Restriction by Non-Contraction*

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February 2, 2015

1 Implication

An operation of *implication*[1] has traditionally been associated with at least six different (possibly related) *functions* (which I'll refer to as 'the *Functions*'). Taking an implicational

*Earlier versions of the material in this paper have been presented in 2012 at 7th SLMFCE 2012 Conference (University of Santiago de Compostela); in 2013, at the PSSA 2013 Conference in Salt Rock (University of KwaZulu-Natal), at the 4th World Congress on Universal Logic (Federal University of Rio de Janeiro), at a NIP Research Seminar (University of Aberdeen), at a research seminar at the University of the Azores in Ponta Delgada, at the MCMP Workshop *Truth and Paradox* in Munich (Ludwig Maximilian University) and at the LOGOS Semantic Paradoxes and Vagueness Seminar (University of Barcelona). I'd like to thank all these audiences for very stimulating comments and discussions. Special thanks go to JC Beall, Franz Berto, Ross Brady, Filippo Ferrari, Andreas Fjellstad, Mikkel Gerken, Ole Hjortland, Dan López de Sa, Pepe Martínez, Julien Murzi, Sergi Oms, Francesco Paoli, Matteo Plebani, Dave Ripley, Rui Sampaio da Silva, Gonçalo Santos, Paula Sweeney, Crispin Wright and an anonymous referee. As on other occasions, I owe for this investigation a special debt to Hartry Field, who first impressed on me the difficulties that restricted quantification creates for non-classical theories of truth. At different stages during the writing of the paper, I've benefitted from a UNAM Postdoctoral Research Fellowship, from an AHRC Postdoctoral Research Fellowship, from the FP7 Marie Curie Intra-European Research Fellowship 301493 on *A Non-Contractive Theory of Naive Semantic Properties: Logical Developments and Metaphysical Foundations* (NTNSP) and from the FCT Research Fellowship IF/01202/2013 on *Tolerance and Instability: The Substructure of Cognitions, Transitions and Collections* (TI), as well as from partial funds from the project CONSOLIDER-INGENIO 2010 CSD2009-00056 of the Spanish Ministry of Science and Innovation on *Philosophy of Perspectival Thoughts and Facts* (PERSP), from the FP7 Marie Curie Initial Training Network 238128 on *Perspectival Thoughts and Facts* (PETAF), from the project FFI2011-25626 of the Spanish Ministry of Science and Innovation on *Reference, Self-Reference and Empirical Data* and from the project FFI2012-35026 of the Spanish Ministry of Economy and Competition on *The Makings of Truth: Nature, Extent, and Applications of Truthmaking.*

[1]Throughout, I use '*implication*' and its relatives to denote the operation expressed by a *conditional connective* (such as 'if', while I use '*conditional*' and its relatives as applying to those sentences that have

proposition expressed by the conditional $\varphi \to \psi$, these can informally be expressed as:

SUFFICIENT CONDITION$^{\Rightarrow}$ Be at least as weak as φ being a sufficient condition for ψ;

SUFFICIENT CONDITION$^{\Leftarrow}$ Be at least as strong as φ being a sufficient condition for ψ;

NO REFUTATION$^{\Rightarrow}$ Follow from its not being the case that φ holds while ψ does not;[2]

NO REFUTATION$^{\Leftarrow}$ Entail that it is not the case that φ holds while ψ does not;

DEDUCTION THEOREM Be at least as weak as φ entailing ψ;

MODUS PONENS Together with φ, logically necessitate ψ.[3]

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a conditional connective as main connective), '*follow from*' and its relatives to express the relation of *logical consequence* (broadly understood so as to encompass for example the "logic of truth") and '*entail*' and its relatives to express the *converse* relation. I use '*equivalence*' and its relatives to denote *two-way* entailment. I use '*be at least as* {*weak/strong*} *as*' and its relatives to express a particularly strict kind of entailment, whose distinctive features I assume to be relevant only for section 5 (see the explanation in fn 41). I use '*logically necessitate*' and its likes to express an entailment strong enough as to preserve logical necessity (the point of this typically vacuous qualification will be clear again in section 5).

[2]Throughout, I identify φ *not holding* with $\neg\varphi$ *holding*. This identification might well be problematic in other areas of philosophy of logic and language (see Zardini [2014h] for a recent discussion relevant for the semantic paradoxes), but, given that 'φ' holds' is another way of saying ''φ' is true', the identification is in fact accepted by almost all theories I'll be concerned with (and greatly simplifies my own discussion). One might initially have thought that precisely the semantic paradoxes give a reason against the identification, on the grounds that they should be diagnosed as dealing with sentences that are *neither true nor false* (see e.g. van Fraassen [1968]). Notoriously, such a diagnosis runs into problems with the so-called "*strengthened*" Liar sentence 'This sentence is not true'. For, if that sentence is neither true nor false, it is not true. However, if one accepts 'P', one should presumably accept ''P' is true' as well, and so 'This sentence is not true' would be true after all. Less notoriously, such a diagnosis runs into problems *already* with the *original* Liar sentence 'This sentence is false'. For, if that sentence is neither true nor false, it is not false. However, if one accepts 'It is not the case that P', one should presumably accept ''P' is false' as well, and so 'This sentence is false' would be false after all.

[3]The traditional *extensional/intensional* distinction is arguably too loose and coarse-grained to do much theoretical work in the study of implications. However, to connect our issues with that venerable tradition, it might be helpful to notice that, while **NO REFUTATION$^{\Rightarrow}$** falls squarely on the extensional side and **SUFFICIENT CONDITION$^{\Leftarrow}$** falls squarely on the intensional side, **NO REFUTATION$^{\Leftarrow}$** only has a more-extensional-than-intensional flavour and **SUFFICIENT CONDITION$^{\Rightarrow}$**, **DEDUCTION THEOREM** and **MODUS PONENS** only have a more-intensional-than-extensional flavour.

Classical logic's implication, defined for example in terms of negation and conjunction,[4] satisfies all the Functions,[5] and so virtually does the implication at work in a philosophically prominent *non-classical* logic like *intuitionist* logic (*modulo* the addition of a double negation in the consequent in the case of **NO REFUTATION**$^{\Rightarrow}$). Once one turns however to certain kinds of logics motivated by the *semantic paradoxes*, one is typically faced with systems where the Functions are essentially fragmented across different operations.

Say that a theory of truth is *naive* iff it validates the rules:

T-INTRODUCTION $\varphi \vdash T\ulcorner\varphi\urcorner$ holds;

T-ELIMINATION $T\ulcorner\varphi\urcorner \vdash \varphi$ holds.[6]

Under minimal assumptions about the expressive richness of the language which I throughout take for granted, naive theories have to reject full classical logic on pain of being

[4]Let's fix on some terminology and notation. Given a standard logic **L**, I'll call an implication from the semantic value of φ to the semantic value of ψ '*material*' iff it is equivalent in **L** to the overall operation on the semantic value of φ and the semantic value of ψ expressed by $\neg(\varphi \,\&\, \neg\psi)$. I'll express a material implication with the conditional connective $\supset$. I'll call an implication '*non-material*' iff it is not material. I'll express a non-material implication with the conditional connective $\leadsto$. I'll express an implication which, for all that has been said, could be either material or non-material with the conditional connective $\rightarrow$.

[5]Contrary to the other Functions, **SUFFICIENT CONDITION**$^{\Rightarrow}$ and **SUFFICIENT CONDITION**$^{\Leftarrow}$ crucially make use of an apparently *loose* notion (that of being a sufficient condition). One can however give *precise sufficient or necessary criteria* for the application of that notion, and thereby still (possibly non-conclusively) precisely test whether an implication satisfies these two Functions. For example, I assume throughout that a precise sufficient criterion for φ being a sufficient condition for ψ is that φ is identical to ψ, and that a precise necessary criterion for φ being a sufficient condition for ψ is that it is not the case φ is logically necessary while ψ is logically absurd. In fact, to keep the discussion focussed on the relevant issues, throughout I make the simplifying assumption that these two tests determine whether an implication satisfies **SUFFICIENT CONDITION**$^{\Rightarrow}$ and **SUFFICIENT CONDITION**$^{\Leftarrow}$ respectively (and so, in particular, I ignore the otherwise important worry that an implication may not satisfy **SUFFICIENT CONDITION**$^{\Leftarrow}$ because it fails to satisfy some constraint of *relevance* that may be operative in the loose notion of being a sufficient condition). Relatedly, I think it's compelling to understand the notion of being a sufficient condition so that φ entailing ψ is a special case of φ being a sufficient condition for ψ and so that φ together with φ being a sufficient condition for ψ logically necessitates ψ. From the first compelling claim it follows that **SUFFICIENT CONDITION**$^{\Rightarrow}$ implies **DEDUCTION THEOREM** and from the second compelling claim it follows that **SUFFICIENT CONDITION**$^{\Leftarrow}$ implies **MODUS PONENS**. However, since those two compelling claims are nevertheless to some extent debatable (and have in fact been debated), in order not to prejudge those debates in the very set-up of the paper our official list of the Functions includes **DEDUCTION THEOREM** and **MODUS PONENS** as separate items. More generally, considering the Functions as coming in the three pairs successively listed above in the text, I think it's appealing to see the first Function of each pair as a specific variation on a basic theme, and to see the second Function of each pair as a specific variation on the converse basic theme.

[6]Throughout, $\vdash$ expresses the relation of logical consequence. As will become apparent below, in order to achieve the required generality, such relation is assumed to be both *multiple-premise* and *multiple-conclusion*, and, in order to accommodate for certain substructural logics, both premises and conclusions are assumed to be put together into *multisets*. Moreover, T is a truth predicate and '$\ulcorner\varphi\urcorner$' refers to a name for φ available in the object language (I assume that every sentence has at least one such name).

subject to the semantic paradoxes. In fact, naive theories often escape such paradoxes either by *rejecting both a paradoxical sentence and its negation*—and so, as a natural consequence, by rejecting the law of *excluded middle* $\oslash \vdash \varphi \vee \neg\varphi$—[7]or by *accepting both a paradoxical sentence and its negation*—and so, as a natural consequence, by rejecting the law of *non-contradiction* $\varphi \,\&\, \neg\varphi \vdash \oslash$ (I've questioned in Zardini [2014g] whether the alleged "natural" consequences are really that plausible, but I won't go into such issues here).[8] Let's call the former kind of theory (developed e.g. by Kripke [1975]; Brady [2006]; Field [2008]) '*analetheic*' and the latter kind (developed e.g. by Priest [2006]; Beall [2009]) '*dialetheic*'.

The distinctive features of analetheic and dialetheic theories of truth prevent material implication from satisfying all the Functions. As for analetheic theories, $\varphi \supset \varphi$ is accepted by everyone in this debate to be equivalent with $\varphi \vee \neg\varphi$. φ is however a sufficient condition for φ (see fn 5), and so material implication can no longer satisfy **SUFFICIENT CONDITION**$^{\Rightarrow}$. One might here simply bite the bullet: on reflection, if one thinks that implication requires some broad sort of *dependence* of the consequent on the antecedent, a *reflexivity* claim like $\varphi \rightarrow \varphi$ actually becomes problematic. The problem is that that is only, as it were, "the tip of the bullet": for the same reason for which $\varphi \supset \varphi$ has to go, so have to go $\varphi \supset \neg\neg\varphi$, $\varphi \supset \varphi \vee \psi$, $\varphi \supset (\psi \supset \varphi \,\&\, \psi)$ etc., and so the analetheist is pressed to search for other implications that do a better job in that respect. But the room for improvement here is *essentially limited*, as the considerations just advanced show that *analetheic theories cannot accept any implication that satisfies both* **SUFFICIENT CONDITION**$^{\Rightarrow}$ *and* **NO REFUTATION**$^{\Leftarrow}$: since φ is a sufficient condition for φ, if $\rightarrow$ satisfies **SUFFICIENT CONDITION**$^{\Rightarrow}$ $\varphi \rightarrow \varphi$ will hold, but, if $\rightarrow$ also satisfies **NO REFUTATION**$^{\Leftarrow}$, $\neg(\varphi \,\&\, \neg\varphi)$ and so $\varphi \vee \neg\varphi$ will also hold.

As for dialetheic theories, φ and $\varphi \supset \psi$ are only a good reason for inferring ψ if it is ruled out that φ both holds and does not hold (otherwise, both φ and $\varphi \supset \psi$ could hold without need for ψ to hold), and so material implication can no longer satisfy **MODUS PONENS**. And here "the tip of the bullet" is enough to press the dialetheist to search for other implications that do a better job in that respect.[9] But

[7]Throughout, faithful to its origins as simply evocative of Latin *vel* (and for lack of a better alternative), I understand $\vee$ as *neutrally* expressing the operation of (inclusive) disjunction that is at play in informal philosophical discussions of the semantic paradoxes, without any prejudice to the details of its logical behaviour (just as I understand $\&$ or $\neg$ as *neutrally* expressing the operations of conjunction and negation respectively that are at play in those discussions, without any prejudice to the details of their logical behaviour). In particular, to forestall a natural association that the symbol might trigger for those familiar with *lattices*, I emphasise that I do *not* presuppose that disjunction has "*join* behaviour" (in fact, in the theory that I'll eventually recommend in section 6, disjunction does fail to satisfy some basic "join principles" like for example the rule of *subidempotency* $\varphi \vee \varphi \vdash \varphi$). Thanks to an anonymous referee for advice about notation and for urging this clarification.

[8]$\oslash$ is the *empty* multiset and, as usual, $\oslash \vdash \varphi$ can be read as saying that φ is *logically necessary* whereas $\varphi \vdash \oslash$ can be read as saying that φ is *logically absurd*.

[9]Beall [2014] tries to resist the pressure, by challengingly arguing that inference according to *modus ponens* for material implication does not require material implication to satisfy **MODUS PONENS**. Basically, and slightly rephrased, Beall endorses a *normative acceptance/rejection principle* (essentially due to Restall [2005]) to the effect that, if $\Gamma \vdash \Delta$ holds, one should not accept all the members of Γ

the room for improvement here is also *essentially limited*, as the considerations just advanced show that *dialetheic theories cannot accept any implication that satisfies both* **NO REFUTATION**$^{\Rightarrow}$ *and* **MODUS PONENS**: if φ both holds and does not hold, if $\rightarrow$ satisfies **NO REFUTATION**$^{\Rightarrow}$ φ and $\varphi \rightarrow \psi$ will hold (with ψ arbitrary), but, if $\rightarrow$ also satisfies **MODUS PONENS**, ψ will also hold.

As I've already hinted at, both analetheists and dialetheists have reacted to the previous problems about the bad behaviour of material implication by introducing in their systems new non-material implications that can satisfy some of the Functions that their material implications cannot satisfy (as observed in the last two paragraphs, 'some of the Functions that their material implications cannot satisfy' cannot be strengthened to 'all the Functions', and so both analetheic and dialetheic theories of truth essentially require a fragmentation of the Functions). As for analetheic theories, material implication typically keeps satisfying **SUFFICIENT CONDITION**$^{\Leftarrow}$, **NO REFUTATION**$^{\Rightarrow}$ and **NO REFUTATION**$^{\Leftarrow}$ (sometimes, as in Kripke [1975] or Field [2008], also **MODUS PONENS**), while a new non-material implication typically satisfies **SUFFICIENT CONDITION**$^{\Rightarrow}$, **SUFFICIENT CONDITION**$^{\Leftarrow}$ and **MODUS PONENS** (sometimes, as in Field [2008], also **NO REFUTATION**$^{\Rightarrow}$).[10] As for dialetheic theories, material implication typically keeps satisfying **SUFFICIENT CONDITION**$^{\Rightarrow}$, **NO REFUTATION**$^{\Rightarrow}$, **NO REFUTATION**$^{\Leftarrow}$ and **DEDUCTION THEOREM**, while a new non-material implication typically satisfies **SUFFICIENT CONDITION**$^{\Rightarrow}$, **SUFFICIENT CONDITION**$^{\Leftarrow}$, **NO REFUTATION**$^{\Leftarrow}$ and **MODUS PONENS**.

At a first glance, it is not clear that there is anything terribly bad in thinking that the functions traditionally played by one and the same operation (classical material implica-

and reject all the members of Δ, and notes that, in his favoured dialetheic logic, although $\varphi, \varphi \supset \psi \vdash \psi$ does not hold, $\varphi, \varphi \supset \psi \vdash \psi, \varphi \,\&\, \neg\varphi$ does. He then surmises that, given a suitable defeasible principle to the effect that one should reject contradictions, that suffices in a standard situation for one to infer according to *modus ponens* (in particular, for one to infer ψ given one's acceptance of $\varphi, \varphi \supset \psi$). But clearly it does not, since all that package delivers is simply (harmlessly ignoring issues of scope) that, in such a situation, one should *not reject* ψ, not that one should *accept* it. The problem briefly surfaces when Beall apparently feels the need to assume further that "[...] you want to expand your theory 'according to logic'—you want to add to your theory, not merely avoid logically clashing with it" (Beall [2014], p. 7). However, even if that further assumption is added to the package, it still does not even follow that one should either accept ψ or accept $\varphi \,\&\, \neg\varphi$ (let alone that one should accept ψ). (Compare: by logic, I should not reject both 'The number of stars in the universe is even' and 'It is not the case that the number of stars in the universe is even', but from that it does not follow that, if I want to expand my present theory "according to logic", I should accept either of those sentences—presumably, I should rather accept the logical consequences of what I already accept, which include neither of those sentences.) As Beall would put it, one can "choose" either to accept ψ or to accept $\varphi \,\&\, \neg\varphi$ (from which, given Beall's package, it then does follow that one should accept ψ), but nothing in Beall's package accounts for why that *choice* is not merely a *whim*. I emphasise that I don't take this point to go against the broad spirit of Beall's proposal, but I do take it to show that the proposal needs to rely on a *stronger* normative principle than the acceptance/rejection principle Beall settles for (obviously, I regard the point as an instance of the general problem that that acceptance/rejection principle *puts no pressure whatsoever on accepting any logical consequence of anything one accepts*; we'll see in fn 43 another case in which reliance on the principle might lead astray).

[10]No, I haven't forgotten **DEDUCTION THEOREM**, which, for reasons that will become clear in section 4, is just really hard to get in analetheic theories of truth.

tion) are actually played by at least two different operations. Isn't that just one of the most glaring examples of the obtusity so characteristic of classical thinking? However, in this paper I'll argue that *the existence of a certain very familiar and all-important operation—restricted quantification—does in fact require the combination of all the Functions*. In particular, I'll argue that such combination is jointly required by certain extremely plausible principles of restricted quantification, and that a wide variety of naive theories of truth cannot validate all those principles. I'll then investigate in the relevant respects the theory of restricted quantification available in my favoured naive theory, showing that it validates all those principles.

2 Restricted Quantification

The world is so varied that we only rarely wish to say so much as that *every object whatsoever* is G[11]—given the extreme variety of worldly objects, that would usually be *too strong to be true*. Much more often, we rather wish to *limit* our claim only to a certain specific collection of objects, the Fs,[12] and so merely say that *every* F is G. We wish to express a *restricted universal quantification*. Dually, the world is so varied that we only rarely wish to say merely that *something or other* is G—given the extreme variety of worldly objects, that would usually be *too weak to be interesting*. Much more often, we rather wish to *apply* our claim to a certain specific collection of objects, the Fs, and so say that *some* F is G. We wish to express a *restricted particular quantification*. Conversely, the world is so varied that we only rarely wish to say so much as that *nothing whatsoever* is G—given the extreme variety of worldly objects, that would usually be *too strong to be true*. Much more often, we rather wish to *limit* our claim only to a certain specific collection of objects, the Fs, and so merely say that *no* F is G. We wish to express a *restricted null quantification*.

The usual quantifiers of logics are not immediately fit for these purposes, since both their *model theory* and *proof theory* only make sense if they are supposed to quantify over every object in the *domain of discourse*—i.e., roughly, the collection of objects *concerned some way or other by one's talk in a certain situation* (which is *more stable* than the collection of objects *one is quantifying over at a certain point of one's talk in a certain situation*: for example, when one does arithmetic the domain of discourse is typically the set of natural numbers, even when one says that every even number is divisible by 2 while every odd number is not, and so even one is quantifying over the even numbers at one point and over the odd numbers at another point.). One might think that one could obviate to this problem by imposing the condition that the domain of discourse consist

[11]Throughout, I use the copula in such constructions only for ease of expression—I don't mean to suggest that a verb phrase like, for example, 'reached the summit' is in some sense equivalent to some verb phrase of the form 'is such-and-such'.

[12]There is an issue as to whether one restricts, *more extensionally*, to *collections of objects* or, *more intensionally*, to *properties* (see Stanley and Szabó [2000], p. 252). Throughout, I use either 'collection'-talk or 'property'-talk merely on the basis of which of the two is more presentationally convenient on a given occasion, but all the points I'll be making are eventually neutral with respect to this issue.

in the collection of Fs, and say that every F is G by saying that everything is G, thereby exploiting the restriction that has thus been imposed on the domain of discourse over which 'everything' quantifies.[13] That measure would indeed allow us to say that every F is G, but it would do so only at the cost of fixing *our whole talk in a certain situation* to be about the Fs, which would seem to go against the *flexibility* that seems demanded by our restrictedly quantifying talk (for example, sometimes we wish to *relax* the current restriction, as when we wish to say that [every F and H is G but it is not the case that every F is G],[14] and some other times we wish to *tighten* the current restriction, as when we wish to say that [it is not the case that every F is G but every F and H is G]). One could rejoin that, contrary to what this objection presupposes, restrictions on the domain of discourse are highly volatile and can change *mid-sentence*. There are possible language users who may indeed find that a convenient way of talking and communicating (either in English or, if English does not allow mid-sentence context shift, in some other language that does allow such a shift; see Zardini [2014a] for an initial investigation into such a kind of language). However, for language users such as we are, for whom it would often be extremely clumsy to mark non-linguistically in talk and communication all the required fluctuations in the restriction on the domain of discourse, a restriction on quantification that is *at least in part syntactically realised* rather than *wholly non-syntactically achieved by imposing a contextual restriction on the domain of discourse* is often much more convenient. In fact, the problem with the latter kind of restriction is not just *convenience*, but *expressivity*: to mention a well-known style of example (going back at least as far as Heim [1991], p. 508), it is hard to see how 'everything' in 'In every suitcase, everything is in order' can be interpreted in the intended way (which requires the things that are supposed to be in order to vary with the suitcases) by simply appealing to a contextual restriction on the domain of discourse.

Let's assume then that *restricted quantification* will be *at least in part syntactically realised*, and let's further assume that such realisation will involve *variable binding* of the obvious kind. We can thus write the resulting paradigmatic constructions of restricted quantification considered by *traditional syllogistic* 'Every F is G', 'Some F is G' and 'No F is G'[15,16] as $\mathcal{E}\xi(F\xi, G\xi)$, $\mathcal{S}\xi(F\xi, G\xi)$ and $\mathcal{N}\xi(F\xi, G\xi)$ respectively (see fn 56 for some

[13]For conciseness, I only develop a bit the dialectic of this paragraph with respect to restricted universal quantification. Dual and converse considerations would of course apply for restricted particular quantification and restricted null quantification respectively.

[14]Throughout, I use square brackets to disambiguate constituent structure.

[15]Throughout, *pace* Strawson [1952] and the ensuing *presuppositional* theory of determiners (see e.g. McCawley [1972]), I assume an understanding of 'Every F is G' and 'No F is G' such that they both can be true even if there are no Fs.

[16]To make this completely explicit, it should by now be clear that I understand the *unrestricted-quantification/restricted-quantification* distinction to be the distinction between quantifying over every object in the domain of discourse and quantifying only over some proper subdomain of that domain. The domain of discourse may itself not include absolutely every object, and so the unrestricted-quantification/restricted-quantification distinction is different from the *absolute-generality/relative-generality* distinction. The restriction may not be syntactically realised, and so the unrestricted-quantification/restricted-quantification distinction is different from the *unary-quantifier/binary-quantifier* distinction (see fn 18 for more details on the last distinction). With regard to such understanding of the unrestricted-quantification/restricted-quantification distinction, I should also note that it is actually very

use of the remaining paradigmatic construction of restricted quantification considered by traditional syllogistic 'Not every F is G').[17] These two assumptions by themselves do not commit us to any specific representation of the syntactic structure of the last three formulae (or of their semantics). There is however a logically standard—and relatively conservative—way of representing such restricted quantifications, which makes do with just the usual *unrestricted quantifiers* $\forall$ and $\exists$, the trick basically consisting in their governing appropriate *conditional* or *conjunctive* connectives:

`Translation`

(i) $\mathcal{E}\xi(\varphi, \psi)$ is represented as $\forall\xi(\varphi \to \psi)$;

(ii) $\mathcal{S}\xi(\varphi, \psi)$ is represented as $\exists\xi(\varphi \mathbin{\&} \psi)$;

(iii) $\mathcal{N}\xi(\varphi, \psi)$ is represented as $\neg\exists\xi(\varphi \mathbin{\&} \psi)$.

(putting arbitrary formulae φ and ψ for the atomic formulae in one free variable $F\xi$ and $G\xi$). I'll assume that `Translation` is correct; in the case of `Translation` (i), this is to be understood in the sense that, for some but possibly not every conditional connective $\to$ available in the relevant theory, $\mathcal{E}\xi(\varphi, \psi)$ is represented in the theory as $\forall\xi(\varphi \to \psi)$ (see fn 56 for some discussion of `Translation` (iii)).[18]

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important for the purposes of this paper to understand quantifying claims restricted to the Fs as claims which, *talking only about the Fs, simply say that everything/something/nothing etc. is G*: in such claims, the only property that is *predicated* is the property of being G, while the property of being F rather fixes the *subject* of the predication. I think that, once restricted quantification is understood along these very natural and appealing lines, many principles that I'll be discussing and that might still have been resisted when cast in terms of implication become virtually irresistible (see for example my justification of **RESTRICTED UNIVERSAL INSTANTIATION**$^{\text{r}}$ in section 3). Thanks to Gonçalo Santos for urging these clarifications.

[17] I say '*at least in part* syntactically realised' because, for example, an utterance of 'Every student has arrived' is in turn naturally understood as being *further restricted* to the set of contextually relevant students, and the status of such further restriction is a matter of debate (see the discussion in Stanley and Szabó [2000]).

[18] While extremely plausibly *truth-conditionally* correct, `Translation` suffers from a certain *intensional* artificiality. For, say, it represents a restricted universal quantification $\mathcal{E}\xi(F\xi, G\xi)$ as *predicating a property for every object whatsoever* (namely, the property of not being both F and $\neg G$), while, intuitively, such a quantification *only predicates a property for every object that is F* (namely, the property of being G). Moreover, especially in linguistically oriented investigations, it is now preferred to represent, say, $\mathcal{E}\xi(\varphi, \psi)$ not with the usual *unary* quantifier $\forall$, but with a *binary* quantifier $\forall^*$, which can be semantically defined along the following somehow rough lines:

($\forall^*$) $\forall^*\xi\langle\varphi; \psi\rangle$ is true relative to an assignment ass iff $\{x : \varphi$ is true relative to $\mathsf{ass}_{x/\xi}\} \subseteq \{x : \psi$ is true relative to $\mathsf{ass}_{x/\xi}\}$

(where, as usual, $\mathsf{ass}_{x/\xi}$ is the assignment that differs from ass at most for assigning x to ξ). Quantifiers such as $\forall^*$ belong to the theory of *generalised quantifiers* (initiated by Mostowski [1957] and Lindström [1966] and first applied to natural languages by Barwise and Cooper [1981]), which contemplates quantifiers expressing relations of arbitrary arity among relations of arbitrary arity ($\forall^*$ being the special case of the quantifier expressing the binary relation of *inclusion* between properties). Contrary to attempts at defining it in terms of a unary quantifier, the theory of generalised quantifiers can give a straightforward

For our purposes, it will be useful to give a (non-exhaustive) list of what seem to me extremely plausible principles for restricted quantification (which I'll refer to as 'the *Principles*'):[19]

SELF-INCLUSION$^{\mathrm{l}}$ $\oslash \vdash \mathcal{E}\xi(\varphi, \varphi)$ holds;

SELF-INCLUSION$^{\mathrm{c}}$ $\mathcal{E}\xi(\varphi, \varphi) \vdash \oslash$ does not hold;

NO ALL-IN-NOTHING INCLUSION If $\oslash \vdash \varphi$ and $\psi \vdash \oslash$ hold, then $\oslash \vdash \mathcal{E}\xi(\varphi, \psi)$ does not hold;

NO COUNTER-INSTANCE$^{\Rightarrow}$ $\neg\mathcal{S}\xi(\varphi, \neg\psi) \vdash \mathcal{E}\xi(\varphi, \psi)$ and $\neg\mathcal{S}\xi(\varphi, \psi) \vdash \mathcal{E}\xi(\varphi, \neg\psi)$ hold;

NO COUNTER-INSTANCE$^{\Leftarrow}$ $\mathcal{E}\xi(\varphi, \psi) \vdash \neg\mathcal{S}\xi(\varphi, \neg\psi)$ and $\mathcal{E}\xi(\varphi, \neg\psi) \vdash \neg\mathcal{S}\xi(\varphi, \psi)$ hold;

RESTRICTED UNIVERSAL GENERALISATION$^{\mathrm{mr}}$ If $\Gamma, \varphi \vdash \Delta, \psi$ holds and ξ does not occur free in Γ or Δ, then $\mathsf{qua}(\Gamma) \vdash \mathsf{qua}(\Delta), \mathcal{E}\xi(\varphi, \psi)$ holds;[20]

RESTRICTED UNIVERSAL GENERALISATION$^{\mathrm{mc}}$ If $\varphi \vdash \psi$ holds, then $\mathcal{E}\xi(\varphi, \psi) \vdash \oslash$ does not hold;

RESTRICTED UNIVERSAL INSTANTIATION$^{\mathrm{r}}$ $\varphi_{\tau/\xi}, \mathcal{E}\xi(\varphi, \psi) \vdash \psi_{\tau/\xi}$ holds;

RESTRICTED UNIVERSAL INSTANTIATION$^{\mathrm{ml}}$ If $\oslash \vdash \varphi_{\tau/\xi}$ and $\oslash \vdash \mathcal{E}\xi(\varphi, \psi)$ hold, then $\oslash \vdash \psi_{\tau/\xi}$ holds.

representation of the natural-language quantification 'Most Fs are Gs'. And the success of the theory at treating the latter quantification (and other similar cases) plus considerations of uniformity are one of the main reasons for why, in these linguistically oriented investigations, it is now preferred to represent $\mathcal{E}\xi(\varphi, \psi)$ as $\forall^*\xi\langle\varphi; \psi\rangle$. (Not that this by itself solves the first problem for `Translation` mentioned in this fn.) However, while, in a classical framework, the intended semantics of $\forall^*$ can straightforwardly be formulated by a clause like ($\forall^*$), once one moves to a non-classical framework substantial complications may be required in order to achieve the same effect (especially if one is working with a naive theory of truth), and these complications will most likely *repeat the pattern and options* exhibited by the search for an implication satisfying `Translation` (i). Moreover, the theory of generalised quantifiers does not at all put into question that `Translation` is at least *truth-conditionally correct*—in fact, at least in a classical framework, it implies that. For these reasons, I think that, for our purposes, we can safely conduct our discussion under the assumption of `Translation`.

[19]As usual, φ_{τ_0/τ_1} is the result of substituting τ_0 for all free occurrences of τ_1 in φ, a result that possibly involves renaming variables in φ so that τ_0 is free for τ_1 in φ (formulae are identified up to α-conversion). Also, the indicative superscripts 'l', 'c', 'r' and 'm' are inspired respectively from '*law*' (a sentence that is logically necessary), '*consistent*' (a sentence that is not logically absurd), '*rule*' (an argument from premises to conclusions) and '*metarule*' (an implication among claims about $\vdash$).

[20]Where $\mathsf{qua}(\Gamma)$ is some natural function of Γ. Given the variety with which non-classical logics (and in particular substructural ones) treat *side* premises and conclusions, it is reasonable to leave this much of a leeway in the formulation of the relevant principles. This being noted, the reader may safely assume until section 6 that `qua` just is *identity*. In fact, the particular applications of `qua`-involving principles made in this paper do *not* have side premises or conclusions, although I still offer the official formulations with side premises and conclusions in order to give a better sense of the *extent* of such principles.

The Principles are so plausible that, in this paper, I'll *directly* proceed to use them as a touchstone for theories of restricted quantification (I don't mean to deny that they can also be provided some *argumentative justification*, and, in effect, I'll do so explicitly for **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ and **RESTRICTED UNIVERSAL GENERALISATION**$^{\mathrm{mr}}$ in sections 3 and 4 respectively and more implicitly for **NO ALL-IN-NOTHING INCLUSION** and **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{ml}}$ in section 5). Clearly, under `Translation` (i)–(ii), each Function corresponds to a set of Principles,[21] and so it is no surprise that, under `Translation` (i)–(ii), classical logic (whose implication, as I've mentioned in section 1, satisfies all the Functions) validates all the Principles.

3 Analetheic and Dialetheic Restricted Quantification

We've already established in section 1 that neither analetheic nor dialetheic theories of truth can allow for the combination of *intensional* and *extensional* Functions **SUFFICIENT CONDITION**$^{\Rightarrow}$, **NO REFUTATION**$^{\Rightarrow}$, **NO REFUTATION**$^{\Leftarrow}$ and **MODUS PONENS**. At a first glance, that would seem merely an unproblematic if peculiar feature of those theories. It now needs to be pointed out that, far from that, such combination is absolutely crucial to the workings of restricted quantification as spelt out by the Principles.

Clearly, under `Translation` (i), **SELF-INCLUSION**$^{\mathrm{l}}$ requires the operative implication to satisfy **SUFFICIENT CONDITION**$^{\Rightarrow}$, thus creating trouble, in an analetheic theory of truth, for material implication representing restricted universal quantification. Things don't improve much once we turn to the typical non-material implication employed by analetheic theories, for, as we've seen in section 1, that implication does not satisfy **NO REFUTATION**$^{\Leftarrow}$, and so, under `Translation` (i)–(ii), these theories will not validate **NO COUNTER-INSTANCE**$^{\Leftarrow}$.

In fact, the situation here is *unimprovably bad* for analetheic theories, for, as we've seen in section 1, these theories cannot accept any implication that satisfies both **SUFFICIENT CONDITION**$^{\Rightarrow}$ and **NO REFUTATION**$^{\Leftarrow}$. Back then, this might have seemed to the reader merely an unproblematic if peculiar feature of those theories: *prima facie*, there do not seem to be substantial reasons for requiring that one and the same implication satisfy both Functions, and the fact that classical material implication does it might *prima facie* be ascribed merely to the obliteration of apparent differences which is so typical of classical logic. So much so that, in the intuitive if rough-and-ready sense mentioned in fn 3, while one Function (**SUFFICIENT CONDITION**$^{\Rightarrow}$) has a more-intensional-than-extensional flavour, the other one (**NO REFUTATION**$^{\Leftarrow}$) has

[21] In fact, although the *correspondence* is not so strict as to amount to an *equivalence*, for the purposes of the issues and theories discussed in this paper it can be assumed (and I'll do so assume) that, under `Translation` (i)–(ii), each Function *stands or falls together* with the relevant set of Principles.

a more-extensional-than-intensional flavour. We now see that this *combination of intensional and extensional Functions*, far from simply being an unlikely cross-breed generated by the hybris of classical logic, is actually of the essence for a very familiar and all-important operation (restricted quantification) that is so pervasive and crucial in our thought and talk. Because analetheic theories cannot accept any implication that satisfies both **SUFFICIENT CONDITION**$^{\Rightarrow}$ and **NO REFUTATION**$^{\Leftarrow}$, it follows that, under `Translation` (i)–(ii), they simply cannot validate both **SELF-INCLUSION**$^{\mathrm{l}}$ (or even **SELF-INCLUSION**$^{\mathrm{c}}$)[22] and **NO COUNTER-INSTANCE**$^{\Leftarrow}$, and this—I reckon—is a great cost of these theories.[23]

It might be suggested that restricted universal quantification, no less than implication, *also* splits up into at least two different operations, a more extensional one and a more intensional one. For example, there certainly is an inclination to read 'Every body exerts gravity' as stating some sort of *law-like* correlation between the property of being a body and the property of exerting gravity, and so as requiring more for its truth than the circumstance that, *as a matter of fact*, every body just so happens to exert gravity. And, if this suggestion is correct, it might then be objected that this offers the beginnings of an argument for adopting in the case of restricted quantification a "divide-and-conquer" strategy analogous to the one that analetheic theories are used to pursuing in the case of implication. In reply, there is no need to enter the (very interesting but also very difficult) debate as to whether there really is an operation of "law-like" restricted universal quantification besides an operation of "as-a-matter-of-fact" restricted universal quantification, nor any need to insist that **NO COUNTER-INSTANCE**$^{\Leftarrow}$ would seem to be no less plausible for the former than it is for the latter (if there is a law-like correlation between the property of being F and the property of being G, one would expect that there are no counter-instances to it; arguably simply put in other words, a law-like restricted universal quantification would seem to entail the corresponding as-a-matter-of-fact restricted universal quantification).[24] For, granting that both operations exist, this paper should then be understood as *concerned throughout with as-a-matter-of-fact restricted universal quantification*, and *the Principles do not lose any of their extreme plausibility when so disambiguated* (for example, as for **SELF-INCLUSION**$^{\mathrm{l}}$, that Principle does not lose any of its extreme plausibility when so disambiguated as to mean that, as a matter of fact, every F just so happens to be F).[25]

[22]Reason for the parenthetical strengthening: on these theories, for some paradoxical φ, $\varphi \vee \neg\varphi \vdash \oslash$ (and hence $\neg(\varphi \,\&\, \neg\varphi) \vdash \oslash$) holds, and so, if **NO COUNTER-INSTANCE**$^{\Leftarrow}$ holds, even **SELF-INCLUSION**$^{\mathrm{c}}$ has to fail.

[23]In fact, the limitative argument can be run directly in terms of **SELF-INCLUSION**$^{\mathrm{l}}$ and **NO COUNTER-INSTANCE**$^{\Leftarrow}$ (following very much the pattern of the argument I've given in the case of **SUFFICIENT CONDITION**$^{\Rightarrow}$ and **NO REFUTATION**$^{\Leftarrow}$), in which case only `Translation` (ii) is needed. A similar comment applies for other arguments to follow.

[24]A sentence like 'A tiger has four legs' would seem true, even if there may be counter-instances to it. But it seems unlikely that such a sentence expresses a straightforward *universal* quantification rather than a more *hedged* kind of quantification (as witnessed by its contrast with 'Every tiger has four legs'), and so it seems unlikely that we have here the materials for a counterexample to **NO COUNTER-INSTANCE**$^{\Leftarrow}$ (or to **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$).

[25]Thanks to Andreas Fjellstad, Mikkel Gerken, Matteo Plebani and Crispin Wright for raising this

Clearly, under `Translation` (i), **RESTRICTED UNIVERSAL INSTANTIATION**$^{\text{r}}$ requires the operative implication to satisfy **MODUS PONENS**, thus creating trouble, in a dialetheic theory of truth, for material implication representing restricted universal quantification. Things don't improve much once we turn to the typical non-material implication employed by dialetheic theories, for, as we've seen in section 1, that implication does not satisfy **NO REFUTATION**$^{\Rightarrow}$, and so, under `Translation` (i)–(ii), these theories will not validate **NO COUNTER-INSTANCE**$^{\Rightarrow}$.

In fact, the situation here is *unimprovably bad* for dialetheic theories of truth, for, as we've seen in section 1, these theories cannot accept any implication that satisfies both **NO REFUTATION**$^{\Rightarrow}$ and **MODUS PONENS**. Back then, this might have seemed to the reader merely an unproblematic if peculiar feature of those theories: *prima facie*, there do not seem to be substantial reasons for requiring that one and the same implication satisfy both Functions, and the fact that classical material implication once again does it might once more *prima facie* be ascribed merely to the obliteration of apparent differences which is so typical of classical logic. So much so that, in the intuitive if rough-and-ready sense mentioned in fn 3, while one Function (**NO REFUTATION**$^{\Rightarrow}$) falls squarely on the extensional side, the other one (**MODUS PONENS**) has a more-intensional-than-extensional flavour. We now see that also this other *combination of extensional and intensional Functions*, far from simply being an unlikely cross-breed generated by the hybris of classical logic, is actually of the essence for a very familiar and all-important operation (restricted quantification) that is so pervasive and crucial in our thought and talk. Because dialetheic theories cannot accept any implication that satisfies both **NO REFUTATION**$^{\Rightarrow}$ and **MODUS PONENS**, it follows that, under `Translation` (i)–(ii), they simply cannot validate both **NO COUNTER-INSTANCE**$^{\Rightarrow}$ and **RESTRICTED UNIVERSAL INSTANTIATION**$^{\text{r}}$ (or even **RESTRICTED UNIVERSAL INSTANTIATION**$^{\text{ml}}$),[26] and this—I reckon—is a great cost of these theories.

Notice that comments analogous to those made in the third last paragraph apply in this case (with the unremarkable exception that **NO COUNTER-INSTANCE**$^{\Rightarrow}$ is indeed much less plausible for law-like restricted universal quantification than it is for as-a-matter-of-fact restricted universal quantification).[27] Since, at least from the perspective of dialetheic theories of truth, it might be thought that **RESTRICTED UNIVERSAL INSTANTIATION**$^{\text{r}}$ is less plausible for as-a-matter-of-fact restricted universal quantifi-

issue.

[26]Reason for the parenthetical strengthening: on these theories, for some paradoxical φ, both $\oslash \vdash \varphi$ and $\oslash \vdash \neg\varphi$ (and hence $\oslash \vdash \neg(\varphi \,\&\, \neg\psi)$, with ψ arbitrary) hold, and so, if **NO COUNTER-INSTANCE**$^{\Rightarrow}$ holds, even **RESTRICTED UNIVERSAL INSTANTIATION**$^{\text{ml}}$ has to fail.

[27]The exception is (for our purposes) unremarkable because it gives no consolation whatsoever to dialetheic theories of truth, since a weakening of **NO COUNTER-INSTANCE**$^{\Rightarrow}$ with suitably "*necessitated*" premises is just as plausible for law-like restricted universal quantification as it is for as-a-matter-of-fact restricted universal quantification, but the facts appealed to in fn 26 show that, under `Translation` (i)–(ii), dialetheic theories simply cannot validate both such weakening of **NO COUNTER-INSTANCE**$^{\Rightarrow}$ and **RESTRICTED UNIVERSAL INSTANTIATION**$^{\text{r}}$ (or even **RESTRICTED UNIVERSAL INSTANTIATION**$^{\text{ml}}$).

cation than it is for law-like restricted universal quantification (as it might be thought that its failure for the former can be made good sense of by **NO COUNTER-INSTANCE**$^{\Rightarrow}$ together with $\varphi_{\tau/\xi}$ both holding and not holding), let me offer some argumentative justification in its favour. The following version of the *dictum de omni*:

(DO$_0$) ''Every F is G' says at least implicitly of x that it is G' is at least as weak as 'x is F'

is extremely plausible: provided that there are Fs (which one can always infer from x being F), one can always infer that to restrict to the Fs is to say things *of* them, and, provided that something says of them that they are *all* G, one can always infer that that thing says at least implicitly of *each* of them that it is G. But, under minimal assumptions, (DO$_0$) entails **RESTRICTED UNIVERSAL INSTANTIATION**$^{\text{r}}$.[28]

Reflection on restricted quantification brings also further insights into controversial issues concerning the logic of implication which are problematic for analetheic and dialetheic theories of truth. For example, both kinds of theories typically reject for their favoured non-material implication both the principle of *importation*:

(IMP) $\varphi \to (\psi \to \chi) \vdash \varphi \,\&\, \psi \to \chi$ holds

and the converse principle of *exportation*:

(EXP) $\varphi \,\&\, \psi \to \chi \vdash \varphi \to (\psi \to \chi)$ holds.

Consider however the principle of *quantificational import/export*:

(QIMPEXP) $\mathcal{E}\xi(\varphi, \psi \to \chi) \dashv\vdash \mathcal{E}\xi(\varphi \,\&\, \psi, \chi)$ holds

(where the implication in the first sentence is the same as the one that is operative in `Translation` (i)). (QIMPEXP) is not only in itself plausible; it follows if we accept the extremely plausible principle:

(R&/RR) The effect of restricting to the $F \,\&\, G$s is the same as the effect of restricting to the Fs and then restricting to the Gs,

if, as a consequence, we regard 'Every F and G is H' as being tantamount to restricting to the Gs an Fs-restricted quantification (just as 'Every F is H' restricts to the Fs an unrestricted quantification) and, finally, if we impose a "higher-order" analogue of `Translation` (i) for the latter "doubly restricted" quantification. A little more explicitly and formally, the idea is as follows. Generally, and a bit roughly, let $\mathcal{Q}\xi(\varphi_0 \downarrow \varphi_1 \downarrow \varphi_2 \ldots \downarrow$

[28] Thanks to Franz Berto for discussion of this point.

$\varphi_i, \psi)$ express the result of restricting to φ_0-satisfying objects (when assigned to ξ by an assignment that is a ξ-variant of the current assignment), then further restricting to the φ_1-satisfying objects (when assigned to ξ by an assignment that is a ξ-variant of the current assignment), then further restricting to the φ_2-satisfying objects (when assigned to ξ by an assignment that is a ξ-variant of the current assignment) $\ldots$, then further restricting to the φ_i-satisfying objects (when assigned to ξ by an assignment that is a ξ-variant of the current assignment) and then quantifying in the way expressed by $\mathcal{Q}$ (universal, particular, null or what have you) over the resulting objects with ψ being the quantified-in formula. Given (R&/RR), we thus regard $\mathcal{E}\xi(\varphi \,\&\, \psi, \chi)$ as being tantamount to $\mathcal{E}\xi(\varphi \downarrow \psi, \chi)$. We then accept the higher-order analogue of `Translation` (i) to the effect that $\mathcal{E}\xi(\varphi \downarrow \psi, \chi)$ (i.e. a restricted universal quantification that, having restricted to φ-satisfying objects, restricts to ψ-satisfying objects) is represented as $\mathcal{E}\xi(\varphi, \psi \rightarrow \chi)$ (where $\rightarrow$ expresses the same implication as the one that is operative in `Translation` (i)). (QIMPEXP) follows. However, clearly, under `Translation` (i) (QIMPEXP) requires the operative implication to satisfy both (IMP) and (EXP), thus creating trouble, in both analetheic and dialetheic theories, for their favoured non-material implication representing restricted universal quantification.[29]

4 Non-Substructural Restricted Quantification

I now wish to generalise the discussion of the last section to *all* naive theories of truth accepting all the traditional *structural* properties (most saliently, transitivity and contraction). I'm going to argue that, under `Translation` (i)–(ii), none of these theories can validate all the Principles, thereby revealing a great cost of all these theories. In turn, this inability stems from the fact that they cannot allow for the combination of *intensional* Functions **DEDUCTION THEOREM** and **MODUS PONENS** (I'll also offer a variation of the argument in terms of restricted null quantification and negation).

Let's assume that there is a sentence $\mathfrak{e}$ which is $\mathcal{E}x(T\mathfrak{e}, \mathsf{f})$, where f is an arbitrary logical absurdity (like $0 \neq 0$). We start with establishing a first lemma:

$$\dfrac{\dfrac{}{T\mathfrak{e} \vdash \mathcal{E}x(T\mathfrak{e}, \mathsf{f})}\ \textbf{T-ELIMINATION} \qquad \dfrac{}{T\mathfrak{e}, \mathcal{E}x(T\mathfrak{e}, \mathsf{f}) \vdash \mathsf{f}}\ \textbf{RESTRICTED UNIVERSAL INSTANTIATION}^{\mathrm{r}}}{\dfrac{T\mathfrak{e}, T\mathfrak{e} \vdash \mathsf{f}}{T\mathfrak{e} \vdash \mathsf{f}}\ \text{contraction}}\ \text{transitivity}$$

[29] Both analetheic and dialetheic theories also typically reject for their favoured non-material implication the principle of *permutation*:

(PERM) $\varphi \rightarrow (\psi \rightarrow \chi) \vdash \psi \rightarrow (\varphi \rightarrow \chi)$ holds.

Obviously, an argument similar to the one in the text (relying on the higher-order analogue of `Translation` (i) and either (R&/RR) plus the commutativity of conjunction or, more straightforwardly, on the commutativity of restriction) can be run to the effect that restricted universal quantification requires the operative implication to satisfy (PERM), thus creating trouble, in both analetheic and dialetheic theories, for their favoured non-material implication representing restricted universal quantification.

We then proceed to establishing a second lemma:

$$\dfrac{\dfrac{\dfrac{}{T\mathfrak{e} \ \vdash \ \mathsf{f}}\,\text{lemma 1}}{\oslash \ \vdash \ \mathcal{E}x(T\mathfrak{e},\mathsf{f})}\,\textbf{RESTRICTED UNIVERSAL GENERALISATION}^{\text{mr}} \qquad \dfrac{}{\mathcal{E}x(T\mathfrak{e},\mathsf{f}) \ \vdash \ T\mathfrak{e}}\,\textbf{T-INTRODUCTION}}{\oslash \ \vdash \ T\mathfrak{e}}\,\text{transitivity}$$

Putting the two lemmas together, we get a catastrophe:

$$\dfrac{\dfrac{}{\oslash \ \vdash \ T\mathfrak{e}}\,\text{lemma 2} \qquad \dfrac{}{T\mathfrak{e} \ \vdash \ \mathsf{f}}\,\text{lemma 1}}{\oslash \ \vdash \ \mathsf{f}}\,\text{transitivity}$$

(let's call this argument 'paradox A').

Some theorists are likely to try to block paradox A by rejecting **RESTRICTED UNIVERSAL GENERALISATION**$^{\text{mr}}$. However, that principle is extremely plausible, even more so in the special case in which, with $\Gamma = \Delta = \oslash$, it reduces to the principle that, if $\varphi \vdash \psi$ holds, then $\oslash \vdash \mathcal{E}\xi(\varphi, \psi)$ holds (which is all is in fact needed in paradox A)—if the property of being F entails the property of being G, every F is G.

Moreover, **RESTRICTED UNIVERSAL GENERALISATION**$^{\text{mr}}$ is the obvious modification for restricted universal quantification of the metarule:

UNRESTRICTED UNIVERSAL GENERALISATION If $\Gamma \vdash \Delta, \varphi$ holds and ξ does not occur free in Γ or Δ, then $\mathsf{qua}(\Gamma) \vdash \mathsf{qua}(\Delta), \forall\xi\varphi$ holds,

which is one of the basic metarules for unrestricted universal quantification, not only in a classical framework, but also in naive theories of truth. Over and above the extreme plausibility of **RESTRICTED UNIVERSAL GENERALISATION**$^{\text{mr}}$, I now wish to stress that it is difficult to see how it can be rejected in a principled way once its companion has been accepted.

It is usual to gloss informally **UNRESTRICTED UNIVERSAL GENERALISATION** (in the particular case in which, say, φ is $G\xi$) by saying that, if one has derived $G\xi$ making *no assumption* about ξ, one can derive $\forall\xi G\xi$.[30] However, the way in which quantificational reasoning proceeds in classical logic and in naive theories of truth makes it clear that it is in effect implicitly assumed about ξ that it denotes some object or other that is in the domain of discourse (which is the domain of the unrestricted quantifiers $\forall$ and $\exists$). That is reflected most notably in the fact that one is allowed to instantiate an unrestricted universal quantification with ξ and unrestrictedly to generalise particularly on ξ, both of which would make little sense if it were not implicitly assumed about ξ that

[30] Sometimes, as in the last sentence in the text, in order to avoid unnecessary clutter I omit mention of side premises or conclusions.

it denotes some object or other that is in the domain of discourse.[31] Hence, the informal gloss on **UNRESTRICTED UNIVERSAL GENERALISATION** can equally well be put by saying that, if one has derived $G\xi$ making *the only assumption* about ξ that it denotes some object or other *that is in the domain of discourse*, one can derive $\forall\xi G\xi$—that is, one can derive that every object *that is in the domain of discourse* is G. But now the analogy with **RESTRICTED UNIVERSAL GENERALISATION**$^{\mathrm{mr}}$ becomes irresistible, for that metarule can be informally glossed (in the particular case in which, say, φ is $F\xi$ and ψ is $G\xi$) by saying that, if one has derived $G\xi$ making *the only assumption* about ξ that it denotes some object or other *that is* F, one can derive $\mathcal{E}\xi(F\xi, G\xi)$—that is, one can derive that every object *that is* F is G.[32] If it is correct to infer that every object *that is in the domain of discourse* is human from the fact that one has derived that x is human making the only assumption about x that *she is in the domain of discourse*, surely it must be equally correct to infer that every object *that is a philosopher* is human from the fact that one has derived that x is human making the only assumption about x that *she is a philosopher*.

It is instructive to compare paradox A with one of the most arresting versions of *Curry's paradox* (see Curry [1942]). Let's assume that there is a sentence $\mathfrak{c}$ which is $T\mathfrak{c} \to \mathsf{f}$. We start with establishing a third lemma:

$$\dfrac{\dfrac{\dfrac{}{T\mathfrak{c} \ \vdash \ T\mathfrak{c} \to \mathsf{f}}\,\textbf{T-ELIMINATION} \qquad \dfrac{}{T\mathfrak{c}, T\mathfrak{c} \to \mathsf{f} \ \vdash \ \mathsf{f}}\,\textbf{MODUS PONENS}}{T\mathfrak{c}, T\mathfrak{c} \ \vdash \ \mathsf{f}}\,\text{transitivity}}{T\mathfrak{c} \ \vdash \ \mathsf{f}}\,\text{contraction}$$

We then proceed to establishing a fourth lemma:

[31]Think for example of a standard classical derivation of $\forall x(Fx \vee Gx)$ from $\forall y Fy$. From $\forall y Fy$ holding by assumption it follows by *unrestricted universal instantiation* that Fx holds, and so, by addition, that $Fx \vee Gx$ holds. That depends on no explicit assumptions about x, and so it follows by **UNRESTRICTED UNIVERSAL GENERALISATION** that $\forall x(Fx \vee Gx)$ holds (on the assumption that $\forall y Fy$ holds). Clearly, the unrestricted-universal-instantiation step in this derivation is only good if it is implicitly assumed about x that it denotes some object or other that is in the domain of discourse of $\forall$. Or think for example of a standard classical derivation of $\forall x \exists y(Fy \supset Fx)$. From $Fx \supset Fx$ holding in virtue of being logically necessary it follows by *unrestricted particular generalisation* that $\exists y(Fy \supset Fx)$ holds. That depends on no explicit assumptions about x, and so it follows by **UNRESTRICTED UNIVERSAL GENERALISATION** that $\forall x \exists y(Fy \supset Fx)$ holds. Clearly, the unrestricted-particular-generalisation step in this derivation is only good if it is implicitly assumed about x that it denotes some object or other that is in the domain of discourse of $\exists$.

[32]Notice that an analogous argumentative justification of a principle in terms of its being the obvious modification for restricted quantification of a basic principle of unrestricted quantification can be given for the other principles of *instantiation* or *no counter-instantiation* failing in the theories of truth discussed in section 3 and in this section. (I should perhaps stress that such argumentative justification does rely on the relevant principle of unrestricted quantification being *basic* and so *extremely abstract*: I see no reason in favour of a *completely general* expectation that less basic and more concrete principles of unrestricted quantification have a corresponding valid obvious modification for restricted quantification—in fact, I'll mention in fn 60 what I take to be a counterexample to such expectation.)

$$\dfrac{\dfrac{\dfrac{}{T\mathfrak{c} \vdash \mathsf{f}}\,\text{lemma 3}}{\oslash \vdash T\mathfrak{c} \to \mathsf{f}}\,\textbf{DEDUCTION THEOREM} \qquad \dfrac{}{T\mathfrak{c} \to \mathsf{f} \vdash T\mathfrak{c}}\,\textbf{T-INTRODUCTION}}{\oslash \vdash T\mathfrak{c}}\,\text{transitivity}$$

Putting the two lemmas together, we get a catastrophe:

$$\dfrac{\dfrac{}{\oslash \vdash T\mathfrak{c}}\,\text{lemma 4} \qquad \dfrac{}{T\mathfrak{c} \vdash \mathsf{f}}\,\text{lemma 3}}{\oslash \vdash \mathsf{f}}\,\text{transitivity}$$

Paradox A is of course fully analogous to Curry's paradox, with **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ and **RESTRICTED UNIVERSAL GENERALISATION**$^{\mathrm{mr}}$ playing the same role in paradox A as **MODUS PONENS** and **DEDUCTION THEOREM** respectively play in Curry's paradox. In fact, the relationship is even stronger, for, clearly, under `Translation` (i) **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ and **RESTRICTED UNIVERSAL GENERALISATION**$^{\mathrm{mr}}$ require the operative implication to satisfy **MODUS PONENS** and **DEDUCTION THEOREM** respectively.

The typical answer that non-substructural naive theories of truth give to Curry's paradox consists in rejecting **DEDUCTION THEOREM** for all their implications (most of them) that satisfy **MODUS PONENS**. In some cases, the situation can be improved a little by adding some implication that does satisfy **DEDUCTION THEOREM**. For example, we've seen in section 1 that, in a dialetheic theory, material implication will typically do so. But the room for improvement here is essentially limited, as Curry's paradox shows that *non-substructural naive theories cannot accept any implication that satisfies both* **DEDUCTION THEOREM** *and* **MODUS PONENS**. I think it's fair to say that this deeply counterintuitive consequence marks an important limitation of all such theories (see Zardini [2011], pp. 516-517; [2013b]; [2014c]; [2014g]; [2014h] for some discussion). Notice that the two Functions in question *fall on the same side of the extensional/intensional divide* (in the intuitive if rough-and-ready sense mentioned in fn 3, they both have an intensional-more-than-extensional flavour), and so the strategy consisting in the fragmentation of the Functions looks even less *prima facie* appealing here than in the cases discussed in section 3.

Still, even such a deeply counterintuitive consequence may after all be accepted, especially if it only ran against intuition and did not compromise other parts of one's logical theory. Unfortunately, consideration of the absolutely vital logical principles of universal generalisation and instantiation shows that the combination of intensional Functions in question is not only intuitively compelling, but is also of the essence for a very familiar and all-important operation (restricted quantification) that is so pervasive and crucial in our thought and talk. Because non-substructural naive theories of truth cannot accept any implication that satisfies both **DEDUCTION THEOREM** and **MODUS PONENS**, it follows that, under `Translation` (i), they simply cannot validate both **RESTRICTED UNIVERSAL GENERALISATION**$^{\mathrm{mr}}$ and **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ and this—I reckon—is a great cost of these theories.

There is an interesting variation on paradox A, which replaces **RESTRICTED UNIVERSAL GENERALISATION**$^{\text{mr}}$ with the equally (i.e. extremely) plausible principle:

UNIVERSAL-PREDICATION CLOSURE If $\Gamma, \psi \vdash \Delta, \chi$ holds and ξ does not occur free in Γ or Δ, then $\mathsf{qua}(\Gamma), \mathcal{E}\xi(\varphi, \psi) \vdash \mathsf{qua}(\Delta), \mathcal{E}\xi(\varphi, \chi)$ holds.

Using this principle, we proceed to establishing a fifth lemma:

$$\dfrac{\dfrac{}{\oslash \vdash \mathcal{E}x(T\mathfrak{e}, T\mathfrak{e})}\,\textbf{SELF-INCLUSION}^{\text{l}} \qquad \dfrac{\dfrac{}{T\mathfrak{e} \vdash \mathsf{f}}\,\text{lemma 1}}{\mathcal{E}x(T\mathfrak{e}, T\mathfrak{e}) \vdash \mathcal{E}x(T\mathfrak{e}, \mathsf{f})}\,\textbf{UNIVERSAL-PREDICATION CLOSURE}}{\oslash \vdash \mathcal{E}x(T\mathfrak{e}, \mathsf{f})}\,\text{transitivity}$$

This gives us the left-hand intermediate conclusion of the subargument establishing lemma 2 in paradox A. Using then the rest of that subargument plus the final subargument of paradox A, we get that $\oslash \vdash \mathsf{f}$ holds (let's call this paradox 'paradox B').

Some theorists are likely to try to block paradox B by rejecting **UNIVERSAL-PREDICATION CLOSURE**. However, that principle is extremely plausible, even more so in the special case in which, with $\Gamma = \Delta = \oslash$, it reduces to the principle that, if $\psi \vdash \chi$ holds, then $\mathcal{E}\xi(\varphi, \psi) \vdash \mathcal{E}\xi(\varphi, \chi)$ holds (which is all is in fact needed in paradox B)—if the property of being G entails the property of being H, every F being G entails every F being H.

Moreover, **UNIVERSAL-PREDICATION CLOSURE** is the obvious modification for restricted universal quantification of the metarule:

SINGULAR-PREDICATION CLOSURE If $\Gamma, \psi \vdash \Delta, \chi$ holds and ξ does not occur free in Γ or Δ, then $\mathsf{qua}(\Gamma), \psi_{\tau/\xi} \vdash \mathsf{qua}(\Delta), \chi_{\tau/\xi}$ holds,

which is one of the usual metarules for predication achieved *via* the employment of singular terms (rather than quantifiers), not only in a classical framework, but also in naive theories of truth. Over and above the extreme plausibility of **UNIVERSAL-PREDICATION CLOSURE**, I now wish to stress that it is difficult to see how it can be rejected in a principled way once its companion has been accepted.

It is natural to gloss informally **SINGULAR-PREDICATION CLOSURE** (in the particular case in which, say, ψ is $G\xi$, χ is $H\xi$ and τ is a) by saying that, if the property of being G entails the property of being H, a being G entails a being H. However, the ultimate extreme plausibility of such principle does not seem to depend on the use of a *singularly referring term* such as a (like 'Ann'). Letting aa be a *plurally referring term* (like 'Ann and Bill'), it is equally (i.e. extremely) plausible that, if the property of being G entails the property of being H, aa being G entails aa being H. As an intermediate case between *referring terms* and *quantifiers*, we can consider *singular* and *plural definite descriptions* (like 'the philosopher' and 'the philosophers'): it is equally (i.e. extremely) plausible that, if the property of being G entails the property of being H, *the* F(s) being G entails *the* F(s) being H. Moving on to *quantifiers* (like 'some philosopher'), it is equally (i.e. extremely) plausible that, if the property of being G entails the property of being H, *some* F being G entails *some* F being H. Indeed, taking the *unrestricted* universal quantifier ('everything'), it is equally (i.e. extremely) plausible that, if the property of being G entails the property of being H, *everything* being G entails *everything* being H. But now the analogy with **UNIVERSAL-PREDICATION CLOSURE** becomes irresistible, for that metarule can be informally glossed (in the particular case in which, say, ψ is $G\xi$, χ is $H\xi$ and φ is $F\xi$) by saying that, if the property of being G entails the property of being H, *every* F being G entails *every* F being H.[33] If it is correct to infer that *the* philosophers, or *some* philosopher, or *everything* being human entails *the* philosophers, or *some* philosopher, or *everything* being an animal from the fact that the property of being human entails the property of being an animal, surely it must be equally correct to infer from that fact that *every philosopher* being human entails *every philosopher* being an animal.[34]

A paradox analogous to paradox A exploits some of the corresponding principles of generalisation and instantiation for $\mathcal{N}$:

RESTRICTED NULL GENERALISATION$^{\text{mr}}$ If $\Gamma, \varphi, \psi \vdash \Delta$ holds and ξ does not occur free in Γ or Δ, then $\mathsf{qua}(\Gamma) \vdash \mathsf{qua}(\Delta), \mathcal{N}\xi(\varphi, \psi)$ holds;

RESTRICTED NULL GENERALISATION$^{\text{mc}}$ If $\varphi, \psi \vdash \oslash$ holds, then $\mathcal{N}\xi(\varphi, \psi) \vdash \oslash$ does not hold;

[33]Of course, this is not to say that analogous metarules where universal quantification is replaced by some other kind of quantification always hold. For example, the fact that the property of being G entails the property of being H does not imply that few Fs being G entails few Fs being H. Speaking a bit roughly, in the theory of generalised quantifiers (see fn 18) the fact that a metarule analogous to **UNIVERSAL-PREDICATION CLOSURE** for a binary quantifier $\mathcal{Q}$ holds is known as the fact that $\mathcal{Q}$ is "*upwards-monotonic in its second argument*". Paradox B shows that, in non-substructural naive theories of truth, binary universal quantification cannot be upwards-monotonic in its second argument. (That being noted, I hasten to add that I haven't written this paper to vindicate all the claims usually made in the theory of generalised quantifiers, see especially fn 61.)

[34]In fact, although **UNIVERSAL-PREDICATION CLOSURE** has just been supported in a very different way from the way in which **RESTRICTED UNIVERSAL GENERALISATION**$^{\text{mr}}$ has earlier been supported, the two metarules stand in very tight relationships. Given the transitivity of $\vdash$ and the transitivity of $\mathcal{E}$ in the form $\mathcal{E}\xi(\varphi, \psi), \mathcal{E}\xi(\psi, \chi) \vdash \mathcal{E}\xi(\varphi, \chi)$, **RESTRICTED UNIVERSAL GENERALISATION**$^{\text{mr}}$ implies **UNIVERSAL-PREDICATION CLOSURE**, while, given the transitivity of $\vdash$ and **SELF-INCLUSION**$^{\text{l}}$, **UNIVERSAL-PREDICATION CLOSURE** implies **RESTRICTED UNIVERSAL GENERALISATION**$^{\text{mr}}$ (and so no wonder that paradox B, contrary to paradox A, needs **SELF-INCLUSION**$^{\text{l}}$). More speculatively, I conjecture that both metarules are rooted in a conception according to which *logical consequence* is a sufficient condition for *being* (where being is said as being of *predication*), in the sense that, if the property of being F *entails* the property of being G, that is a sufficient condition for every F *being* G (from which **RESTRICTED UNIVERSAL GENERALISATION**$^{\text{mr}}$ immediately follows, and, by the transitivity of being of predication (reflected, at least partly, by the transitivity of $\mathcal{E}$ mentioned above), so does **UNIVERSAL-PREDICATION CLOSURE**). (Interestingly, assuming very plausibly that the same tie—being of predication—is involved in both singular and universal predication, **RESTRICTED UNIVERSAL INSTANTIATION**$^{\text{r}}$ itself can ultimately be seen as simply yet another manifestation of the transitivity of being of predication.)

RESTRICTED NULL INSTANTIATION$^{\mathrm{r}}$ $\varphi_{\tau/\xi}, \psi_{\tau/\xi}, \mathcal{N}\xi(\varphi, \psi) \vdash \oslash$ holds;

RESTRICTED NULL INSTANTIATION$^{\mathrm{mc}}$ If $\oslash \vdash \varphi_{\tau/\xi}$ and $\oslash \vdash \psi_{\tau/\xi}$ hold, then $\oslash \vdash \mathcal{N}\xi(\varphi, \psi)$ does not hold.

Let's assume that there is a sentence $\mathfrak{n}$ which is $\mathcal{N}x(T\mathfrak{n}, \mathtt{t})$, where $\mathtt{t}$ is an arbitrary logical necessity (like $0 = 0$). We start with establishing a sixth lemma:

$$\dfrac{\dfrac{}{T\mathfrak{n} \ \vdash \ \mathcal{N}x(T\mathfrak{n}, \mathtt{t})}\textbf{T-ELIMINATION} \qquad \dfrac{}{T\mathfrak{n}, \mathtt{t}, \mathcal{N}x(T\mathfrak{n}, \mathtt{t}) \ \vdash \ \oslash}\textbf{RESTRICTED NULL INSTANTIATION}^{\mathrm{r}}}{\dfrac{T\mathfrak{n}, \mathtt{t}, T\mathfrak{n} \ \vdash \ \oslash}{T\mathfrak{n}, \mathtt{t} \ \vdash \ \oslash}\text{contraction}}\text{transitivity}$$

We then proceed to establishing a seventh lemma:

$$\dfrac{\dfrac{\dfrac{}{T\mathfrak{n}, \mathtt{t} \ \vdash \ \oslash}\text{lemma 6}}{\oslash \ \vdash \ \mathcal{N}x(T\mathfrak{n}, \mathtt{t})}\textbf{RESTRICTED NULL GENERALISATION}^{\mathrm{mr}} \qquad \dfrac{}{\mathcal{N}x(T\mathfrak{n}, \mathtt{t}) \ \vdash \ T\mathfrak{n}}\textbf{T-INTRODUCTION}}{\oslash \ \vdash \ T\mathfrak{n}}\text{transitivity}$$

Putting the two lemmas together, we get a catastrophe:

$$\dfrac{\dfrac{}{\oslash \ \vdash \ T\mathfrak{n}}\text{lemma 7} \qquad \dfrac{}{T\mathfrak{n}, \mathtt{t} \ \vdash \ \oslash}\text{lemma 6}}{\mathtt{t} \ \vdash \ \oslash}\text{transitivity}$$

(let's call this argument 'paradox C').

Some theorists are likely to try to block paradox C by rejecting **RESTRICTED NULL GENERALISATION**$^{\mathrm{mr}}$.[35] However, that principle is extremely plausible, even more so in the special case in which, with $\Gamma = \Delta = \oslash$, it reduces to the principle that, if $\varphi, \psi \vdash \oslash$ holds, then $\oslash \vdash \mathcal{N}\xi(\varphi, \psi)$ holds (which is all is in fact needed in paradox C)—if the property of being F is inconsistent with the property of being G, no F is G. Moreover, if we envisage a primitive unrestricted null quantifier, the principle admits of an argumentative justification analogous to the one I've developed for **RESTRICTED UNIVERSAL GENERALISATION**$^{\mathrm{mr}}$.

[35] Some other theorists, likely to be attracted by dialetheic theories of truth, may be more tempted to reject **RESTRICTED NULL INSTANTIATION**$^{\mathrm{r}}$ than they are to reject **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$. But such theorists should consider, among other things, that, just as **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ can be argumentatively justified by appeal to an extremely plausible version of the *dictum de omni* (see section 3), **RESTRICTED NULL INSTANTIATION**$^{\mathrm{r}}$ can be argumentatively justified by appeal to an extremely plausible version of the *dictum de nullo* (according to which ''No F is G' denies at least implicitly of x that it is G' is at least as weak as 'x is F').

Similarly to how the principles of restricted universal generalisation and instantiation require the combination of functions of *implication* that cannot but be fragmented by non-substructural naive theories of truth, so the principles of restricted null generalisation and instantiation require the combination of functions of *negation* that cannot but be fragmented by non-substructural naive theories. For, under `Translation` (iii), **RESTRICTED NULL GENERALISATION**$^{\mathrm{mr}}$ requires the operative negation to satisfy the metarule from $\varphi, \psi \vdash \oslash$ to $\oslash \vdash \neg(\varphi \,\&\, \psi)$, which in turn requires $\neg\varphi$ to satisfy a function traditionally associated with negation:

REDUCTION Be at least as weak as φ being logically absurd.[36]

And, under `Translation` (iii), **RESTRICTED NULL INSTANTIATION**$^{\mathrm{r}}$ requires the operative negation to satisfy the rule $\varphi, \psi, \neg(\varphi \,\&\, \psi) \vdash \oslash$, which in turn requires $\neg\varphi$ to satisfy another function traditionally associated with negation:

EXCLUSION Be logically incompatible with φ.[37]

A version of the *Liar paradox* shows that *non-substructural naive theories of truth cannot accept any negation that satisfies both* **REDUCTION** *and* **EXCLUSION**.[38] I think it's fair to say that this deeply counterintuitive consequence marks an important limitation of all such theories (see Zardini [2011], p. 514; [2014e]; [2014g]; [2014h] for some discussion). Still, even such a deeply counterintuitive consequence may after all be accepted, especially if it only ran against intuition and did not compromise other parts of one's logical theory. Unfortunately, consideration of the absolutely vital logical principles of null generalisation and instantiation shows that the combination of functions of negation in question is not only intuitively compelling, but is also of the essence for a very familiar and all-important operation (restricted quantification) that is so pervasive and crucial in our thought and talk. Because non-substructural naive theories cannot accept any negation that satisfies both **REDUCTION** and **EXCLUSION**, it follows that, under `Translation` (iii), they simply cannot validate both **RESTRICTED NULL GENERALISATION**$^{\mathrm{mr}}$ and **RESTRICTED NULL INSTANTIATION**$^{\mathrm{r}}$ and this—I reckon—is a great cost of these theories.

There is an interesting variation on paradox C, which replaces **RESTRICTED NULL GENERALISATION**$^{\mathrm{mr}}$ with the equally compelling metarule:

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[36]Reason: all naive theories of truth I know of envisage a sentence χ such that φ is intersubstitutable with $\varphi \,\&\, \chi$ (at least in extensional contexts). Thus, if $\varphi \vdash \oslash$ holds, by monotonicity $\varphi, \chi \vdash \oslash$ holds, and so, by the metarule in question, $\oslash \vdash \neg(\varphi \,\&\, \chi)$ holds, and hence, by intersubstitutability, $\oslash \vdash \neg\varphi$ holds.

[37]Reason: all naive theories of truth I know of envisage a sentence χ as in fn 36 and with the related property that $\oslash \vdash \chi$ holds. Thus, by the metarule in question, $\varphi, \chi, \neg(\varphi \,\&\, \chi) \vdash \oslash$ holds, and so, by $\oslash \vdash \chi$ and transitivity, $\varphi, \neg(\varphi \,\&\, \chi) \vdash \oslash$ holds, and hence, by intersubstitutability, $\varphi, \neg\varphi \vdash \oslash$ holds.

[38]Let's assume that there is a sentence $\mathfrak{l}$ that is $\neg T\mathfrak{l}$. By **EXCLUSION**, $T\mathfrak{l}, \neg T\mathfrak{l} \vdash \oslash$ holds, and so, by **T-ELIMINATION** and transitivity, $T\mathfrak{l}, T\mathfrak{l} \vdash \oslash$ holds, and hence, by contraction, $T\mathfrak{l} \vdash \oslash$ holds. Moreover, from $T\mathfrak{l} \vdash \oslash$ holding it follows by **REDUCTION** that $\oslash \vdash \neg T\mathfrak{l}$ holds, and so, by **T-INTRODUCTION** and transitivity, that $\oslash \vdash T\mathfrak{l}$ holds. Putting these two pieces together, by transitivity $\oslash \vdash \oslash$ holds.

NULL-PREDICATION COUNTER-CLOSURE If $\Gamma, \neg\psi \vdash \Delta, \neg\chi$ holds, and ξ does not occur free in Γ or Δ, then $\mathsf{qua}(\Gamma), \mathcal{N}\xi(\varphi, \psi) \vdash \mathsf{qua}(\Delta), \mathcal{N}\xi(\varphi, \chi)$ holds.

Using this principle, we start with establishing an eighth lemma:

$$\dfrac{\dfrac{\dfrac{}{T\mathfrak{n} \vdash \mathcal{N}x(T\mathfrak{n}, \mathsf{t})}\ \text{T-ELIMINATION} \qquad \dfrac{}{T\mathfrak{n}, \mathcal{N}x(T\mathfrak{n}, \mathsf{t}) \vdash \neg\mathsf{t}}\ \text{variation of RESTRICTED NULL INSTANTIATION}^{\mathrm{r}}}{T\mathfrak{n}, T\mathfrak{n} \vdash \neg\mathsf{t}}\ \text{transitivity}}{T\mathfrak{n} \vdash \neg\mathsf{t}}\ \text{contraction}$$

We then proceed to establishing a ninth lemma:

$$\dfrac{\dfrac{}{\oslash \vdash \mathcal{N}x(T\mathfrak{n}, \neg T\mathfrak{n})}\ \text{variation of SELF-INCLUSION}^{\mathrm{l}} \qquad \dfrac{\dfrac{\dfrac{}{T\mathfrak{n} \vdash \neg\mathsf{t}}\ \text{lemma 8}}{\neg\neg T\mathfrak{n} \vdash \neg\mathsf{t}}\ \text{uncontroversial logic if any}}{\mathcal{N}x(T\mathfrak{n}, \neg T\mathfrak{n}) \vdash \mathcal{N}x(T\mathfrak{n}, \mathsf{t})}\ \text{NULL-PREDICATION COUNTER-CLOSURE}}{\oslash \vdash \mathcal{N}x(T\mathfrak{n}, \mathsf{t})}\ \text{transitivity}$$

This gives us the left-hand intermediate conclusion of the subargument establishing lemma 7 in paradox C. Using then the rest of that subargument plus the final subargument of paradox C, we get that $\mathsf{t} \vdash \oslash$ holds (let's call this paradox 'paradox D').

Paradox D makes use of the same principles as paradox C save for using **NULL-PREDICATION COUNTER-CLOSURE** and (a variation of) **SELF-INCLUSION**^{l} instead of **RESTRICTED NULL GENERALISATION**$^{\mathrm{mr}}$ (plus using a variation of **RESTRICTED NULL INSTANTIATION**$^{\mathrm{r}}$). **NULL-PREDICATION COUNTER-CLOSURE** has an extreme intuitive plausibility, even more so in the special case in which, with $\Gamma = \Delta = \oslash$, it reduces to the principle that, if $\neg\psi \vdash \neg\chi$ holds, then $\mathcal{N}\xi(\varphi, \psi) \vdash \mathcal{N}\xi(\varphi, \chi)$ holds (which is all is in fact needed in paradox D)—if the property of being $\neg G$ entails the property of being $\neg H$, then no F being G entails no F being H. Moreover, the principle admits of an argumentative justification analogous to the one I've developed for **UNIVERSAL-PREDICATION CLOSURE**.

5 Non-Transitive Restricted Quantification

I think that the impossibility results of section 4 give us reason to investigate naive theories of truth that escape paradoxes A–D—that is, in effect, *substructural* theories. *Transitivity* was heavily made use of in all these paradoxes, so a natural suggestion at this point would be to restrict that structural metarule. In fact, it has always seemed quite likely that the family of non-transitive logics developed in Zardini [2008a]; [2008b]; [2009]; [2014b]; [2014f]; [2014i] for dealing with *vagueness* could be used to obtain a non-transitive naive theory of *truth*, and that this is in fact so has recently been established by Ripley [2012].[39] In the following, I focus on Ripley's theory (which would seem among the most promising non-transitive theories as far as restricted quantification is concerned) and assume that its extension to restricted quantification uses material implication for representing restricted universal quantification (see Ripley [2013], pp. 156–157; I'll call the theory so understood '**R**'). I should stress that the discussion in this section is meant to be even more tentative and explorative than that in other sections: in many respects, non-transitive logics are a very peculiar beast, and, although we're now clear about the formal properties of at least some of them, we're still at the early stages of their philosophical understanding (I've made my own initial attempt in this sense in Zardini [2014d]).

Like the other non-transitive logics belonging to the family developed in my works referenced in the last paragraph, **R** can very roughly be understood as identifying the validity of an argument with the fact that, whenever all of its premises are "*very good*", some of its conclusions are at least "*good enough*". However, in the context of the semantic paradoxes, **T-INTRODUCTION** and **T-ELIMINATION** together with some basic properties of negation in **R** force *certain specific sentences as well as their negation to be good enough but not very good* (roughly, such sentences are those that are "paradoxical"). Generally, this circumstance has the consequence that **R** treats *paradoxical premises* very much like *analetheic* theories do and *paradoxical conclusions* very much like *dialetheic* theories do. More specifically, letting π be a paradoxical sentence, given the properties of disjunction in **R** this circumstance has the consequences that $\pi \vee \neg\pi$ cannot be very good and so is logically absurd (as in analetheic theories) and that $\neg\pi \vee \psi$ (with ψ arbitrary) as well as π must be good enough and so are logically necessary (as in dialetheic theories). As might be expected from our discussion in section 3, these features determine that material implication in **R** does not satisfy all the Functions and so that **R** does not validate all the Principles.

More in detail, this is so in at least four respects. Firstly, **R** is not only *naive*, but also *transparent* (see fn 39), and so, using lemmas 2 and 1 of paradox A, we can still conclude without transitivity to $\oslash \vdash_{\mathbf{R}} T\mathfrak{e}$ and $T\mathfrak{e} \vdash_{\mathbf{R}} \oslash$ respectively (the latter following from $T\mathfrak{e} \vdash_{\mathbf{R}} \mathsf{f}$). Because transitivity fails in **R**, that does not imply the catastrophical $\oslash \vdash_{\mathbf{R}} \oslash$, but it does imply that **SELF-INCLUSION**$^{\text{c}}$ fails in **R**, since it does imply that $\mathcal{E}x(T\mathfrak{e}, T\mathfrak{e}) \vdash_{\mathbf{R}} \oslash$ holds.[40] Thus, for some property F, in **R** it is logically absurd that every F is F. But, *pace* **R**, for every property F, it is clearly not logically absurd, and it is indeed *true-only* (i.e. it *is true* and it *fails to be false*), that every F is F.

[39] Before these works, Alan Weir already developed a non-transitive theory of truth (and sets) with possible application to vagueness too (see Weir [2005] for a recent presentation). However, Weir's theory does validate the *final* instances of transitivity used in paradoxes A–D as well as those instances used in lemmas 5 and 9, since on that theory—contrary to the theories of my works referenced above in the text and Ripley [2012]—transitivity holds *in the absence of side premises that are not logically necessary* (as for the other instances of transitivity, these are unproblematic at least given the principle of *transparency* according to which φ is intersubstitutable with $T\ulcorner\varphi\urcorner$ (at least in extensional contexts), a principle which is importantly stronger than *naivety* and which holds in Ripley's but not in Weir's theory).

[40] Reason: $T\mathfrak{e} \vdash_{\mathbf{R}} \oslash$ holds. Moreover, $\oslash \vdash_{\mathbf{R}} T\mathfrak{e}$ holds, and so, by the properties of negation in **R**, $\neg T\mathfrak{e} \vdash_{\mathbf{R}} \oslash$ holds. Putting these two pieces together, by the properties of disjunction in **R** $T\mathfrak{e} \vee \neg T\mathfrak{e} \vdash_{\mathbf{R}} \oslash$ holds. Since, if ξ does not occur free in φ, $\forall\xi\varphi$ and φ are intersubstitutable in **R**, it follows that $\forall\xi(T\mathfrak{e} \vee \neg T\mathfrak{e}) \vdash_{\mathbf{R}} \oslash$ holds, from which, under `Translation` (i), the fact follows.

Secondly, there is little solace in observing that, although $\mathcal{E}x(T\mathfrak{e}, T\mathfrak{e}) \vdash_{\mathbf{R}} \oslash$ unfortunately holds, at least we have that $\oslash \vdash_{\mathbf{R}} \mathcal{E}x(T\mathfrak{e}, T\mathfrak{e})$ holds too, for, since $\oslash \vdash_{\mathbf{R}} T\mathfrak{e}$ and $T\mathfrak{e} \vdash_{\mathbf{R}} \oslash$ hold, that implies failure of **NO ALL-IN-NOTHING INCLUSION**. Thus, for some property F that every object whatsoever must exemplify by logical necessity and some property G that every object whatsoever cannot exemplify on pain of logical absurdity, in **R** it is logically necessary that every F is G. But, *pace* **R**, for every such properties F and G, it is clearly not logically necessary, and it is indeed *false-only* (i.e. it *is false* and it *fails to be true*), that every F is G. Relatedly, since $\oslash \vdash_{\mathbf{R}} T\mathfrak{e}$ holds, so does $\oslash \vdash_{\mathbf{R}} \forall x T\mathfrak{e}$ (see fn 40), and so the **R**-theorist should presumably think that $\forall x T\mathfrak{e}$ holds. But, using lemma 1 and **RESTRICTED UNIVERSAL GENERALISATION**$^{\mathrm{mr}}$, we can still conclude without transitivity to $\oslash \vdash_{\mathbf{R}} \mathcal{E}x(T\mathfrak{e}, \mathsf{f})$, so that the **R**-theorist should presumably think that $\mathcal{E}x(T\mathfrak{e}, \mathsf{f})$ holds. Thus, the **R**-theorist is committed to thinking that every object such that $T\mathfrak{e}$ holds—*and every object whatsoever is like that*—is such that f holds. "Suffixing" rather than "prefixing", the **R**-theorist is committed to thinking that every object whatsoever is such that $T\mathfrak{e}$ holds—*and every object like that is such that* f *holds*. Delving deeper into this point, the following version of *Barbara*:

(BARB) 'Every F {is such as to be/exemplifies a property that is a way of being /is one of the objects that are} H' is at least as weak as[41] 'Every F is G and every G is H'

is extremely plausible (in all of its *higher-order*, *property-theoretic* and *plural* variants): provided that every F is G and every G is H, one can always infer that every F is a G which is H,[42] from which (BARB) presumably follows. But it follows from (BARB) and the two [things mentioned above that the **R**-theorist should presumably think] that every

[41]Now that non-transitivity is in focus, I should make explicit that, throughout, I use '*at least as* {*weak/strong*} *as*' to mean that, in spite of the general failure of transitivity, the entailment in question is so *strict* that it can always be chained with another entailment to yield an entailment.

[42]That is indeed extremely plausible, but not entirely uncontroversial. I think that one interesting source of resistance comes from the usual account of *restrictive relative clauses* (see e.g. Quine [1960], pp. 109–111), according to which the interpretation of the restrictive relative clause 'which is H' is something along the lines of $\lambda\xi H\xi$, so that the interpretation of the restricted noun phrase 'G which is H' is something along the lines of $\lambda\xi(G\xi \,\&\, H\xi)$. On such account, 'Every F is a G which is H' is problematically strong in a non-transitive framework (or in the non-contractive framework developed in section 6), since from it, extremely plausibly, one can in turn always infer 'Every F is H', which can be problematic in a non-transitive framework (or since it is tantamount to 'Every F is G and H', which can be problematic in the non-contractive framework developed in section 6). But the usual account can easily be *modified* so as to make it more congenial to a non-transitive framework (and to the non-contractive framework developed in section 6): the interpretation of the restrictive relative clause 'which is H' can be taken to be something along the lines of $\lambda\Xi\xi(\Xi\xi \,\&\, (\Xi\xi \supset H\xi))$, so that the interpretation of the restricted noun phrase 'G which is H' can be taken to be something along the lines of $\lambda\xi(G\xi \,\&\, (G\xi \supset H\xi))$ (which is intersubstitutable with $\lambda\xi(G\xi \,\&\, H\xi)$ in classical logic and in many other non-substructural logics, but which is not typically such in a non-transitive logic and not such in the non-contractive logic of section 6). (Notice that, for essentially the same reason, the *non-restrictive* reading of 'Every F is a G which is H', on which it is presumably equivalent with something like 'Every F is a G and that G is H', is also problematically strong, thus revealing a further interesting difference between restrictive and non-restrictive readings of relative clauses.) It's time for natural-language semantics to become sensitive to substructural distinctions!

object whatsoever {is such as to be/exemplifies a property that is a way of being/is one of the things that are} such that f holds. It's true that, given **R**'s relevant weakness, all these consequences strictly speaking still do *not entail* that every object whatsoever is such that f holds, but, to me at least, the consequences are *just about as rebarbative* all the same. In a broad variation of the latter point, since the **R**-theorist is committed to thinking that every object whatsoever is such that $T\mathfrak{e}$ holds, she is committed to thinking that, say, I am such that $T\mathfrak{e}$ holds. But then, by (DO_0), she is committed to thinking that $\mathcal{E}x(T\mathfrak{e}, \mathsf{f})$ says at least implicitly of me that I am such that f holds, which is presumably tantamount to saying at least implicitly that f holds. Thus, the **R**-theorist is committed to thinking that *a sentence holds which says at least implicitly that* f *holds*.

Thirdly, **RESTRICTED UNIVERSAL GENERALISATION**$^{\mathrm{mc}}$ fails in **R**, since $T\mathfrak{e} \vdash_{\mathbf{R}} \mathsf{f}$ and $\mathcal{E}x(T\mathfrak{e}, \mathsf{f}) \vdash_{\mathbf{R}} \oslash$ hold (the latter following by transparency from $T\mathfrak{e} \vdash_{\mathbf{R}} \oslash$). Thus, for some property F entailing some property G, in **R** it is logically absurd that every F is G. But, *pace* **R**, for every such properties F and G, it is clearly not logically absurd, and it is indeed true-only, that every F is G (an analogous point holds for failure of **RESTRICTED NULL GENERALISATION**$^{\mathrm{mc}}$ in **R**).

Fourthly, **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{ml}}$ fails in **R**, since $\oslash \vdash_{\mathbf{R}} T\mathfrak{e}$ and $\oslash \vdash_{\mathbf{R}} \mathcal{E}x(T\mathfrak{e}, \mathsf{f})$ hold but $\oslash \vdash_{\mathbf{R}} \mathsf{f}$ does not. Thus, for some property F that every object whatsoever must exemplify by logical necessity and some property G that not every object whatsoever must exemplify by logical necessity, and that indeed no object whatsoever can exemplify on pain of logical absurdity, in **R** it is logically necessary that every F is G. But, *pace* **R**, for every such properties F and G, it is clearly not logically necessary, and it is indeed false-only, that every F is G. Relatedly, the fact that **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{ml}}$ fails in **R** in turn casts serious doubts on the alleged significance of the fact that **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ nevertheless holds in **R**: if *the laws of logic themselves do not satisfy the principle of restricted universal instantiation*, in what sense is **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ nevertheless a valid rule? Delving deeper into this point, the *logical* idea of *universal instantiation* can be seen as intimately connected with and arguably grounded in the *semantic* idea that *a true universal quantification cannot have false-only instances*. Yet, we've seen in the second last paragraph that the **R**-theorist should presumably think that [it is true that $\mathcal{E}x(T\mathfrak{e}, \mathsf{f})$ holds] and that every object whatsoever is such that $T\mathfrak{e}$ holds; moreover, the **R**-theorist should also presumably think that it is false-only that every, or indeed even some, object whatsoever is such that f holds. Thus, given another equally (i.e. extremely) plausible version of the *dictum de omni*:

(DO_1) ''x is G' is an instance of 'Every F is G'' is at least as weak as 'x is F',

the **R**-theorist is committed to thinking that a certain true universal quantification has false-only instances, indeed that it has only false-only instances. This is not only in itself extremely problematic; as I've just indicated, it also means to reject the very semantic idea with which universal instantiation is intimately connected and in which it is arguably grounded. And that in turn casts even more serious doubts on the alleged significance of the fact that **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ nevertheless holds in **R**, since its instance with $T\mathfrak{e}$ for φ and f for ψ has true premises and a false-only conclusion: if **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ can have true premises and a false-only conclusion, in what sense is it nevertheless a valid rule (similar points hold for failure of **RESTRICTED NULL INSTANTIATION**$^{\mathrm{mc}}$ in **R**)?[43] (Notice that the problem raised by this point is even further exacerbated by the fact that 'true' and 'false-only' can be replaced throughout by 'logically necessary' and 'logically absurd' respectively.)[44]

[43]This latter point (in particular from "Delving deeper into this point. . . "), as well as the latter point in the second last paragraph (plus some other scattered comments), may seem to depend on the (in my view, very plausible) assumption that, if $\oslash \vdash_{\mathbf{R}} \varphi$ holds, the **R**-theorist should *accept* φ. As I understand him, however, Ripley [2013] postulates a distinction between *two kinds of acceptance and rejection* (kinds which I'll label with subscripts 0 and 1 respectively) such that the assumption just mentioned holds for acceptance$_1$ but not for acceptance$_0$. How do then acceptance$_0$ and rejection$_0$ fare with respect to these points? As far as I can tell, they do not fare substantially better. For, given that rejection$_0$ is such that, if $\oslash \vdash_{\mathbf{R}} \varphi$ holds, the **R**-theorist should *not reject*$_0$ φ, these points can be recast with almost equal force *substituting obligation not to reject*$_0$ *for obligation to accept*$_1$. For example, the very last comment made in the text can be recast by asking in what sense is **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ a valid rule if it can have premises that should not be rejected$_0$ and a conclusion that should be rejected$_0$—quite generally, if one should not reject$_0$ φ or ψ (nor their conjunction), this is presumably so because, in the relevant sense, it is *possible* that φ and ψ hold, and, if one should reject$_0$ χ, this is presumably so because, in the relevant sense, it is *impossible* that χ holds, and presumably φ and ψ cannot entail χ if it is possible that φ and ψ hold but it is not possible that χ holds. Following Ripley [2013], it might still be observed that **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ may be a "valid" rule in the sense that *one should not accept*$_0$ *all its premises and reject*$_0$ *all its conclusions* (in the offending case discussed in the text, the **R**-theorist thinks in effect that one should not accept$_0$ either premise), but the fact that this obligation is for example *compatible*, as we've just seen, with the joint obligation [not to reject$_0$ all the premises] and [to reject$_0$ all the conclusions] raises a serious issue whether the sense in question is strong enough as to capture the spirit of **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ and of the other Principles as well as, more generally, the understanding of logical consequence that is at play in informal philosophical discussions of the semantic paradoxes when it is debated what follows from what. Zooming out from the dialectic concerning **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$, acceptance$_0$ and rejection$_0$ fare substantially worse than acceptance$_1$ and rejection$_1$ with respect to *other* Principles. For, given that acceptance$_0$ is such that, if $\varphi \vdash_{\mathbf{R}} \oslash$ holds, the **R**-theorist should *not accept*$_0$ φ, the **R**-theorist should not accept$_0$, say, $\mathcal{E}x(T\mathfrak{e}, T\mathfrak{e})$ (since $\mathcal{E}x(T\mathfrak{e}, T\mathfrak{e}) \vdash_{\mathbf{R}} \oslash$ holds), and, although, say, $T\mathfrak{e} \vdash_{\mathbf{R}} \mathsf{f}$ holds, she should not accept$_0$ $\mathcal{E}x(T\mathfrak{e}, \mathsf{f})$ (since $\mathcal{E}x(T\mathfrak{e}, \mathsf{f}) \vdash_{\mathbf{R}} \oslash$ holds). That puts in jeopardy the spirit of **SELF-INCLUSION**$^{\mathrm{l}}$ and **RESTRICTED UNIVERSAL GENERALISATION**$^{\mathrm{mr}}$ (which certainly includes that the respective universal quantifications be at least accepted as true!). (Some of the issues touched on from "Following Ripley [2013]. . . " have actually already emerged in the discussion of the normative acceptance/rejection principle mentioned in fn 9.) Thanks to Dave Ripley and an anonymous referee for crucial help with this fn.

[44]It may be worth noting that, for reasons I can't go into in this paper, I don't think that any of the problems discussed in this section applies in the case of the use of non-transitive logics for dealing with vagueness made in my work referenced above in the text.

6 Non-Contractive Restricted Quantification

I think that the results of section 5 give us reason to investigate an alternative substructural approach. *Contraction* too was heavily made use of in all of paradoxes A–D, so a natural suggestion at this point would be to restrict that structural metarule. In fact, in Zardini [2011]; [2013a]; [2013b]; [2014c]; [2014e]; [2014g]; [2014i], I myself have developed a specific non-contractive approach to the semantic paradoxes, and it is precisely that approach that, in this section, I'd like to bring to bear on the problems discussed in this paper.

For reasons that will presently become apparent, I'll call '$\mathbf{IKT}^{\omega}$' the non-contractive naive theory of truth within which I'd like to develop a theory of restricted quantification. We can specify the *background logic* of $\mathbf{IKT}^{\omega}$ as the smallest logic containing as *axiom* the *structural rule*:

$$\frac{}{\varphi \ \vdash_{\mathbf{IKT}^{\omega}} \ \varphi}\ \text{I}$$

and suitably *closed* under the *structural metarules*:

$$\frac{\Gamma_0 \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta}{\Gamma_0, \Gamma_1 \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta}\ \text{K-L} \qquad \frac{\Gamma \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_0}{\Gamma \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_0, \Delta_1}\ \text{K-R}$$

$$\frac{\Gamma_0 \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_0, \varphi \qquad \Gamma_1, \varphi \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_1}{\Gamma_0, \Gamma_1 \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_0, \Delta_1}\ \text{S}$$

and under the *operational metarules*:

$$\frac{\Gamma \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta, \varphi}{\Gamma, \neg\varphi \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta}\ \neg\text{-L} \qquad \frac{\Gamma, \varphi \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta}{\Gamma \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta, \neg\varphi}\ \neg\text{-R}$$

$$\frac{\Gamma, \varphi, \psi \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta}{\Gamma, \varphi \,\&\, \psi \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta}\ \&\text{-L} \qquad \frac{\Gamma_0 \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_0, \varphi \qquad \Gamma_1 \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_1, \psi}{\Gamma_0, \Gamma_1 \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_0, \Delta_1, \varphi \,\&\, \psi}\ \&\text{-R}$$

$$\frac{\Gamma_0, \varphi \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_0 \qquad \Gamma_1, \psi \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_1}{\Gamma_0, \Gamma_1, \varphi \vee \psi \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_0, \Delta_1}\ \vee\text{-L} \qquad \frac{\Gamma \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta, \varphi, \psi}{\Gamma \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta, \varphi \vee \psi}\ \vee\text{-R}$$

$$\frac{\Gamma_0 \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_0, \varphi \qquad \Gamma_1, \psi \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_1}{\Gamma_0, \Gamma_1, \varphi \supset \psi \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta_0, \Delta_1}\ \supset\text{-L} \qquad \frac{\Gamma, \varphi \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta, \psi}{\Gamma \ \vdash_{\mathbf{IKT}^{\omega}} \ \Delta, \varphi \supset \psi}\ \supset\text{-R}$$

$$\frac{\Gamma, \varphi_{\upsilon_0/\xi}, \varphi_{\upsilon_1/\xi}, \varphi_{\upsilon_2/\xi} \ldots \ \vdash_{\mathbf{IKT}^\omega} \ \Delta}{\Gamma, \forall\xi\varphi \ \vdash_{\mathbf{IKT}^\omega} \ \Delta} \ \forall\text{-L}$$

$$\frac{\Gamma_0 \ \vdash_{\mathbf{IKT}^\omega} \ \Delta_0, \varphi_{\upsilon_0/\xi} \qquad \Gamma_1 \ \vdash_{\mathbf{IKT}^\omega} \ \Delta_1, \varphi_{\upsilon_1/\xi} \qquad \Gamma_2 \ \vdash_{\mathbf{IKT}^\omega} \ \Delta_2, \varphi_{\upsilon_2/\xi} \ldots}{\bigsqcup_{0 \leq i < \omega} (\Gamma_i) \ \vdash_{\mathbf{IKT}^\omega} \ \bigsqcup_{0 \leq i < \omega} (\Delta_i), \forall\xi\varphi} \ \forall\text{-R}$$

$$\frac{\Gamma_0, \varphi_{\upsilon_0/\xi} \ \vdash_{\mathbf{IKT}^\omega} \ \Delta_0 \qquad \Gamma_1, \varphi_{\upsilon_1/\xi} \ \vdash_{\mathbf{IKT}^\omega} \ \Delta_1 \qquad \Gamma_2, \varphi_{\upsilon_2/\xi} \ \vdash_{\mathbf{IKT}^\omega} \ \Delta_2 \ldots}{\bigsqcup_{0 \leq i < \omega} (\Gamma_i), \exists\xi\varphi \ \vdash_{\mathbf{IKT}^\omega} \ \bigsqcup_{0 \leq i < \omega} (\Delta_i)} \ \exists\text{-L}$$

$$\frac{\Gamma \ \vdash_{\mathbf{IKT}^\omega} \ \Delta, \varphi_{\upsilon_0/\xi}, \varphi_{\upsilon_1/\xi}, \varphi_{\upsilon_2/\xi} \ldots}{\Gamma \ \vdash_{\mathbf{IKT}^\omega} \ \Delta, \exists\xi\varphi} \ \exists\text{-R}$$

With respect to the metarules for the *unrestricted* quantifiers $\forall$ and $\exists$, a couple of clarifications are in order. Firstly, '$\upsilon_0, \upsilon_1, \upsilon_2 \ldots$' and its likes refer to a designated complete enumeration of the denumerable set of singular terms of the language and its likes (so that for example $\forall$-R amounts in effect to a kind of ω-rule). Secondly, $\bigsqcup$ expresses the operation of "union" of (countably many) multisets that is the analogue of the operation of union of sets (in particular, numbers of occurrences of the same member in different arguments of the operation are summed (up to countable infinity) in the resulting "multiset union", so that, for example, $\bigsqcup_{0 \leq i < 2} (\Gamma_i) = \Gamma_0, \Gamma_1$).

As for the *theory of truth* proper, we add to the background logic of $\mathbf{IKT}^\omega$ the following metarules for T:

$$\frac{\Gamma, \varphi \ \vdash_{\mathbf{IKT}^\omega} \ \Delta}{\Gamma, T\ulcorner\varphi\urcorner \ \vdash_{\mathbf{IKT}^\omega} \ \Delta} \ T\text{-L} \qquad\qquad \frac{\Gamma \ \vdash_{\mathbf{IKT}^\omega} \ \Delta, \varphi}{\Gamma \ \vdash_{\mathbf{IKT}^\omega} \ \Delta, T\ulcorner\varphi\urcorner} \ T\text{-R}$$

As proved in Zardini [2011], the end result is a consistent naive (indeed, transparent) theory of truth with several interesting properties that distinguish it from its rivals (for example, in contrast to analetheic and dialetheic theories, both the law of excluded middle and the law of non-contradiction hold in $\mathbf{IKT}^\omega$; I refer the reader to my works referenced in the second last paragraph for an extended examination of this and other features of $\mathbf{IKT}^\omega$).

Before embarking on an investigation of the theory of restricted quantification available in $\mathbf{IKT}^\omega$, it's important to make explicit a substantial limitation of the discussion to follow. Clearly, $\forall$-R and $\exists$-L only make sense with respect to the intended meaning of $\forall$ and $\exists$ as *objectual* quantifiers if every object in the domain of discourse is referred to by a singular term of the language, which in turn implies that those objects are *countably many.* That is indeed a substantial limitation, but it is multiply warranted for our purposes. Firstly, focus on the countable case will greatly simplify our discussion. Secondly, the problems of restricted quantification presented in sections 3–5 that arise for the other naive theories of truth make no assumption about the cardinality of the domain of discourse, and so it will already be a great advantage for $\mathbf{IKT}^\omega$ that it outperforms its rivals over countable domains. Thirdly, although the matter certainly deserves proper investigation (better left for another occasion), there would seem to be at present no reasons to expect that things will turn out to be substantially different over uncountable domains of discourse.

Following one of the guiding thoughts of this paper, and as a first preliminary before investigating the theory of restricted quantification proper available in $\mathbf{IKT}^\omega$, let's notice that the material implication expressed by $\supset$ in $\mathbf{IKT}^\omega$ satisfies all the Functions. This feature of material implication in $\mathbf{IKT}^\omega$ is rather unique among naive theories of truth, and is an early indication that $\mathbf{IKT}^\omega$ can validate all the Principles. As a second preliminary before investigating the theory of restricted quantification proper available in $\mathbf{IKT}^\omega$, let's notice that also the following principles hold in $\mathbf{IKT}^\omega$:

(MP) $\varphi, \varphi \supset \psi \vdash_{\mathbf{IKT}^\omega} \psi$ holds;

(MAT) $\varphi \supset \psi$ is intersubstitutable with $\neg(\varphi \,\&\, \neg\psi)$;

(NEUTR) If $\Gamma, \varphi \vdash_{\mathbf{IKT}^\omega} \Delta, \psi$ holds and ξ does not occur free in Γ or Δ, then $\Gamma, \varphi_{\tau/\xi} \vdash_{\mathbf{IKT}^\omega} \Delta, \psi_{\tau/\xi}$ holds;

(UUI) $\forall\xi\varphi \vdash_{\mathbf{IKT}^\omega} \varphi_{\tau/\xi}$ holds;

(QDUAL) $\forall\xi\neg\varphi$ is intersubstitutable with $\neg\exists\xi\varphi$

(notice that (NEUTR) is in effect **SINGULAR-PREDICATION CLOSURE** taking `qua` to be identity).

With so much by way of preliminaries, we can now proceed to verify that, under `Translation` (i)–(ii), $\mathbf{IKT}^\omega$ validates all the Principles:

SELF-INCLUSION$^{\mathrm{l}}$: By I and $\supset$-R, for every i $\oslash \vdash_{\mathbf{IKT}^\omega} \varphi_{\upsilon_i/\xi} \supset \varphi_{\upsilon_i/\xi}$ holds, and so, by $\forall$-R and `Translation` (i), $\oslash \vdash_{\mathbf{IKT}^\omega} \mathcal{E}\xi(\varphi, \varphi)$ holds;

SELF-INCLUSION$^{\mathrm{c}}$: We've shown above that $\oslash \vdash_{\mathbf{IKT}^\omega} \forall\xi(\varphi \supset \varphi)$ holds. If $\mathcal{E}\xi(\varphi, \varphi) \vdash_{\mathbf{IKT}^\omega} \oslash$ also held, under `Translation` (i) $\forall\xi(\varphi \supset \varphi) \vdash_{\mathbf{IKT}^\omega} \oslash$ would hold, and so, by S, $\oslash \vdash_{\mathbf{IKT}^\omega} \oslash$ would hold, which it does not (see Zardini [2011], p. 532);

NO ALL-IN-NOTHING INCLUSION: If $\oslash \vdash_{\mathbf{IKT}^\omega} \varphi$ holds, then, if $\oslash \vdash_{\mathbf{IKT}^\omega} \mathcal{E}\xi(\varphi, \psi)$ holds, under `Translation` (i), (UUI), S and (MP), $\oslash \vdash_{\mathbf{IKT}^\omega} \psi$ holds. But, if $\psi \vdash_{\mathbf{IKT}^\omega} \oslash$ also held, by S $\oslash \vdash_{\mathbf{IKT}^\omega} \oslash$ would hold, which again it does not;

NO COUNTER-INSTANCE$^{\Rightarrow}$: By I and `Translation` (ii), $\neg\mathcal{S}\xi(\varphi, \neg\psi) \vdash_{\mathbf{IKT}^\omega} \neg\exists\xi(\varphi \,\&\, \neg\psi)$ holds, and so, by (QDUAL), (MAT) and `Translation` (i), $\neg\mathcal{S}\xi(\varphi, \neg\psi) \vdash_{\mathbf{IKT}^\omega} \mathcal{E}\xi(\varphi, \psi)$ holds. A similar argument holds for $\neg\mathcal{S}\xi(\varphi, \psi) \vdash_{\mathbf{IKT}^\omega} \mathcal{E}\xi(\varphi, \neg\psi)$;

NO COUNTER-INSTANCE$^{\Leftarrow}$: Arguments analogous to those given for **NO COUNTER-INSTANCE$^{\Rightarrow}$** hold;

RESTRICTED UNIVERSAL GENERALISATION$^{\mathrm{mr}}$: If $\Gamma, \varphi \vdash_{\mathbf{IKT}^\omega} \Delta, \psi$ holds and ξ does not occur free in Γ or Δ, by $\supset$-R $\Gamma \vdash_{\mathbf{IKT}^\omega} \Delta, \varphi \supset \psi$ holds, and so, by (NEUTR), for every i $\Gamma \vdash_{\mathbf{IKT}^\omega} \Delta, \varphi_{\upsilon_i/\xi} \supset \psi_{\upsilon_i/\xi}$ holds, and hence, by $\forall$-R, $\mathsf{ome}(\Gamma) \vdash_{\mathbf{IKT}^\omega} \mathsf{ome}(\Delta), \forall\xi(\varphi \supset \psi)$ holds,[45] wherefore, under `Translation` (i), $\mathsf{ome}(\Gamma) \vdash_{\mathbf{IKT}^\omega} \mathsf{ome}(\Delta), \mathcal{E}\xi(\varphi, \psi)$ holds.

RESTRICTED UNIVERSAL GENERALISATION$^{\mathrm{mc}}$: If $\varphi \vdash_{\mathbf{IKT}^\omega} \psi$ holds, we've shown above that $\oslash \vdash_{\mathbf{IKT}^\omega} \forall\xi(\varphi \supset \psi)$ holds. Thus, if $\mathcal{E}\xi(\varphi, \psi) \vdash_{\mathbf{IKT}^\omega} \oslash$ also held, under `Translation` (i) and S $\oslash \vdash_{\mathbf{IKT}^\omega} \oslash$ would hold, which again it does not;

RESTRICTED UNIVERSAL INSTANTIATION$^{\mathrm{r}}$: By (UUI) and `Translation` (i), $\mathcal{E}\xi(\varphi, \psi) \vdash_{\mathbf{IKT}^\omega} \varphi_{\tau/\xi} \supset \psi_{\tau/\xi}$ holds, and so, by (MP) and S, $\varphi_{\tau/\xi}, \mathcal{E}\xi(\varphi, \psi) \vdash_{\mathbf{IKT}^\omega} \psi_{\tau/\xi}$ holds;

RESTRICTED UNIVERSAL INSTANTIATION$^{\mathrm{ml}}$: If $\oslash \vdash_{\mathbf{IKT}^\omega} \varphi_{\tau/\xi}$ and $\oslash \vdash_{\mathbf{IKT}^\omega} \mathcal{E}\xi(\varphi, \psi)$ hold, then, since, by **RESTRICTED UNIVERSAL INSTANTIATION$^{\mathrm{r}}$**, $\varphi_{\tau/\xi}, \mathcal{E}\xi(\varphi, \psi) \vdash_{\mathbf{IKT}^\omega} \psi_{\tau/\xi}$ holds, by S $\oslash \vdash_{\mathbf{IKT}^\omega} \psi_{\tau/\xi}$ holds.

Moreover, it can be also verified along similar lines that, under `Translation`, $\mathbf{IKT}^\omega$ validates all the additional principles of restricted quantification introduced in sections 3–5.

Although the theory of restricted quantification available in $\mathbf{IKT}^\omega$ is very satisfactory in these respects, there is at least one respect in which it may be regarded as problematic. For it's easy to see that, under `Translation` (i), the principle of *constancy*:

(CONST) $\mathcal{E}\xi(\varphi, \psi) \vdash \mathcal{E}\xi(\varphi, \psi \,\&\, \varphi)$ holds

has to fail in $\mathbf{IKT}^\omega$.[46] However, (CONST) is *prima facie* plausible, and so its failure may seem like an important cost of $\mathbf{IKT}^\omega$.

But, on reflection, at least one kind of example to be found in natural languages puts pressure on (CONST). For (CONST) would seem to get into trouble with verbs expressing *functions that things could play.* 'Every piece of wood that makes 4 chairs makes 1 bed' seems true, but 'Every piece of wood that makes 4 chairs makes 1 bed and makes 4 chairs'[47] does not seem so; 'Every lion that needs 4 steaks per day needs 1 joint of roast

[45] Where $\mathsf{ome}(\Gamma)$ is the ω-fold "multiset union" of Γ with itself—that is, the multiset in which every member of Γ occurs denumerably many times and nothing else occurs any positive number of times.

[46] Reason: putting φ for ψ in (CONST), $\mathcal{E}\xi(\varphi, \varphi) \vdash_{\mathbf{IKT}^\omega} \mathcal{E}\xi(\varphi, \varphi \,\&\, \varphi)$ would hold. But, by **SELF-INCLUSION$^{\mathrm{l}}$**, $\oslash \vdash_{\mathbf{IKT}^\omega} \mathcal{E}\xi(\varphi, \varphi)$ holds, and so, by S and `Translation` (i), $\oslash \vdash_{\mathbf{IKT}^\omega} \varphi \supset (\varphi \,\&\, \varphi)$ would hold, which it does not (see Zardini [2011], p. 519).

[47] Strictly speaking, the verb phrase here should be 'makes 1 bed and is a piece of wood that makes 4 chairs', but for readability's sake I employ in this and the other examples a more compressed and natural form.

beef per day' seems true, but 'Every lion that needs 4 steaks per day needs 1 joint of roast beef per day and needs 4 steaks per day' does not seem so; 'Every quantity of energy that heats 4 houses moves 1 train' seems true, but 'Every quantity of energy that heats 4 houses moves 1 train and heats 4 houses' does not seem so.

Let me propose an attractive (for me at least) way of making sense of this kind of example, while stressing that, as will become obvious, the following discussion only scratches at the surface of so many complex issues that I simply offer it in the spirit of making some first small steps in the direction of a new possible application of substructural logics to natural-language semantics. The basic workings behind the examples arguably operate already at the level of *singular* predication, so let's focus on that. Focussing on the first example, suppose, to fix ideas, that every piece wood that makes 4 chairs makes 1 bed and *vice versa*, and suppose further that w is a piece of wood big enough so that it "could" make 4 chairs and big enough so that it "could" make 1 bed, but not big enough so that it "could" make 4 chairs and 1 bed at the same time (henceforth assuming an understanding of the relevant occurrences of 'could' weak enough so that 'could make' and its likes are not subject to the issues we're discussing for 'makes' and its likes—an understanding that is arguably artificial, see fn 52). We may then accept 'w makes 4 chairs'. What about 'w makes 1 bed and makes 4 chairs'? By the extremely plausible principle of *collection*:

(COLL) If x makes i Ks and makes j Ls, x makes i Ks and j Ls,

the latter sentence entails 'w makes 1 bed and 4 chairs'. And, since the last sentence is manifestly false, by (COLL) (and contraposition) 'w makes 1 bed and makes 4 chairs' is false too, which explains why 'Every piece of wood that makes 4 chairs makes 1 bed and makes 4 chairs' is false. It is because of pieces of wood like w that (CONST) fails.

A natural objection to the reasoning of the last paragraph claims that 'and' (or similar expressions) in constructions like 'w makes 1 bed and makes 4 chairs' does not function to express conjunction (i.e. the usual operation on pairs of semantic values of sentences), so that the kind of example discussed is not after all relevant for (CONST), which instead does concern conjunction.[48] The claim may be supported by appeal to well-known cases of "collective" readings like 'Ann and Bill lifted the piano', where, according to orthodoxy, 'and' functions as an *aggregator* rather than as an *operator*: it can be contended that a similar analysis could be provided for 'w makes 1 bed and 4 chairs', and that, *via* (COLL), such analysis could then be extended to 'w makes 1 bed and makes 4 chairs'. There is an interesting issue whether, when so supported, this line of objection should still envisage also a "real-conjunction" reading of 'w makes 1 bed and makes 4 chairs' (which would seem predicted by standard syntax and semantics), and, if so, what it should say about it. Let's set that issue aside, however, and focus instead on the fact that this strategy for accounting for the felt strength of 'w makes 1 bed and makes 4 chairs' does not seem to be *general enough*. For suppose that w is not only big enough so that it could make 4

[48]Thanks to Dave Ripley and an anonymous referee for pressing this objection and, more generally, for discussions that led to crucial changes in the material concerning (CONST).

chairs, but also big enough so that it could feed 1,000 termites, although, again, not big enough so that it could make 4 chairs and feed 1,000 termites at the same time. Then 'w feeds 1,000 termites and makes 4 chairs' seems false for essentially the same reason as 'w makes 1 bed and makes 4 chairs' does (and could thus just as well be used against (CONST)), but it is very unclear how the strategy in question can account for its felt strength.

The central claim of the previous objection may however be supported in a less committing way by simply contending that *both* 'w makes 4 chairs' *and* 'w makes 1 bed' are true, and inferring from this that, since 'w makes 1 bed and makes 4 chairs' is false, 'and' (or similar expressions) in such a construction does not function as a conjunctive connective (for it is extremely plausible that conjunction satisfies the rule of *adjunction* $\varphi, \psi \vdash \varphi \,\&\, \psi$).[49] Again, let's set aside the interesting issue whether, when so supported, this line of objection should still envisage also a "real-conjunction" reading of 'w makes 1 bed and makes 4 chairs' (which would still seem predicted by standard syntax and semantics), and, if so, what it should say about it, and ask instead whether it is really the case that, as contended, both 'w makes 4 chairs' and 'w makes 1 bed' are true. Presented with the facts of the matter about w's properties, it would certainly be reasonable to come to accept 'w makes 4 chairs'. It would also certainly be reasonable to come to accept 'w makes 1 bed'. But, it seems to me, such reasonableness is *defeasible*. In particular, if one has already accepted 'w makes 4 chairs', it would no longer seem reasonable to accept 'w makes 1 bed' while continuing at the same time to accept also 'w makes 4 chairs'. This claim about *non-cotenability* seems to me in itself rather plausible, but I know from several conversations that it is less than uncontroversial, so let me try to buttress it further with a series of observations.

Firstly, the non-cotenability emerges more clearly in some *other* examples of the same kind: supposing that Simba is voracious enough so that it could need 4 steaks per day and voracious enough so that it could need 1 joint of roast beef per day, although not voracious enough so that it could need 4 steaks and 1 joint of roast beef per day at the same time, a zoo administrator should not accept at the same time both 'Simba needs 4 steaks per day' and 'Simba needs 1 joint of roast beef per day'. Secondly, the non-cotenability emerges more clearly even in our running example by *stacking up further functions* that w can play: given the facts of the matter about w's properties, one should not accept at the same time all of 'w makes 4 chairs', 'w makes 1 bed', 'w makes 8 baby chairs', 'w feeds 1,000 termites', 'w provides fire for 6 barbecues', 'w produces 2 lbs of paper', 'w carbonises into 10 lbs of coal', 'w takes up 1 square metre of ground', 'w offers support for many lichens and fungi', 'w is a good weight for ambitious body-builders'... Thirdly, in a situation in which one is interested in what to use to produce 4 chairs rather than in what to use

[49] Henceforth, I'll implicitly take for granted the principles of λ-*abstraction* and λ-*conversion*. I suppose that, in particular, some may want to resist such λ-abstractions as the one from 'w makes 4 chairs and w makes 1 bed' to 'w (is such that it) makes 4 chairs and makes 1 bed'. I think that that would be an interesting alternative strategy for accounting for the kind of example discussed in the text, a strategy whose main distinctive feature would be that of considering both 'w makes 4 chairs' and 'w makes 1 bed' (and so 'w makes 4 chairs and w makes 1 bed') true (contrary to the view that I'll be developing in reply to the objection in the text).

to produce 1 bed, one naturally accepts 'w makes 4 chairs', given which an assertion of 'w makes 1 bed' would be felt to *clash* with what one is accepting, to the point that the cheeky reply 'w does not make 1 bed, it makes 4 chairs' would sound appropriate, contrary to what would be expected if 'w makes 4 chairs' and 'w makes 1 bed' were co-tenable[50] (those tempted here by some sort of pragmatic manoeuvre, should first consider that no such behaviour is exhibited by an assertion of, say, 'One possible way to use w would be to make 1 bed out of it', which would at worst come across as a tendentious statement of the facts of the matter). Fourthly, starting from the same situation, when an equal interest arises in what to use to produce 1 bed the normal reaction is to assert something to the effect of *weakening* what was previously accepted rather than to the effect of *strengthening* it, and so to assert something like 'Well OK, either w makes 4 chairs or it makes 1 bed' rather than something like 'Very well then, it is also the case that w makes 1 bed', contrary to what would be expected if one were now accepting at the same time both 'w makes 4 chairs' and 'w makes 1 bed'. Fifthly, suppose that one sells w in a box for DIY customers. It would be felt as a cheat and indeed as straightforwardly false to write on the box *both* 'Makes 4 chairs' *and* 'Makes 1 bed'. But such assertions would both be true if both 'w makes 4 chairs' and 'w makes 1 bed' were true.

If the discussion of the previous objection is on the right track, it certainly raises a pressing issue. If, presented with the facts of the matter about w's properties, one cannot accept at the same time both 'w makes 4 chairs' and 'w makes 1 bed', *what should one accept*? It seems to me that *that will depend on the kind of situation in which one is*, and in particular on *what information would be useful* in that situation. Thus, in *ordinary* situations in which one is interested in what to use to produce 4 chairs, one would naturally accept 'w makes 4 chairs' and would not accept 'w makes 1 bed'. Conversely, in *ordinary* situations in which one is interested in what to use to produce 1 bed, one would naturally accept 'w makes 1 bed' and would not accept 'w makes 4 chairs'. In situations of *sustained theoretical reasoning* in which one is no more interested in what to use to produce 4 chairs than one is in what to use to produce 1 bed (since one is interested in neither), one would accept neither 'w makes 4 chairs' nor 'w makes 1 bed'. For, since the truth of 'Every piece of wood that makes 4 chairs makes 1 bed' becomes salient in such situations, and since in such situations there is much more pressure, all other things being equal, to follow through the valid arguments licenced by what one accepts (especially so when the relevant entailment becomes salient), by **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ one would have to accept 'w makes 1 bed', which, as I've argued in the last two paragraphs, cannot be accepted at the same time as 'w makes 4 chairs'; however, courtesy of the converse true claim 'Every piece of wood that makes 1 bed makes 4 chairs', analogous considerations apply if one accepts 'w makes 1 bed', so that one would be caught in an endless and pointless oscillation between accepting 'w makes 4 chairs' and not accepting 'w makes 1 bed' on the one hand and accepting 'w makes 1 bed' and not accepting 'w makes 4 chairs' on the other hand. Therefore, in situations of sustained

[50]Relatedly, given that one accepts 'w makes 4 chairs' it would seem disingenuous to lead someone else to accept 'w makes 1 bed', contrary to what would be expected if 'w makes 4 chairs' and 'w makes 1 bed' were co-tenable.

theoretical reasoning in which one is no more interested in what to use to produce 4 chairs than one is in what to use to produce 1 bed, it seems more reasonable to demur from accepting 'w makes 4 chairs' (and from accepting 'w makes 1 bed'), and fall back on accepting the more neutral 'Either w makes 4 chairs or w makes 1 bed'[51] (and, if one wishes to accept at the same time a sentence "more directly about" w making 4 chairs and a sentence "more directly about" w making 1 bed, one can accept at the same time both 'One possible way to use w would be to make 4 chairs out of it' and 'One possible way to use w would be to make 1 bed out of it').[52]

[51]Very interestingly, that disjunction is naturally read as entailing *either* of 'w makes 4 chairs' and 'w makes 1 bed' (but not *both* of them), in which case it would be in technical parlance a kind of "*free-choice disjunction*" (more specifically, in entailing (either but) not both disjuncts, it would arguably pattern with free-choice disjunctions in, say, *deontic* contexts rather than with free-choice disjunctions in, say, *epistemic* contexts). (Thus, if one accepts 'Either w makes 4 chairs or w makes 1 bed' under this reading, one accepts something that we may suppose saliently entails 'w makes 4 chairs' and saliently entails 'w makes 1 bed'. Given what I myself have said in the text, in a situation of sustained theoretical reasoning isn't there then a strong pressure to accept, say, 'w makes 4 chairs'? No, because, as I've put it, in such a situation the pressure is to the effect of following through the valid arguments licenced by what one accepts *all other things being equal*. But, since, in this particular case, by following through the valid argument from 'Either w makes 4 chairs or w makes 1 bed' to 'w makes 4 chairs' one would lose the theoretically attractive neutral position afforded by the free-choice disjunction, arbitrarily privileging that argument over the equally valid argument from 'Either w makes 4 chairs or w makes 1 bed' to 'w makes 1 bed' and then getting caught in the endless and pointless oscillation noted in the text, all other things are *not* equal.) Free-choice disjunctions have long been known to occur especially when they govern *modal* disjuncts, and so it is no surprise that they occur in our context, since the use of simple-present forms on which I'm focussing arguably does involve some sort of modal component ('w makes 4 chairs' can reasonably be paraphrased as 'w can make 4 chairs' under a natural reading of the latter, see fn 52). $\mathbf{IKT}^{\omega}$'s $\vee$ is clearly an inappropriate interpretation for the free-choice reading of 'Either w makes 4 chairs or w makes 1 bed', since $\varphi \vee \psi \vdash_{\mathbf{IKT}^{\omega}} \varphi$ does not hold. A rather obvious and very attractive alternative, both in the case of the kind of example discussed in the text and in the case of deontic contexts, is to interpret the free-choice reading using the *additive* "conjunction" that is available in non-contractive logics. (Barker [2010], following Lokhorst [1997], proposes to use a non-contractive logic for interpreting free-choice permissions. That specific proposal consists however in a construction with implication and additive "disjunction" rather than in straightforwardly using additive "conjunction": such interpretation has the unappealing features that K-L and K-R need to be rejected and that, as Barker [2010], pp. 24–26 himself notices, certain free-choice readings remain unaccounted for.) The proposed interpretation of free-choice permission provides an illuminating model of the patterns of reasoning arising in the kind of example discussed in the text: just as one starts with the neutral 'You may have English breakfast or you may have continental breakfast', and then, as a consumer of that information, one can—according to one's preferences—infer either 'You may have English breakfast' or 'You may have continental breakfast' (but not both) in order to guide one's actions, so one starts with the neutral 'w makes 4 chairs or w makes 1 bed', and then, as a consumer of that information, one can—according to one's preferences—infer either 'w makes 4 chairs' or 'w makes 1 bed' (but not both) in order to guide one's actions. This is of course not the place to pursue these issues further, but it was important here to give a sense of how the kind of example discussed in the text can plausibly be seen as relating to better-known linguistic phenomena (compare the discussion of related issues in Zardini [2014i]).

[52]Notice that simple *modalisations* of the relevant sentences along the lines of 'w can make 4 chairs' would not seem to offer a sentence "more directly about" w making 4 chairs and a sentence "more directly about" w making 1 bed that one can accept at the same time, for, contrary to what textbook theories of modality might suggest and as I've already implied in fn 51, *under a natural reading such modalisations would actually seem to exhibit, in the relevant respects, the same behaviour of the corresponding not overtly modal sentences.*

In order to acquire further understanding of the import of the kind of example under discussion, it will now pay to devote some comments to Beall et al. [2006], one of the few works I know of concerning restricted quantification in the context of naive theories of truth (I'll henceforth refer to the authors as 'BBHPR'). These authors list a series of plausible principles of restricted quantification (including some of the Principles), some of which concern the logical relations of the traditional *square of opposition* that are usually claimed in modern times to be (only) acceptable under the assumption that *the relevant properties have something that exemplifies them.*

Let's take for example the relation of *subalternation* between 'Every F is G' and 'Some F is G'. Now, if we understand the assumption that the property expressed by φ has something that exemplifies it as having the form $\exists\xi\varphi$ (as BBHPR do and as it's natural to do in *modern classical logic*), so that what is claimed to hold is in effect $\mathcal{E}\xi(\varphi, \psi), \exists\xi\varphi \vdash \mathcal{S}\xi(\varphi, \psi)$, the claim is arguably mistaken in view of the examples just presented. On the one hand, 'Every piece of wood that makes 4 chairs makes 1 bed' (an instance of the first premise) is true. On the other hand, 'Some piece of wood that makes 4 chairs makes 1 bed' (the relevant instance of the conclusion) is false if we suppose that there is no piece of wood big enough so that it could make 4 chairs and 1 bed at the same time (a supposition that I'll henceforth refer to with '`Supposition`'). For, contraposing on (COLL), `Supposition` entails that there is no piece of wood that makes 4 chairs and makes 1 bed, from which it in turn follows, under `Translation` (ii), that 'Some piece of wood that makes 4 chairs makes 1 bed' is false. And, if the first premise of $\mathcal{E}\xi(\varphi, \psi), \exists\xi\varphi \vdash \mathcal{S}\xi(\varphi, \psi)$ is true and the conclusion false, if that rule is valid it is natural to conclude that the second premise ('Something makes 4 chairs') is false,[53] and so, *a fortiori*, that 'w makes 4 chairs' is false. But that is arguably too strong: not only would it prevent us from accepting 'w makes 4 chairs' (contrary to what I've argued in the second last paragraph we should be able to do in certain ordinary situations); an analogous argument would also allow us to conclude that 'w makes 1 bed' too is false, and the joint falsity of these two sentences would contradict the apparent truth of their (non-free-choice) disjunction 'Either w makes 4 chairs or w makes 1 bed'.

If we instead understand the assumption that the property expressed by φ has something that exemplifies it as having the form $\mathcal{S}\xi(\varphi, \varphi)$ (as it's natural to do in *traditional syllogistic*),[54] so that what is claimed to hold is in effect $\mathcal{E}\xi(\varphi, \psi), \mathcal{S}\xi(\varphi, \varphi) \vdash \mathcal{S}\xi(\varphi, \psi)$,

[53] Since the first premise is also true-only and the conclusion also false-only, if that rule is valid it is also natural to conclude that the second premise is false-only, even in the kind of *paraconsistent* framework assumed by BBHPR (which is in other respects inhospitable to *antilogisms*).

[54] For one thing, traditional syllogistic typically works with restrictedly quantified sentences like $\mathcal{S}\xi(\varphi, \varphi)$ rather than with unrestrictedly quantified sentences like $\exists\xi\varphi$. More importantly, if the aim is to find an argument in the vicinity of subalternation that can be justified by appeal to syllogistic forms of reasoning that are uncontroversial even in modern times, the by far most natural route goes *via* $\mathcal{E}\xi(\varphi, \psi), \mathcal{S}\xi(\chi, \varphi) \vdash \mathcal{S}\xi(\chi, \psi)$ (an argument form known as *Darii* in traditional syllogistic), which, putting φ for χ, yields precisely $\mathcal{E}\xi(\varphi, \psi), \mathcal{S}\xi(\varphi, \varphi) \vdash \mathcal{S}\xi(\varphi, \psi)$. Notice that such route is unavailable if, instead of $\mathcal{S}\xi(\varphi, \varphi)$, we work with $\exists\xi\varphi$ massaged into particular-affirmative form as, say, $\mathcal{S}\xi(\xi = \xi, \varphi)$. For then *Darii* only yields $\mathcal{E}\xi(\varphi, \psi), \mathcal{S}\xi(\xi = \xi, \varphi) \vdash \mathcal{S}\xi(\xi = \xi, \psi)$, while $\mathcal{E}\xi(\varphi, \psi), \mathcal{S}\xi(\xi = \xi, \varphi) \vdash \mathcal{S}\xi(\varphi, \psi)$ does not even have the form of a syllogism in the first place (although it is of course a valid rule in modern classical logic).

the claim is no longer threatened by the examples just presented. For, although a reasoning analogous to the one in the last paragraph can conclude that 'Some piece of wood that makes 4 chairs makes 4 chairs' is false given `Supposition`, under `Translation` (ii) the falsity of that sentence does not have in $\mathbf{IKT}^{\omega}$ the negative consequences that were observed in the last paragraph (in $\mathbf{IKT}^{\omega}$, the falsity of 'Some piece of wood makes 4 chairs and makes 4 chairs' does not entail the falsity of 'w makes 4 chairs'). In fact, 'Some piece of wood that makes 4 chairs makes 4 chairs' seems anyways false given `Supposition`, since, under `Translation` (ii), it is equivalent with 'Some piece of wood makes 4 chairs and makes 4 chairs', which, by (COLL), entails 'Some piece of wood makes 4 chairs and 4 chairs', which, since 4 chairs and 4 chairs are 8 chairs, is in turn arguably equivalent with 'Some piece of wood makes 8 chairs', which is false given `Supposition`. Indeed, $\mathcal{E}\xi(\varphi,\psi), \mathcal{S}\xi(\varphi,\varphi) \vdash \mathcal{S}\xi(\varphi,\psi)$ does seem a valid rule, since, as I've remarked in fn 54, it is a special case of the extremely plausible syllogistic mood *Darii*. It is thus pleasing to observe that, under `Translation` (i)–(ii), $\mathbf{IKT}^{\omega}$ validates *Darii* in general and so that rule in particular (and does not validate the rule criticised in the last paragraph). More generally, once we understand the assumption that the property expressed by φ has something that exemplifies it as having the form $\mathcal{S}\xi(\varphi,\varphi)$, under `Translation` $\mathbf{IKT}^{\omega}$ validates all the principles of restricted quantification listed by BBHPR as desirable.[55,56]

[55]A corresponding claim holds once we understand the assumption that the property expressed by φ has something that exemplifies it as the assumption that the system $\vdash$ in question is such that, for some closed singular term τ, $\oslash \vdash \varphi_{\tau/\xi}$ holds, so that what is claimed to hold is in effect that, if $\vdash$ is extended so that, for some closed singular term τ, $\oslash \vdash \varphi_{\tau/\xi}$ holds, then $\vdash$ validates the relevant principle.

[56]It is worth mentioning that BBHPR actually reject `Translation` (iii) (while accepting `Translation` (i)–(ii)). They do so because, in the kind of *paraconsistent* framework they assume, under `Translation` (iii) the subalternation $\mathcal{N}\xi(\varphi,\psi), \exists\xi\varphi \vdash \mathcal{S}\xi(\varphi,\neg\psi)$ does not hold (for essentially the same reason for why material implication in their framework does not satisfy **MODUS PONENS**). They thus propose to represent $\mathcal{N}\xi(\varphi,\psi)$ as $\forall\xi(\varphi \rightsquigarrow \neg\psi)$ instead (where $\rightsquigarrow$ is a non-material conditional connective which they also use for representing restricted universal quantification and which does satisfy **MODUS PONENS**). But it is not clear to me that in this case the cure is better than the disease, since such rejection of `Translation` (iii) has the startling consequence that $\neg\exists\xi(\varphi \,\&\, \psi) \vdash \mathcal{N}\xi(\varphi,\psi)$ does not hold: on BBHPR's proposal, even if nothing whatsoever is both F and G (and so even if it not the case that some F is G, and so even if there are no counter-instances to 'No F is G'), it does not follow that no F is G (thus, on the proposal, 'Some F is G' and 'No F is G' are no longer *contradictories* in the sense that the falsity of the former does no longer entail the truth of the latter, which goes against both the traditional square of opposition and its modern remains). A related objectionable feature of BBHPR's proposal which concerns directly our discussion is that, although it does yield $\oslash \vdash \mathcal{E}\xi(\varphi,\psi), \mathcal{S}\xi(\varphi,\neg\psi)$ and similar dilemmas, it does not validate the arguably more fundamental **NO COUNTER-INSTANCE**$^{\Rightarrow}$: on BBHPR's proposal, even if it is not the case that some F is not G, and so even if there are no counter-instances to 'Every F is G', it does not follow that every F is G (thus, on the proposal, 'Some F is not G' and 'Every F is G' are no longer *contradictories* in the sense that the falsity of the former does no longer entail the truth of the latter, which again goes against both the traditional square of opposition and its modern remains). (A similar "dilemma-without-rule" situation arises for the extent to which BBHPR's proposal validates the *subcontrariety* between $\mathcal{S}\xi(\varphi,\psi)$ and $\mathcal{S}\xi(\varphi,\neg\psi)$.) Another related objectionable feature of BBHPR's proposal which concerns less directly our discussion is that, as they themselves note (p. 589), in their paraconsistent framework *contraposition of restricted universal quantification* is jointly incompatible with *monotonicity of restricted universal quantification over unrestricted universal quantification* ($\forall\xi\varphi \vdash \mathcal{E}\xi(\psi,\varphi)$) and **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$. *Prima facie* not unreasonably, BBHPR opt for rejecting contraposition of restricted universal quantification, but, given BBHPR's representation of restricted universal and null quantification, failure of contraposition

The foregoing discussion of (CONST) enables us to make sense of the failure of other two rules in the theory of restricted quantification available in $\mathbf{IKT}^{\omega}$: $\mathcal{E}\xi(\varphi, \varphi \supset \psi) \vdash \mathcal{E}\xi(\varphi, \psi)$ and $\exists\xi\varphi \vdash \mathcal{S}\xi(\varphi, \varphi)$. The failure of the first rule may be regarded as problematic as it may suggest some violation of at least the spirit of **MODUS PONENS**: if every F is such that, if it is F, it is G, doesn't the spirit of **MODUS PONENS** require that every F is indeed G? It arguably doesn't. For, arguably, what it only requires is that, if some object x is both H_0 and H_1, and the property of being H_0 and the property of being H_1 together entail by **MODUS PONENS** the property of being H_2, x is H_2 (presumably, this has nothing to do in particular with **MODUS PONENS**, so that corresponding requirements hold for other principles). So arguably, in our case, what the spirit of **MODUS PONENS** only requires is that, if every F is both [such that, if it F, it is G] and [such that it is F], every F is G. But 'Every F is both [such that, if it F, it is G] and [such that it is F]' only follows from 'Every F is such that, if it F, it is G' if (CONST) holds. Thus, if (CONST) fails (and I've argued that it does), the spirit of **MODUS PONENS** is not violated by the failure of $\mathcal{E}\xi(\varphi, \varphi \supset \psi) \vdash \mathcal{E}\xi(\varphi, \psi)$.[57]

of restricted universal quantification has as startling consequence failure of *conversion of restricted null quantification* ($\mathcal{N}\xi(\varphi, \psi) \vdash \mathcal{N}\xi(\psi, \varphi)$; given the natural connection between restricted universal and null quantification $\mathcal{E}\xi(\varphi, \neg\psi) \dashv\vdash \mathcal{N}\xi(\varphi, \psi)$, conversion of restricted null quantification is probably the strongest reason in favour of contraposition of restricted universal quantification). Unsurprisingly, an analogous situation holds for analetheic theories of truth, as can be appreciated from at least two "Pseudo-Scottish" arguments analogous to those available for dialetheic theories. As for the first argument, if one rejects φ for every object whatsoever, or if φ is logically absurd for every object whatsoever, by *vacuity of restricted universal quantification* one accepts $\mathcal{E}\xi(\varphi, \psi)$ (with ψ arbitrary), and so, by contraposition of restricted universal quantification, one accepts $\mathcal{E}\xi(\neg\psi, \neg\varphi)$, and hence, if one accepts $\neg\psi_{\tau/\xi}$, by **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ one accepts $\neg\varphi_{\tau/\xi}$, wherefore one does not reject it. As for the second argument, a natural version thereof can be presented by writing the fourth paradigmatic construction of restricted quantification considered by traditional syllogistic 'Not every F is G' (*restricted non-universal quantification*) as $\mathcal{O}\xi(F\xi, G\xi)$ (and by assuming in this context that, if $\Gamma \vdash \Delta$ holds, then, just as "going forwards" one should [accept that some member of Δ holds if one accepts that every member of Γ holds], "going backwards" one should [reject that every member of Γ holds if one rejects that some member of Δ holds]). By *non-vacuity of restricted non-universal quantification*, if one rejects φ for every object whatsoever one rejects $\mathcal{O}\xi(\varphi, \psi)$ (with ψ arbitrary), and so, by *contraposition of restricted non-universal quantification*, one rejects $\mathcal{O}\xi(\neg\psi, \neg\varphi)$, and hence, if one also rejects $\neg\varphi_{\tau/\xi}$, by *restricted non-universal instantiation* (${\chi_0}_{\tau/\xi} \vdash {\chi_1}_{\tau/\xi}, \mathcal{O}\xi(\chi_0, \chi_1)$) one rejects $\neg\psi_{\tau/\xi}$, wherefore one does not accept it (restricted non-universal quantification could naturally be represented at the level at which `Translation` operates by using the *unconditional* connective discussed in Zardini [2014g]).

[57]Ripley [2013], p. 157 puts forth a broadly related objection focussing on the fact that, under `Translation` (i), $\mathcal{E}\xi(\varphi, \mathcal{E}\xi(\varphi, \psi)) \vdash \mathcal{E}\xi(\varphi, \psi)$ does not hold in $\mathbf{IKT}^{\omega}$ (nor in many other naive theories of truth). In the notation of this paper, he asks: "[. . .] all Fs are such that the conclusion holds, and what besides all the Fs could matter for whether all Fs are Gs?". However, the conclusion is something *absolute* to the effect that every F is G, not something *relative* to the effect that "it" is G, and, for any conclusion of the former kind, whether every F is such that it holds should by default be no more relevant for its holding than, for every other property H, whether every H is such that it holds. In fact, under `Translation` (i), in $\mathbf{IKT}^{\omega}$ the relevance of any restricted universal quantification for the holding of conclusions of the former kind follows in the relevant respects the extremely plausible pattern familiar from classical logic: for any conclusion P of the former kind, if a certain object a is F, then every F is such that P only if P (by **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$); if instead there are no Fs, every F is such that P no matter whether P (by **NO COUNTER-INSTANCE**$^{\Rightarrow}$ or vacuity of

The failure of the second rule I regard as something that, even *prima facie*, lies more on the quirky rather than disturbing side. And the quirk becomes fully intelligible once we realise that (as I've argued in the last paragraph) the reason why (CONST) fails also implies that there is a crucial difference in logical strength between $\exists\xi\varphi$ and $\mathcal{S}\xi(\varphi,\varphi)$.

Another way to look at the failure of $\exists\xi\varphi \vdash \mathcal{S}\xi(\varphi,\varphi)$ relies on the observation, implicit in much of the foregoing discussion of (CONST), that, in the kind of example under discussion, *judging twice the same sentence is more committing than judging it once*: if one judges twice that w makes 4 chairs, one can use one of these two judgements to *derive* that w makes 1 bed (given that every piece of wood that makes 4 chairs makes 1 bed) while also using the other judgement to *uphold* that w makes 4 chairs, given which one would then extremely plausibly both be committed to accepting 'w makes 1 bed' and be committed to accepting 'w makes 4 chairs', which I've argued in the sixth and seventh last paragraphs to be an extremely problematic set of commitments (contrary to the commitment to accepting 'w makes 4 chairs').[58] And what goes for *judging* also goes for other acts like *supposing*, *denying*, *inferring* etc. Now, arguably, if one supposes 'Some F is F', truth-conditionally one at least indirectly supposes twice of some (supposed) object x that x is F, and so one's overall supposition is more committing than one's supposition that x is F. Again, $\exists\xi\varphi \vdash \mathcal{S}\xi(\varphi,\varphi)$ fails. This latter perspective on the failure of $\exists\xi\varphi \vdash \mathcal{S}\xi(\varphi,\varphi)$ is also useful because it allows us to see that, since presumably the act of *restricting* follows in this respect the same pattern as the acts mentioned above, the deftness of the theory of restricted quantification available in $\mathbf{IKT}^{\omega}$ is further confirmed by the fact that, together with (R&/RR), it entails that restricting twice to the Fs is more committing than restricting once to the Fs (since it entails that restricting to the F & Fs is more committing than restricting to the Fs).[59,60]

restricted universal quantification). Thus, since $\mathcal{E}\xi(\varphi,\mathcal{E}\xi(\varphi,\psi))$ alone clearly does not even entail $\exists\xi\varphi$, it would not seem that Ripley's thought offers enough materials for arguing that $\mathcal{E}\xi(\varphi,\mathcal{E}\xi(\varphi,\psi)) \vdash \mathcal{E}\xi(\varphi,\psi)$ holds (notice that, under `Translation` (i), by **RESTRICTED UNIVERSAL INSTANTIATION**$^{\mathrm{r}}$ $\mathcal{E}\xi(\varphi,\mathcal{E}\xi(\varphi,\psi)),\varphi_{\tau/\xi} \vdash \mathcal{E}\xi(\varphi,\psi)$ does hold in $\mathbf{IKT}^{\omega}$).

[58] Contrary to misguided parodies of these ideas which I've sometimes come across, this is obviously not to deny that, if one has nothing better to do, one can token in thought and talk 'w makes 4 chairs' as many times as one pleases without increase in commitment, as long as such tokenings are understood to be *repetitions of the same old judgement* rather than *realisations of fresh new judgements*, and so to *lack an autonomous cognitive life*.

[59] Another worry about (CONST) should at least be mentioned given its relevance for the semantic paradoxes. Essentially by **SELF-INCLUSION**$^{\mathrm{l}}$, *transparent* theories of truth entail the existence of "*self-negating*" properties: properties whose exemplification consists in their not being exemplified, and so such that *everything that exemplifies them doesn't and* vice versa. But, given the extremely plausible principle that *nothing exemplifies a contradiction*, (CONST) is virtually incompatible with the existence of such properties.

[60] Field [2014] is a more recent work concerning restricted quantification in the context of naive theories of truth. With one exception, under `Translation` (i) $\mathbf{IKT}^{\omega}$ validates all the principles of restricted quantification listed by Field as desirable (what Field's own theory does too) with all of their contrapositives and exportations and without introducing two different implications (what Field's own theory does not). The one exception is $\mathcal{E}\xi(\varphi,\psi),\mathcal{E}\xi(\varphi,\chi) \vdash \mathcal{E}\xi(\varphi,\psi\,\&\,\chi)$, which, since, by **SELF-INCLUSION**$^{\mathrm{l}}$ and transitivity, it entails (CONST), is actually at least as problematic as (CONST) itself. (Under `Translation` (i), $\mathbf{IKT}^{\omega}$ does validate $\mathcal{E}\xi(\varphi_0,\varphi_1),\mathcal{E}\xi(\varphi_2,\varphi_3) \vdash \mathcal{E}\xi(\varphi_0\,\&\,\varphi_2,\varphi_1\,\&\,\varphi_3)$, which is arguably the salient truth in the vicinity left unscathed by the failure of (CONST).) Field's own theory is analetheic,

Summing up our discussion of (CONST), although that principle is *prima facie* plausible, it (and other related rules that fail in the theory of restricted quantification available in $\mathbf{IKT}^{\omega}$) is put under pressure by at least one kind of example to be found in natural languages.[61] A case can thus be made that $\mathbf{IKT}^{\omega}$'s failure to declare (CONST) unrestrictedly valid should after all be seen not as a cost, but as yet another advantage of the theory of restricted quantification available in $\mathbf{IKT}^{\omega}$.

7 Conclusion

In this paper, I've argued that certain features traditionally associated with implication, diverse and unrelated as they may seem, are actually all essential to the very familiar and all-important operation of restricted quantification, and that this circumstance creates problems for non-substructural as well as for non-transitive naive theories of truth. I've then investigated in the relevant respects the theory of restricted quantification available in my favoured non-contractive theory $\mathbf{IKT}^{\omega}$, showing that it does not suffer from the problems I've identified for the other theories and arguing that it can successfully address an important problem concerning restricted quantification that arises specifically for it. Naive truth succeeds in restricting by failing to contract.

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and so it is subject to the problems developed in sections 3 and 4 and in fn 56 (Field treats explicitly only restricted universal quantification assuming `Translation` (i), but does comment in passing (p. 154) on the truth-conditional plausibility of `Translation` as a whole). In particular, under `Translation` and its higher-order analogues, Field's theory does not validate **NO COUNTER-INSTANCE**$^{\Leftarrow}$, (R&/RR), **RESTRICTED UNIVERSAL GENERALISATION**$^{\mathrm{mr}}$, **RESTRICTED UNIVERSAL GENERALISATION**$^{\mathrm{mc}}$, **UNIVERSAL-PREDICATION CLOSURE**, **RESTRICTED NULL GENERALISATION**$^{\mathrm{mr}}$, **RESTRICTED NULL GENERALISATION**$^{\mathrm{mc}}$, **NULL-PREDICATION COUNTER-CLOSURE**, vacuity of restricted universal quantification and, taking $\mathcal{O}\xi(\varphi, \psi)$ to be tantamount to $\neg\mathcal{E}\xi(\varphi, \psi)$, restricted non-universal instantiation.

[61] (CONST) is in effect a crucial component of the "determiner universal" proposed by Barwise and Cooper [1981], pp. 178–179 and taken up by much of the subsequent literature on generalised quantifiers in natural languages. At least one natural idea behind that proposed universal is that a binary quantifier should act as a proper *restrictor*, and so *only look at the objects that exemplify the property expressed by the formula in its first argument to check if they behave in a certain way with respect to the property expressed by the formula in its second argument.* However, contrary to (CONST), that idea is *not* threatened by the kind of example discussed in the text: for instance, 'Every piece of wood that makes 4 chairs makes 1 bed' only looks at the pieces of wood that make 4 chairs to check if each of them makes 1 bed (which they do). It's understandable that one tries at a first pass to cash out such idea by imposing the proposed universal, but to do so is to ignore the possibility that the objects the quantifier looks at behave in a certain way with respect to the property expressed by the formula in its second argument *only by not exemplifying the property expressed by the formula in its first argument* (in which case, in a sense, the *predication* operated by the formula in the second argument *goes beyond* the *restriction* operated by the formula in the first argument).

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