that the sentence is determined to carry by the semantic rules of the language. Surely, even for indexical sentences, one could try to come up with some concocted propositions that are specific enough as to capture at least part of the meaning of the sentence: for example, the proposition ⟨Someone is uttering ‘I am British’ and that person is British⟩ seems to be a specific enough necessary consequence of the truth of any utterance of ‘I am British’. Yet, of course, it does not necessitate the truth of any such utterance: someone other than the utterer may be uttering ‘I am British’ and be British, while the utterer is not.

A more incisive point in this direction is constituted by the remark that, given Bradwardine’s theory of utterance truth, definition 2 would indeed entail that even ‘I am British’ says<sup>S</sup> something: this is so because, by (C), every utterance of it says at least every logical truth, and so, by definition 2, ‘I am British’ itself says<sup>S</sup> every logical truth. This second attempt, based on definition 2, at specifying what ‘I am British’ says<sup>S</sup> is however not without its problems. For it would have the unwelcome consequence of making it (non-vacuously) true that every proposition said<sup>S</sup> by ‘I am British’ is also said<sup>S</sup> by ‘Someone is British’ and *vice versa* (since the strongest proposition said<sup>S</sup> by every utterance of ‘I am British’ is arguably ⟨Someone is British⟩). Indeed, this second attempt would have the even more unwelcome consequence that what is said<sup>S</sup> by ‘I am British’ is not inconsistent with what is said<sup>S</sup> by its syntactic negation ‘I am not British’ (since the strongest proposition said<sup>S</sup> by every utterance of ‘I am British’ is arguably ⟨Someone is British⟩ while the strongest proposition said<sup>S</sup> by every utterance of ‘I am not British’ is arguably ⟨Someone is not British⟩, and these two propositions are obviously consistent with one another). As I suspect that attempts of both of these kinds are misguided, I wish to leave open the possibility that some sentences (such as, paradigmatically, indexical ones) do not say<sup>S</sup> anything.

Having specified what saying<sup>S</sup> is, we also need to know something about which specific propositions are said<sup>S</sup> by which specific sentences. As we have already been doing in the case of what is said by an utterance, we will be relying on intuitive judgements about what is said<sup>S</sup> by a sentence, in particular helping ourselves to acceptable instances of the principle of *sentence saying<sup>S</sup> disquotation*:

(SSD) ‘*P*’ says<sup>S</sup> that *P*

(which, in its full generality, is incorrect, unless ‘I am British’ says<sup>S</sup> that I, Elia Zardini, am British—a matter we don’t need to settle here). It will also prove helpful to define the following notion:

**Definition 3.** A sentence is *contentful* iff, for some  $P$ , it says<sup>S</sup> that  $P$ .

Having dealt with these preliminaries, we can impose a principle of *closure of saying<sup>S</sup> under consequence* analogous to (C):

(C<sup>S</sup>) If  $\varphi$  says<sup>S</sup> that  $P_0$ , that  $P_1$ , that  $P_2 \dots$  and  $\langle P_0 \rangle, \langle P_1 \rangle, \langle P_2 \rangle \dots$  entail  $\langle Q \rangle$ , then  $\varphi$  says<sup>S</sup> that  $Q$ .

Bradwardine's theory can then be extended to truth<sup>S</sup> in the following natural way:

(BT<sup>S</sup>) For every sentence  $\varphi$ ,  $\varphi$  is true<sup>S</sup> iff:

- (i) For some  $P$ ,  $\varphi$  says<sup>S</sup> that  $P$ ;
- (ii) For every  $P$ , if  $\varphi$  says<sup>S</sup> that  $P$ , then  $P$ ;

(BF<sup>S</sup>) For every sentence  $\varphi$ ,  $\varphi$  is false<sup>S</sup> iff, for some  $P$ ,  $\varphi$  says<sup>S</sup> that  $P$  and it is not the case that  $P$ ,

where the notion of saying<sup>S</sup> is understood as being governed by (C<sup>S</sup>).

Next, observe that the following principle *connecting contentful sentencehood with saying<sup>S</sup>* holds in virtue of definition 3:

(CSS) For every contentful sentence  $\varphi$ , for some  $P$ ,  $\varphi$  says<sup>S</sup> that  $P$ .

The following semantic principles are then easily derivable from (BT<sup>S</sup>), (BF<sup>S</sup>) and (CSS):

**Theorem 4.** *The semantic principles of bivalence<sup>S</sup>:*

(BIV<sup>S</sup>) *For every contentful sentence  $\varphi$ , either  $\varphi$  is true<sup>S</sup> or  $\varphi$  is false<sup>S</sup>*

*and contravalence<sup>S</sup>:*

(CONTRAV<sup>S</sup>) *For every sentence  $\varphi$ , it is not the case that [ $\varphi$  is true<sup>S</sup> and  $\varphi$  is false<sup>S</sup>]*

*hold.*

*Proof.*

- (BIV<sup>S</sup>): assume that  $\varphi$  is a contentful sentence. Then, by (CSS), for some  $P$ ,  $\varphi$  says<sup>S</sup> that  $P$ . Therefore, either, for some  $P$ ,  $\varphi$  says<sup>S</sup> that  $P$  and, for every  $P$ , if  $\varphi$  says<sup>S</sup> that  $P$ , then  $P$ , or, for some  $P$ ,  $\varphi$  says<sup>S</sup> that  $P$  and it is not the case that  $P$ —that is, by (BT<sup>S</sup>) and (BF<sup>S</sup>),  $\varphi$  is either true<sup>S</sup> or false<sup>S</sup>.
- (CONTRAV<sup>S</sup>): assume for *reductio* that  $\varphi$  is a sentence which is both true<sup>S</sup> and false<sup>S</sup>. Then, by (BT<sup>S</sup>) and (BF<sup>S</sup>), for some  $P$ , both  $P$  and it is not the case that  $P$ . Contradiction. By *reductio*,  $\varphi$  is not both true<sup>S</sup> and false<sup>S</sup>.

□

Now, given the extension of Bradwardine’s theory, the following result would seem to be readily available:

**Theorem 5.** (BT<sup>S</sup>), (BIV<sup>S</sup>), (CONTRAV<sup>S</sup>) and an acceptable instance of (SSD) entail that  $\lambda_1$  is false<sup>S</sup>.

*Proof.* Assume for *reductio* that  $\lambda_1$  is true<sup>S</sup>. Given the fact that  $\lambda_1$  is identical with ‘ $\lambda_1$  is false<sup>S</sup>’, the relevant acceptable instance of (SSD):

(SSD <sup>$\lambda_1$  is false<sup>S</sup></sup>) ‘ $\lambda_1$  is false<sup>S</sup>’ says<sup>S</sup> that  $\lambda_1$  is false<sup>S</sup>

yields:

(SSD <sup>$\lambda_1$  is false<sup>S</sup></sup> <sub>$\lambda_1/‘\lambda_1$  is false<sup>S}’</sup></sub>)  $\lambda_1$  says<sup>S</sup> that  $\lambda_1$  is false<sup>S</sup>,

which, together with (BT<sup>S</sup>) and the assumption that  $\lambda_1$  is true<sup>S</sup>, entails that  $\lambda_1$  is false<sup>S</sup>. Contradiction with (CONTRAV<sup>S</sup>). Hence, by *reductio*,  $\lambda_1$  is not true<sup>S</sup>. But, by (SSD <sup>$\lambda_1$  is false<sup>S</sup></sup> <sub>$\lambda_1/‘\lambda_1$  is false<sup>S}’</sup></sub>),  $\lambda_1$  is contentful, and so by (BIV<sup>S</sup>),  $\lambda_1$  is false<sup>S</sup>.

□

At this point, we could, if we wish, continue as we did in the case of  $\mathfrak{L}_0$ , arguing as follows:

Thus, given again ( $\text{SSD}_{\lambda_1/\lambda_1 \text{ is false}^S}^{\lambda_1 \text{ is false}^S}$ ), we have that at least one of the things said<sup>S</sup> by  $\lambda_1$ —namely, that  $\lambda_1$  is false<sup>S</sup>—is the case. In the context of Bradwardine’s theory, this is however still compatible with the possibility that  $\lambda_1$  is false<sup>S</sup>, since it might be that *some other things* said<sup>S</sup> by  $\lambda_1$  are not the case.

We could also make a compelling case that there is indeed a proposition said<sup>S</sup> by  $\lambda_1$  which is neither absolutely nor as-a-matter-of-fact entailed by  $\langle \lambda_1 \text{ is false}^S \rangle$ , and which in addition fails to be the case, putting forth the following theorem:

**Theorem 6.**  $\lambda_1$  says<sup>S</sup> that  $\lambda_1$  is true<sup>S</sup>.

*Proof.* By ( $\text{SSD}_{\lambda_1/\lambda_1 \text{ is false}^S}^{\lambda_1 \text{ is false}^S}$ ),  $\lambda_1$  says<sup>S</sup> that  $\lambda_1$  is false<sup>S</sup>, and, by (C<sup>S</sup>), everything which is absolutely or as-a-matter-of-fact entailed by it. Take the weakest proposition  $\langle P \rangle$ , said<sup>S</sup> by  $\lambda_1$ , which as-a-matter-of-fact entails everything that  $\lambda_1$  says<sup>S</sup> that is not already as-a-matter-of-fact entailed by  $\langle \lambda_1 \text{ is false}^S \rangle$ . Then the weakest proposition, said<sup>S</sup> by  $\lambda_1$ , that as-a-matter-of-fact entails everything that  $\lambda_1$  says<sup>S</sup> is  $\langle P \text{ and } \lambda_1 \text{ is false}^S \rangle$ . By (BF<sup>S</sup>),  $\langle \lambda_1 \text{ is false}^S \rangle$  as-a-matter-of-fact entails that it is not the case that  $[P \text{ and } \lambda_1 \text{ is false}^S]$ , and this together with  $\langle P \rangle$  absolutely entails that it is not the case that  $\lambda_1$  is false<sup>S</sup>, and so, by (BIV<sup>S</sup>), that  $\lambda_1$  is true<sup>S</sup>. But  $\lambda_1$  says<sup>S</sup> that  $\lambda_1$  is false<sup>S</sup> and says<sup>S</sup> that  $P$ , and so, by (C<sup>S</sup>),  $\lambda_1$  says<sup>S</sup> that  $\lambda_1$  is true<sup>S</sup>.

□

We could even argue with Bradwardine as follows.  $\lambda_1$  both says<sup>S</sup> that  $\lambda_1$  is false<sup>S</sup> (by ( $\text{SSD}_{\lambda_1/\lambda_1 \text{ is false}^S}^{\lambda_1 \text{ is false}^S}$ )) and that  $\lambda_1$  is true<sup>S</sup> (by theorem 6). By (CONTRAV<sup>S</sup>), however, it is not the case that  $\lambda_1$  is both true<sup>S</sup> and false<sup>S</sup>, and so either it is not the case that  $\lambda_1$  is true<sup>S</sup> or it is not the case that  $\lambda_1$  is false<sup>S</sup>. Either way, something  $\lambda_1$  says<sup>S</sup> is not going to be the case (namely, either  $\langle \lambda_1 \text{ is true}^S \rangle$  or  $\langle \lambda_1 \text{ is false}^S \rangle$ ). Therefore, by (BF<sup>S</sup>),  $\lambda_1$  is false<sup>S</sup>. It is false<sup>S</sup>, even if, on the face of it, it seems to say<sup>S</sup> only what is the case (namely,  $\langle \lambda_1 \text{ is false}^S \rangle$  and whatever absolutely or as-a-matter-of-fact follows from it), for it also says<sup>S</sup> something more, as revealed in theorem 6.

We could do all this, but, contrary to the case of  $\mathfrak{L}_0$ , now we would just be spinning deeper and deeper into paradox. For, at various crucial points in the exposition of our view, we would be insisting in asserting and accepting, in



thought or talk,<sup>38</sup> the sentence ‘ $\lambda_1$  is false<sup>S</sup>’. Unfortunately, ‘ $\lambda_1$  is false<sup>S</sup>’ *just is*  $\lambda_1$  itself, so that what we would in effect be doing would be to assert and accept a sentence while at the same time (indeed, with those very same acts!) declaring that such a sentence is false<sup>S</sup>. While such a position would not be strictly speaking logically inconsistent, I think it goes without saying that to declare false<sup>S</sup> some sentences that one nevertheless asserts and accepts does a great violence to our notion of falsity<sup>S</sup>.<sup>39</sup>

Indeed, given the extremely plausible principle *connecting falsity<sup>S</sup> with truth<sup>S</sup> of the negation*:

(FN<sup>S</sup>) For every sentence  $\varphi$ ,  $\varphi$  is false<sup>S</sup> iff ‘It is not the case that  $\varphi$ ’ is true<sup>S</sup>,<sup>40</sup>

one would have to declare true<sup>S</sup> a sentence, ‘It is not the case that  $\lambda_1$  is false<sup>S</sup>’, which, on pain of contradiction with ‘ $\lambda_1$  is false<sup>S</sup>’, one has to deny and reject—indeed, one would have to declare true<sup>S</sup> a sentence while asserting and accepting what it negates. Again, while such a position would not be strictly speaking logically inconsistent, I think it also goes without saying that to declare true<sup>S</sup> some sentences that one nevertheless denies and rejects

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<sup>38</sup>I understand the distinction between *asserting* and *accepting* a sentence or an utterance, in thought or talk, as follows. Asserting a sentence or an utterance in thought (talk) is the linguistic correlate of an *occurring mental event* of inner (overt) *judgement*; accepting a sentence or an utterance, in thought or talk, is the linguistic correlate of a *standing mental state* of *belief*. The correlation has its limits: the linguistic correlates are obviously not available to beings who, while capable of judgement and belief, do not master a language (as is arguably the case for many higher animals) and they don’t even need always to be present whenever a language-using being judges or believes that something is the case. Also, I will understand *denial* as the attitude opposite to assertion and *rejection* as the attitude opposite to acceptance.

<sup>39</sup>I should however mention that, recently, Maudlin [2004] has heroically tried to defend the view that one can assert and accept sentences that one also declares to be *untrue<sup>S</sup>*. Assuming very plausibly that falsity<sup>S</sup> implies untrue<sup>S</sup> but not *vice versa*, such a view is strictly weaker than the view discussed in the text, and Maudlin’s arguments in defence of the former cannot straightforwardly be extended to a defence of the latter. Note also that one of the attractions of Bradwardine’s theory was exactly that of blocking the *enquotation* direction of (UTD) (that from ‘ $P$ ’ to ‘ $u$  is true’), without committing one to asserting or accepting something that is false (or even simply untrue). What I’m arguing in the text is in effect that this nice feature of the theory, which sets it ahead of a theory such as Maudlin’s in the case of utterance truth, goes irremediably lost in the case of sentential truth.

<sup>40</sup>(FN<sup>S</sup>) is in fact so extremely plausible that it is taken by many theorists as a *definition* of falsity<sup>S</sup>. Notice that (FN<sup>S</sup>) is perfectly consistent with some sentences being neither true<sup>S</sup> nor false<sup>S</sup> (contrary to the principle according to which  $\varphi$  is false<sup>S</sup> iff  $\varphi$  is not true<sup>S</sup>, which is gotten from (FN<sup>S</sup>) by moving its negation from the object language to the metalanguage).

(and such that one nevertheless asserts and accepts what they negate) does a great violence to our notion of truth<sup>S</sup>. I hasten to add that, while extremely plausible, (FN<sup>S</sup>) actually has to fail if (BT<sup>S</sup>) is to be upheld in full generality in the face of theorem 5. For, given the relevant acceptable instance of (SSD):

(SSD<sup>It is not the case that  $\lambda_1$  is false<sup>S</sup>) ‘It is not the case that  $\lambda_1$  is false<sup>S</sup>’ says<sup>S</sup> that it is not the case that  $\lambda_1$  is false<sup>S</sup>,</sup>

the assumption that ‘It is not the case that  $\lambda_1$  is false<sup>S</sup>’ is true<sup>S</sup> entails that it is not the case that  $\lambda_1$  is false<sup>S</sup>, which contradicts the result of theorem 5 to the effect that  $\lambda_1$  is false<sup>S</sup>.<sup>41,42</sup> There is however really not much solace in rejecting (FN<sup>S</sup>) on these grounds. For such grounds entail, by *reductio*, that ‘It is not the case that  $\lambda_1$  is false<sup>S</sup>’ is not true<sup>S</sup>, and so, by (BIV<sup>S</sup>), that it is false<sup>S</sup>. Thus both  $\lambda_1$  and its negation would be false<sup>S</sup>. Again, while such a position would not be strictly speaking logically inconsistent, I think it goes without saying that to declare false<sup>S</sup> (and not simply untrue<sup>S</sup>) both a sentence and its negation does a great violence to our notions of

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<sup>41</sup>Up to here, this train of thought based on (FN<sup>S</sup>) closely resembles the one developed by Serény [2008], pp. 177–182, who however uses the more problematic principle about falsity<sup>S</sup> mentioned in fn 40, and who also seems to assume that Bradwardine’s theory is in itself a theory of sentential truth rather than of utterance truth.

<sup>42</sup>Clearly, the direction of (FN<sup>S</sup>) that is problematic in the envisaged extension of Bradwardine’s theory is the left-to-right one (in fact, the converse direction is easily seen to follow from (BT<sup>S</sup>) and (BF<sup>S</sup>) together with the extremely plausible assumption that, if a sentence ‘It is not the case that  $\varphi$ ’ says<sup>S</sup> something, then, for some  $P$ , it says<sup>S</sup> [that it is not the case that  $P$ ] and  $\varphi$  says<sup>S</sup> that  $P$ ). It is important to observe that the anomaly manifested in the rejection of (FN<sup>S</sup>) is not something peculiar to the envisaged extension of Bradwardine’s theory; *it is already present in the theory of utterance truth*. To elaborate, for utterance truth (and setting aside questions of existence), the two natural counterparts of (FN<sup>S</sup>) are:

(FN<sup>∀</sup>) For every utterance  $u_0$  of a sentence  $\varphi$ ,  $u_0$  is false iff, for every utterance  $u_1$  of ‘It is not the case that  $\varphi$ ’,  $u_1$  is true;

(FN<sup>∃</sup>) For every utterance  $u_0$  of a sentence  $\varphi$ ,  $u_0$  is false iff, for some utterance  $u_1$  of ‘It is not the case that  $\varphi$ ’,  $u_1$  is true.

Both these principles have to fail (left-to-right) in Bradwardine’s theory. For example, by theorem 2,  $\mathfrak{L}_0$  is false, but, given the relevant acceptable instance of (USD):

(USD<sup>It is not the case that  $\mathfrak{L}_0$  is false</sup>) For every utterance  $u$  of the sentence ‘It is not the case that  $\mathfrak{L}_0$  is false’,  $u$  says that it is not the case that  $\mathfrak{L}_0$  is false,

it follows by (BT) that no utterance of ‘It is not the case that  $\mathfrak{L}_0$  is false’ is true.

falsity<sup>S</sup> and negation.<sup>43</sup> Even worse, ‘Either  $\lambda_1$  is false<sup>S</sup> or it is not the case that  $\lambda_1$  is false<sup>S}</sup>’ is a logical truth (and hence, presumably, at least not false<sup>S</sup>), although, according to the position in question, it would have two false<sup>S</sup> disjuncts. Again, while such a position would not be strictly speaking logically inconsistent, I think it goes without saying that to declare anything but false<sup>S</sup> a disjunction with false<sup>S</sup> disjuncts does a great violence to our notions of falsity<sup>S</sup> and disjunction. One way or the other, the anomalies of the envisaged extension of Bradwardine’s theory will thus go even beyond declaring false<sup>S</sup> some sentences that one nevertheless asserts and accepts.

It is now crucial to see that no analogue of the move presented at the end of section 5 is available in the case of  $\lambda_1$ . For that move relied on the circumstance that it is *one utterance*,  $\mathfrak{T}_0$ , that declares false *another utterance*,  $\mathfrak{L}_0$  (which nevertheless tokens the same sentence and “directly” expresses the same proposition). The situation in the case of  $\lambda_1$  is however substantially different: what the utterance  $\mathfrak{T}_1$  of  $\lambda_1$  the envisaged extension of Bradwardine’s theory leads to declares false<sup>S</sup> is not *another utterance* of  $\lambda_1$ , but  $\lambda_1$  *itself* (that is, the very same sentence  $\mathfrak{T}_1$  is an utterance of). And while, as we’ve seen, there might be some space for arguing that the falsity of  $\mathfrak{L}_0$  needn’t reflect badly on  $\mathfrak{T}_0$  (indeed, could be compatible with the truth of  $\mathfrak{T}_0$ ), there doesn’t seem to be any space for arguing that the falsity<sup>S</sup> of  $\lambda_1$  needn’t reflect badly on  $\mathfrak{T}_1$ , for the falsity<sup>S</sup> of  $\lambda_1$  makes *precisely*  $\mathfrak{T}_1$  an utterance of a false<sup>S</sup> sentence. And while it is strictly speaking logically consistent to hold that an utterance (i.e.  $\mathfrak{T}_1$ ) is true while the sentence the utterance is an utterance of (i.e.  $\lambda_1$ ) is false<sup>S</sup>, even the recourse to such an extreme measure would do nothing to mitigate the fact that in producing  $\mathfrak{T}_1$  one is asserting and accepting a false<sup>S</sup> sentence (i.e.  $\lambda_1$ ).

To illustrate the problem, I have chosen sentences rather than propositions, mainly because, in virtue of their linguistic character, they are in many respects closer to utterances than propositions are (thus facilitating the comparison with the semantic paradoxes for utterance truth). However, I hope it is clear that an analogous dialectic can be run by choosing propositions rather than sentences and developing the relevant semantic paradoxes for

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<sup>43</sup>It also raises the issue as to what extent the spirit if not even the letter of the principle of bivalence<sup>S</sup> is preserved in the envisaged extension of Bradwardine’s theory. Although in such extension, for every contentful sentence  $\varphi$ ,  $\varphi$  is either true<sup>S</sup> or false<sup>S</sup> (i.e. (BIV<sup>S</sup>) holds), that no longer entails that, for every contentful sentence  $\varphi$ , either  $\varphi$  is true<sup>S</sup> or ‘It is not the case that  $\varphi$ ’ is true<sup>S</sup>. Thus, there are some yes/no questions, such as that expressible by using  $\lambda_1$  with interrogative force, to which *there is no true answer* (since both  $\lambda_1$  and its negation are false). That may well be felt to violate what bivalence<sup>S</sup> should really be all about (for one, Tarski [1983], p. 197 considers a close kin to (FN<sup>S</sup>), rather than merely (BIV<sup>S</sup>), the principle that a satisfactory theory of truth should validate).

propositional truth. In the specific case of  $\mathcal{L}_1$ , such a dialectic would lead to the analogous result that the envisaged extension of Bradwardine’s theory would be forced to assert and accept a proposition while at the same time (indeed, with those very same acts!) declaring that such a proposition is false<sup>P</sup>. I leave it to the reader to work out the details and further consequences of such a dialectic.

I do want to mention however at least one specific use to which Liar paradoxes for propositional truth can be put. This emerges in connection with (what seems to me) a rather perverse kind of reaction to the previous Liar paradox for sentential truth on behalf of the envisaged extension of Bradwardine’s theory. The idea would be that, instead of asserting and accepting  $\lambda_1$ , one might try to pull off the effect of thinking and saying that  $\lambda_1$  is false<sup>S</sup> by asserting and accepting a sentence *different* from  $\lambda_1$  that nevertheless says<sup>S</sup> that  $\lambda_1$  is false<sup>S</sup>. As examples, let’s consider an English sentence  $\sigma_0$  different from  $\lambda_1$  such that  $\langle \ulcorner \sigma_0 \urcorner \rangle$ <sup>44</sup> entails  $\langle \lambda_1 \text{ is false}^S \rangle$  ( $\langle \ulcorner \sigma_0 \urcorner \rangle$  might even be logically equivalent and indeed identical with  $\langle \lambda_1 \text{ is false}^S \rangle$  if we choose a sentence like ‘ $\lambda_1$  is in fact false<sup>S</sup>’ which is synonymous with  $\lambda_1$ ) and a Latin sentence  $\sigma_1$  such that  $\langle \ulcorner \sigma_1 \urcorner \rangle$  is identical with  $\langle \lambda_1 \text{ is false}^S \rangle$ . If one asserts and accepts  $\sigma_0$  or  $\sigma_1$  instead of  $\lambda_1$ , one is still asserting and accepting a sentence saying<sup>S</sup> that  $\lambda_1$  is false<sup>S</sup>, but one now avoids the problem of declaring false<sup>S</sup> some sentences that one nevertheless asserts and accepts, since the sentence which one declares false<sup>S</sup> is  $\lambda_1$ , and that is not the sentence that one asserts and accepts (which is either  $\sigma_0$  or  $\sigma_1$ ).

This idea would seem however to face insurmountable difficulties already in the case of Liar paradoxes for sentential truth. Firstly, although we’ve worked up to now with a notion of consequence as a relation among *propositions* (see section 3), there seems to be an equally good notion of consequence as a relation among *sentences* (indeed, such notion has arguably some claim to be by far the most useful one when theorising about consequence, see Zardini [2011c]). However, for many choices of a  $\sigma_0$  with the required properties and on virtually all understandings of the notion of consequence as a relation among sentences,  $\sigma_0$  will entail ‘ $\lambda_1$  is false<sup>S</sup>’. If so, when, on the current strategy, one asserts and accepts  $\sigma_0$ , one asserts and accepts a sentence that one knows full well to entail  $\lambda_1$ , yet one will neither assert nor accept this known consequence of what one does assert and accept. In fact, one will not merely refuse to assert and accept such consequence, one will declare it false<sup>S</sup>. And since the whole point of the exercise was presumably to preserve the truth<sup>S</sup> of what one asserts and accepts, one will end up declaring true<sup>S</sup> the premise

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<sup>44</sup>I’m using ‘ $\ulcorner \varphi \urcorner$ ’ as a sentence-forming operator such that the proposition that ‘ $\ulcorner \varphi \urcorner$ ’ is the proposition “directly” expressed by  $\varphi$ .

(i.e.  $\sigma_0$ ) and false<sup>S</sup> the conclusion (i.e.  $\lambda_1$ ) of a known entailment. Again, while such a position would not be strictly speaking logically inconsistent, I think it goes without saying that to declare true<sup>S</sup> the premise and false<sup>S</sup> the conclusion of a known entailment does a great violence to our notions of truth<sup>S</sup>, falsity<sup>S</sup> and entailment. (Even stronger considerations apply to the case where  $\sigma_0$  is synonymous with  $\lambda_1$ .) Similarly, when, on the current strategy, one asserts and accepts  $\sigma_1$ , one asserts and accepts a sentence that one knows full well to be translated by  $\lambda_1$ , yet one will neither assert nor accept this known translation of what one does assert and accept. In fact, one will not merely refuse to assert and accept such translation, one will declare it false<sup>S</sup>. And since the whole point of the exercise was presumably to preserve the truth<sup>S</sup> of what one asserts and accepts, one will end up declaring true<sup>S</sup> one sentence (i.e.  $\sigma_1$ ) and false<sup>S</sup> its known translation (i.e.  $\lambda_1$ ). Again, while such a position would not be strictly speaking logically inconsistent, I think it goes without saying that to declare true<sup>S</sup> one sentence and false<sup>S</sup> its known translation does a great violence to our notions of truth<sup>S</sup>, falsity<sup>S</sup> and translation.

Secondly, as was expectable from discussions of revenge, the resources introduced by the paradox-avoiding strategy themselves point the way to how the paradox can be so modified as to circumvent the strategy. We again assume that there is some self-referential device ensuring the existence of a sentence,  $\lambda_4$ , which is identical to the sentence ‘Every sentence  $\varphi$  such that what  $\varphi$  says<sup>S</sup> entails  $\langle \ulcorner \lambda_4 \urcorner \rangle$  is false<sup>S</sup>’. Consider an arbitrary sentence  $\psi$  such that what  $\psi$  says<sup>S</sup> entails  $\langle \ulcorner \lambda_4 \urcorner \rangle$ . Assume for *reductio* that  $\psi$  is true<sup>S</sup>. Then, by (C<sup>S</sup>) and (BT<sup>S</sup>),  $\langle \ulcorner \lambda_4 \urcorner \rangle$  is the case—that is, it is the case that every sentence  $\varphi$  such that what  $\varphi$  says<sup>S</sup> entails  $\langle \ulcorner \lambda_4 \urcorner \rangle$  is false<sup>S</sup>. But  $\psi$  by assumption is such a sentence, and so  $\psi$  is false<sup>S</sup>. Contradiction with (CONTRAV<sup>S</sup>). Hence, by *reductio*,  $\psi$  is not true<sup>S</sup>, and so, by (BIV<sup>S</sup>), it is false<sup>S</sup>. But  $\psi$  was arbitrary, and so, by universal generalisation, every sentence  $\varphi$  such that what  $\varphi$  says<sup>S</sup> entails  $\langle \ulcorner \lambda_4 \urcorner \rangle$  is false<sup>S</sup>. As with  $\lambda_1$ , the envisaged extension of Bradwardine’s theory has reached a conclusion,  $\lambda_4$ , that declares itself to be false<sup>S</sup> (more precisely, it declares itself to be false<sup>S</sup> in the same sense in which ‘Every man is mortal’ declares Socrates to be mortal). The problem now is that not only does that most natural conclusion declare itself to be false<sup>S</sup>, it also declares false<sup>S</sup> every sentence  $\varphi$  such that what  $\varphi$  says<sup>S</sup> entails  $\langle \ulcorner \lambda_4 \urcorner \rangle$ , and it does so in virtue of the proposition that it “directly” expresses (i.e.  $\langle \ulcorner \lambda_4 \urcorner \rangle$ ) declaring false<sup>S</sup> every such sentence. If, instead of asserting and accepting  $\lambda_4$ , one thus tried to pull off the effect of thinking and saying that  $\ulcorner \lambda_4 \urcorner$  by asserting and accepting a sentence different from  $\lambda_4$  that nevertheless says<sup>S</sup> that  $\ulcorner \lambda_4 \urcorner$ , one would still assert and accept

a sentence that, saying<sup>S</sup> that  $\ulcorner \lambda_4 \urcorner$ , also says<sup>S</sup> of itself that it is false<sup>S</sup>.<sup>45</sup>

In spite of these difficulties, the current strategy is revealing because it shows that, although they are *more coarse-grainedly* individuated than utterances, sentences still exhibit a *certain fine-grainedness* of individuation that might tempt one into something like the envisaged extension of Bradwardine's theory. For example, an English sentence can be a different sentence from one with which it is synonymous, and a Latin sentence can be a different sentence from an English sentence which is its translation. As we've seen, that might tempt one into perversely thinking that one could assert and accept a sentence synonymous with or translated by  $\lambda_1$  in order to pull off the effect of thinking and saying that  $\lambda_1$  is false<sup>S</sup> without asserting and accepting  $\lambda_1$ . We've also seen some of the difficulties that that strategy faces already within the territory of Liar paradoxes for sentential truth, but it is now crucial to observe that the *further coarsening* of the domain of truth bearers constituted by the ascent from sentences to propositions would seem to foreclose even more categorically any analogous extension of Bradwardine's theory to truth<sup>P</sup>.

For example, as I've mentioned, in the specific case of  $\mathcal{L}_1$  the analogous extension of Bradwardine's theory would be forced to assert and accept a proposition while at the same time declaring that such a proposition is false<sup>P</sup>. Even more clearly than in the case of Liar paradoxes for sentential truth, it would be completely idle to try to obviate to this by asserting and accepting a sentence which is synonymous with or is translated by ' $\mathcal{L}_1$  is false<sup>P</sup>', and which thus says<sup>S</sup> that  $\mathcal{L}_1$  is false<sup>P</sup>—one would simply thereby assert and accept  $\mathcal{L}_1$  all over again. It would seem that the best one could do is to assert and accept a proposition  $\mathcal{P}$  that is logically equivalent with or at least entails  $\mathcal{L}_1$ . But that move is subject to a consideration analogous to the one developed in the case of Liar paradoxes for sentential truth and in fact even more forceful in that it only appeals to a notion of entailment as a relation among propositions. For when, on the current strategy, one asserts and accepts  $\mathcal{P}$ , one asserts and accepts a proposition that one knows full well to entail  $\mathcal{L}_1$ , yet one will neither assert nor accept this known consequence of what one does assert and accept. In fact, one will not merely refuse to assert and accept such consequence, one will declare it false<sup>P</sup>. And since the whole point of the exercise was presumably to preserve the truth<sup>P</sup> of what one asserts and accepts, one will end up declaring true<sup>P</sup> the premise (i.e.  $\mathcal{P}$ ) and false<sup>P</sup> the conclusion (i.e.  $\mathcal{L}_1$ ) of a known entailment. Again, while such a position would not be strictly speaking logically inconsistent, I think it goes

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<sup>45</sup>The paradox presented in this paragraph is a variant for sentences of the quantificational Liar paradox for utterance truth presented in Zardini [2008], pp. 562–563.

without saying that to declare true<sup>P</sup> the premise and false<sup>P</sup> the conclusion of a known entailment does a great violence to our notions of truth<sup>P</sup>, falsity<sup>P</sup> and entailment.

The semantic paradoxes for propositional and sentential truth presented in section 7 are genuine paradoxes, for it does seem that there are truth-like predicates for propositions and sentences and that such predicates obey intersubstitutability principles (and it does seem that the required kind of self-reference exists). As we've seen in this section however, Bradwardine's theory not only is explicitly restricted to utterance truth, but also so crucially relies—in its solution to the semantic paradoxes—on specific features of the targeted truth bearers (utterances) that it is very hard to see how it could be extended to cope with those paradoxes. For one, the most natural extension of the theory in the case of truth<sup>S</sup> (extension which we've gone to some length to spell out) has highly implausible (if not downright incoherent) consequences. Henceforth assuming that such extensions are thus not viable, I conclude that the original theory for utterance truth faces the danger of being either dramatically incomplete or committed to denying that propositions, sentences and other kinds of things that are akin to them in the relevant respects are truth bearers, in the sense that they are in the range of significance of a truth-like predicate.

The second horn of this dilemma could be made more palatable than it seems by adopting a staunch *nominalist* stance (which in effect was adopted e.g. by John Buridan, whose theory shares the problematic features of Bradwardine's). Such a stance would however fit very badly with contemporary semantic and logical theorising, whose investment in the existence of propositions and sentences can hardly be underestimated (and—short of degenerating into a thoroughgoing nominalism denying the existence of numbers—would also be in some tension with such results as the '*arithmetisation of syntax*', showing that talk of numbers can be interpreted as talk of sentences). In this ballpark, there is also the stance of accepting the *ontology* of propositions and sentences while rejecting that the *ideology* of truth applies to them. Again, such a stance would however fit very badly with contemporary semantic and logical theorising, whose investment in the truth-bearerhood of propositions and sentences can hardly be underestimated (and would have the uncomfortable feature of accepting propositions and sentences while rejecting one of their main *raison d'être*).<sup>46</sup>

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<sup>46</sup>I'm aware that some commentators (see e.g. Klima [2004], pp. 106–107, who is explicitly considering Buridan's theory) seem to infer from the fact that a certain solution to the semantic paradoxes requires operating at the level of utterances (as is the case for Bradwardine's or Buridan's theories) the conclusion that propositions, sentences and their

In any event, the problem I've raised has ultimately very little to do with the abstract/concrete distinction (see fn 6). For the problem could be replicated by imagining a situation where one can only entertain thoughts by using utterances, an utterance  $\mathfrak{L}_5$  exists of the sentence ' $\mathfrak{L}_5$  is always false' and no other utterance exists which could be used to entertain the thought that  $\mathfrak{L}_5$  is always false. It's easy to see that Bradwardine's theory entails that, in such a situation,  $\mathfrak{L}_5$  is always false, but the only way one could endorse this thought in that situation would be by asserting or accepting, in thought or talk,  $\mathfrak{L}_5$ —i.e. by asserting and accepting an utterance while at the same time (indeed, with those very same acts!) declaring that such an utterance is false. It is an unexpected and problematic feature of Bradwardine's theory that the coherence of its endorsement depends on such vagaries as what is required to entertain a thought and which concrete objects are there in the world.

Even more importantly, Bradwardine's theory itself, as is evident from (BT) and (BF), has to make use of (possibly higher-order) phrases and of a construction signalling the objective representational correctness of what these phrases express (as 'things are' does in "[...] signifying only ways things are", Bradwardine [2010], 6.2).<sup>47</sup> As fn 35 has already indicated, semantic paradoxes can be constructed on any plausible (committing or non-committing) interpretation of such phrases. Let's briefly see how one of the examples of section 7 could be recast in a higher-order framework. Let's consider the way things are when it is not the case that they are the way considered in the fourth sentence of the second last paragraph of section 8 of such-and-such paper (where 'way' is now understood as a higher-order variable and, again, 'such-and-such' stands in for a definite description that in the actual world picks out this paper). Given the instance of reflexivity 'It is not the case that things are the way considered in the fourth sentence of the second last paragraph of section 8 of such-and-such paper iff it is not the case that things are the way considered in the fourth sentence of the second last paragraph of section 8 of such-and-such paper', intersubstitutability of 'Things are [the way they are when it is not the case that they are the way considered in the fourth sentence of the second last paragraph of section 8

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like cannot be truth bearers. I think, and have briefly argued in the text, that, in addition to its being counterintuitive, that conclusion should be regarded as an extremely problematic consequence of the theories in question, which should give us some pause and make us wonder whether these theories are actually giving the right treatment to the semantic paradoxes (moreover, I'll now proceed to argue in the text that, even confining ourselves to the resources afforded by Bradwardine's theory, the problems we've discussed would still remain in certain particularly recalcitrant forms).

<sup>47</sup> "[...] *significans tantum sicut est*".



of such-and-such paper]’ with ‘It is not the case that things are the way considered in the fourth sentence of the second last paragraph of section 8 of such-and-such paper’<sup>48</sup> yields ‘Things are [the way they are when it is not the case that they are the way considered in the fourth sentence of the second last paragraph of section 8 of such-and-such paper] iff it is not the case that things are the way considered in the fourth sentence of the second last paragraph of section 8 of such-and-such paper’, which, together with the equivalence ‘Things are [the way they are when it is not the case that they are the way considered in the fourth sentence of the second last paragraph of section 8 of such-and-such paper] iff things are the way considered in the fourth sentence of the second last paragraph of section 8 of such-and-such paper’ (guaranteed by empirical facts),<sup>49</sup> yields ‘Things are the way considered in the fourth sentence of the second last paragraph of section 8 of such-and-such paper iff it is not the case that things are the way considered in the fourth sentence of the second last paragraph of section 8 of such-and-such paper’—a (higher-order) classical contradiction.

In closing, I should like to stress that the first horn of the dilemma is no more palatable than the second: for, if a different solution applies to the semantic paradoxes for propositional and sentential truth (and to those for higher-order quantification, as the one sketched in the previous paragraph), then the worry will arise that the same kind of solution could be used to solve the semantic paradoxes for utterance truth, thus rendering Bradwardine’s theory idle.<sup>50</sup>

## 9 Conclusion

Geared in its very foundations to a treatment of the semantic paradoxes, Bradwardine’s theory of truth constitutes an admirable attempt at a solution of one of the most intractable and yet (or thereby) fascinating problems

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<sup>48</sup>The general intersubstitutability principle at work here is the seemingly unassailable one between ‘Things are [the way they are when  $\varphi$ ]’ and  $\varphi$  (where ‘ $\varphi$ ’ can now range over sentential variables as well as sentences).

<sup>49</sup>And underwritten, if you’re happy to talk about identity of what higher-order phrases express, by the identity ‘The way things are when it is not the case that they are the way considered in the fourth sentence of the second last paragraph of section 8 of such-and-such paper is the way considered in the fourth sentence of the second last paragraph of section 8 of such-and-such paper’ (in turn guaranteed by the same empirical facts).

<sup>50</sup>Even though this is not the place to elaborate on the matter, I should note that, with some modifications and qualifications, an argument similar to the overarching argument of this section can be run against the view mentioned in fn 20.

in the history of philosophy and logic (the label of ‘*insolubilia*’ is indeed very apt!). The attempt certainly has a great historical interest, since Bradwardine’s theory deeply influenced many subsequent medieval theories of truth (for example, different as it is in many significant respects, Buridan’s theory preserves some crucial features of Bradwardine’s; see Read [2002] for a critical comparison). More importantly, the attempt has an equally great theoretical interest, since it manages to preserve classical logical and semantic principles without introducing a hierarchy of truth properties and predicates (as notoriously happens in one of the *loci classici* of the contemporary reflection on the semantic paradoxes: Tarski [1983]). It does so by restricting in an unobvious, but relevant and principled way the right-to-left direction of (UTD). We have seen how crucial in such a restriction is the utterance-based approach, which allows to draw fine-grained distinctions that can be then exploited to justify a difference in exemplification of the property of utterance truth—a difference that cuts across two different utterances of the same sentence that “directly” express the same proposition. We have also seen how such difference in exemplification of the property of utterance truth is ultimately grounded in a difference in exemplification of the saying relation—again, two different utterances of the same sentence that “directly” express the same proposition can nevertheless say different things.

Yet, even in the relevant sense of ‘true’ (as denoting a property of objective representational correctness), more things in heaven and earth can be true than are dreamt of in Bradwardine’s philosophy. I have considered in particular propositions and sentences, and showed how semantic paradoxes for propositional and sentential truth arise just as easily and naturally as do the semantic paradoxes for utterance truth with which Bradwardine concerned himself. I have then argued that the fine-grained distinctions that an extension of his strategy to these cases would require are not to be had. Not exhibiting the indefinite reproducibility of tokens typical of utterances, the domain of propositions and the domain of sentences are crucially more coarse-grained than the domain of utterances—while in the case of utterances the Theorist can safely condemn the Liar to falsity, in the case of propositions (or sentences) the Theorist’s condemnation of the Liar to falsity irremediably brings in its wake the condemnation to falsity of the Theorist itself. Thus, relying on features that are specific to utterances and are not present for other kinds of entities that are also plausibly assumed to be truth bearers, Bradwardine’s theory of truth emerges as being objectionably incomplete. Despite its remarkable ingenuity, I think it’s unlikely to have unveiled the real source by which Liars of all kinds (Liar utterances, Liar propositions and

Liar sentences alike) are begotten.<sup>51</sup>

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<sup>51</sup>I do happen to have my own positive view about the semantic paradoxes—I make a start on it in Zardini [2011b].

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