Truth, Demonstration and Knowledge: A Classical Solution to the Paradox of Knowability*

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Abstract

After introducing semantic anti-realism and the paradox of knowability, the paper offers a reconstruction of the anti-realist argument from understanding. The proposed reconstruction validates an unrestricted principle to the effect that truth requires the existence of a certain kind of "demonstration". The paper shows that that principle fails to imply the problematic instances of the original unrestricted feasible-knowability principle but that the overall view underlying the new principle

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still has unrestricted epistemic consequences. Appealing precisely to the paradox of knowability, the paper also argues, against the BHK semantics, for the non-constructive character of the demonstrations envisaged by semantic anti-realism, and contends that, in such setting, one of the most natural arguments for a broadly intuitionist revision of classical logic loses all its force.

1 Semantic Anti-Realism

Semantic anti-realism (henceforth simply '*anti-realism*') is the doctrine that *there is a conceptual connection between truth and our recognition of it*. As qualifiedly applying to a particular discourse D, anti-realism is the doctrine that there is a conceptual connection between the truth of sentences belonging to D and our recognition of it.[1] Henceforth assuming a suitably simplified version of English to be the language used by the discourse in question, this conceptual connection has typically been supposed to be captured by the formulation of an *epistemic constraint* on the notion of truth operating over the discourse, in particular by the principle of *feasible knowability of truth*:

(TFPK) If 'P' is true, it is feasibly knowable that P,

where the *substitution class* of the *substitutional variable* 'P' will henceforth be understood to be restricted to sentences supposed to be included in the scope of the relevant form of anti-realism, and where it will henceforth be left to context to disambiguate exactly which *quotation environment* is activated by single quotation marks (for example, in the case of (TFPK) quotation marks activate an *autonymous* quotation environment).[2]

[1] Call the unqualified form of anti-realism, applying to *whichever* discourse, '*global anti-realism*'. Call a qualified form of anti-realism, applying only to a *particular* discourse D, '*local anti-realism with respect to D*'. Henceforth assuming that sentences talking about knowledge concerning a certain discourse themselves belong to that discourse, the discussion of this paper is insensitive to the distinction between global and local anti-realism.

[2] Notice that, throughout, expressions will be individuated in such a way as to make redundant the usual *relativization to languages* of the truth predicate. Moreover, notice that, if we want to preserve their disquotational character, (TFPK) and subsequent principles must be restricted to *unambiguous* and *non-context-dependent* sentences; alternatively, assuming context also to resolve ambiguity 'true' and its like must be replaced by 'true-in-the-present-context' and its like (see e.g. Zardini [2008], pp. 550–561; Zardini [2012a]; Zardini [2015a], pp. 49–52 for discussions of the damning effects of context dependence on disquotational principles of all kinds). For ease of discussion, I will henceforth ignore ambiguity and context dependence. Furthermore, notice that, throughout, one can replace *substitutional* quantification with, roughly, *propositional* quantification *restricted* to propositions expressed by sentences of English (plus further restrictions that may be operative at the relevant passage). Less restricted feasible-knowability principles using propositional quantification are also in themselves interesting, but are not directly relevant for the topic of this paper, whose foundations lie in what *understanding an expression* ultimately consists in. Finally notice that, although *in general* an epistemic constraint need not go the other way too (consider for example a principle to the effect that truth only requires the existence of some evidence), all the *specific* epistemic constraints considered in this paper use notions that are strong enough to guarantee that their converse directions hold too (although I will not usually make this explicit). Thanks to Gabriel Uzquiano for discussion of some of these points.

Three features usually associated with the *modal epistemic* operator 'it is feasibly knowable that' are worth remarking upon right at the outset. Firstly, the operator is intended to be *factive* in the sense that its being feasibly knowable that P entails that P (call this 'the *factivity constraint*').[3] Secondly, that the knowability in question is a *feasible* one means that the relevant possible situations witness to a claim of feasible knowability are situations concerning beings endowed *with our actual cognitive powers* or, at most, with some *finite extensions* thereof (call this 'the *finitude constraint*').[4] Thirdly, that the knowability in question is a *feasible* one means that the relevant possible situations witness to a claim of feasible knowability are situations in which the available evidence is *constrained by the present state of the actual world*—that is, by the state at which the claim of feasible knowability is to be evaluated (call this 'the *accessibility constraint*').[5]

The rest of this paper is organized as follows. Section 2 introduces a particular, unrestricted form of anti-realism and a problem posed to it by a well-known reasoning, the paradox of knowability. Section 3 offers a reconstruction of an influential argument from understanding yielding, for semantically simple sentences, the (TFPK)-version of anti-realism. Sections 4 and 5 develop the argument further with respect to semantically complex sentences, yielding, for sentences in general, a different version of unrestricted anti-realism to the effect that truth requires the existence of a certain kind of "demonstration". Section 6 shows that that principle fails to imply the problematic instances of (TFPK) but that the overall view underlying the new principle still has unrestricted epistemic consequences. Appealing precisely to the paradox of knowability, section 7 argues, against the BHK semantics, for the non-constructive character of the demonstrations envisaged by anti-realism, and contends that, in such setting, one of the most natural arguments for a broadly intuitionist revision of classical logic loses all its force.

[3]Throughout, I will not essentially presuppose any particular *semantic structure* in the operator 'it is feasibly knowable that' and in its like. I should note though that it is an appealing idea to treat feasible knowability as *some sort of possibility of knowledge*, even though carrying out such idea in detail is no trivial task, especially insofar as satisfying the factivity constraint is concerned, given that possibility is *not in general factive* (see Zardini [2007]; [2012b]; [2015c] for discussion of some of the difficulties involved). Thanks to an anonymous referee for urging this clarification.

[4]I myself am no friend of such and similar constraints (see fn 34 and Zardini [2015b] for some discussion), but I will typically assume it to dispel the likely impression that the view developed in this paper is incompatible with it. Thanks to an anonymous referee for pressing me to be more explicit about this.

[5]To clarify, the cash value of the accessibility constraint is supposed to be that, if it is feasibly knowable that P, it can be known that P by relying only on something that is *present and accessible in the actual world at the present time*. That plausibly implies, for example, that only extant records are admissible evidence for evaluating the feasible knowability of facts concerning the past. The accessibility constraint is thus more controversial than the factivity or finitude constraint (see Dummett [1969] for a seminal discussion). But, as we will see, it is a natural component of the view developed in this paper; moreover, it is worthwhile showing that the relevant form of anti-realism can be stabilized even under its assumption. Thanks to an anonymous referee for comments on an earlier version of the material concerning the accessibility constraint.

2 The Paradox of Knowability

Unrestricted anti-realism is the doctrine that, whichever principle (such as (TFPK)) is epistemically to constrain the truth of sentences belonging to a discourse, *every* instance of it holds.[6] Under some natural assumptions, the (TFPK)-version of unrestricted anti-realism is refuted by the following simple reasoning, originally published in Fitch [1963], pp. 138–139 (but most likely due to Alonzo Church, see Salerno [2009], pp. 34–37) and known as 'the *paradox of knowability*'.

Start with the assumption that it is known[7] that [P and it is not known that P].[8] Then, by distribution of knowledge over conjunction, it is known that P and it is known that it is not known that P. By simplification on the first conjunct, it is known that P. By simplification on the second conjunct, it is known that it is not known that P. By factivity of knowledge, it is not known that P. Contradiction. By *reductio*, it is not known that [P and it is not known that P]. By necessitation, necessarily, it is not known that [P and it is not known that P]. Given that metaphysical necessity (of ignorance) entails feasible necessity (of ignorance), and therefore the negation of feasible possibility (of knowledge), this result, together with the contraposed (TFPK)-version of unrestricted anti-realism, entails that 'P and it is not known that P' is untrue. Together with the principle of *disquotation for truth*:

(DT) 'P' is true iff P,

this yields in turn the result that there are no unknown truths.[9]

[6] Notice that the distinction between unrestricted and restricted anti-realism is *orthogonal* to the distinction between global and local anti-realism (see fn 1). Notice also that the (TFPK)-version of unrestricted anti-realism (and, more generally, similarly disquotational versions of unrestricted anti-realism, see fn 2) might need some restriction at least on some approaches to the *semantic paradoxes* (but not on mine, see e.g. Zardini [2011]). (This seems overlooked by Kallestrup [2007], who oddly seems to take the ensuing epistemic paradox straightforwardly to refute unrestricted anti-realism. The oddity is brought out, for example, by the fact that, within his overall argument, Kallestrup accepts a sub-argument which does not rely on anti-realism and whose conclusion—which he presumably regards as *proved*— is a sentence saying of itself that it is *unknowable*. Kallestrup's argument is inspired by Milne [2007], in turn inspired by Milne [2005]. The whole style of argument is criticized by López de Sa and Zardini [2006]; [2007]; [2011]; Zardini [2015d].) Although it is unclear to me, in case such restriction were indeed needed, exactly what if any substantially new problem for the relevant version of unrestricted anti-realism would be posed by the considerations presented in this section, I will henceforth ignore the semantic paradoxes.

[7] Throughout, read 'it is known that' and its relatives as implicitly existentially quantifying over subjects and times.

[8] Throughout, I use square brackets to disambiguate constituent structure in English.

[9] Use the contraposed right-to-left direction of (DT) to go from the untruth of 'P and it is not known that P' to its negation; use the left-to-right direction of (DT) to go from there to the negation of ''P' is true and it is not known that P'; use substitutional universal generalization to go from there to everything being not both true and unknown. Using an appropriate natural-deduction system, the whole reasoning can be formalized thus:

1	(1)	$\exists st\mathcal{K}_{s,t}(P \wedge \neg\exists st\mathcal{K}_{s,t}P)$	assumption
1	(2)	$\exists st\mathcal{K}_{s,t}P \wedge \exists st\mathcal{K}_{s,t}\neg\exists st\mathcal{K}_{s,t}P$	1 $\exists st\mathcal{K}_{s,t}$-distribution

Classically, this result is equivalent to every truth's being known. If you accept *classical* logic and think that it is not the case that every truth is known, you had better reject the (TFPK)-version of unrestricted anti-realism. If you accept *intuitionist* logic and think that there are unknown truths, you too had better reject the (TFPK)-version of unrestricted anti-realism (and you had better reject it even if you only accept the intuitionistically weaker claim that it is not the case that there are no unknown truths).[10]

Should we take the foregoing reasoning to give us reasons for disbelieving unrestricted anti-realism *in general*? No, for there is at least one way of reconstructing what the advertized connection between truth and our recognition of it consists in which yields a principle different from (TFPK) and a version of unrestricted anti-realism which, contrary to the (TFPK)-version, does not fall prey to the paradox of knowability, or so I shall argue.

3 Anti-Realism and Manifestation

One major line of argument for anti-realism, taking its lead from the *theory of understanding*, runs like this.[11] What does *understanding* an expression ultimately consist in? In some *ability* or other to *use* the expression,[12] otherwise understanding would be an

1	(3)	$\exists st\mathcal{K}_{s,t}P$	2 simplification
1	(4)	$\exists st\mathcal{K}_{s,t}\neg\exists st\mathcal{K}_{s,t}P$	2 simplification
1	(5)	$\neg\exists st\mathcal{K}_{s,t}P$	4 $\exists st\mathcal{K}_{s,t}$-factivity
	(6)	$\neg\exists st\mathcal{K}_{s,t}(P\wedge\neg\exists st\mathcal{K}_{s,t}P)$	3,5 *reductio*
	(7)	$\Box\neg\exists st\mathcal{K}_{s,t}(P\wedge\neg\exists st\mathcal{K}_{s,t}P)$	6 $\Box$-necessitation
	(8)	$\blacksquare\neg\exists st\mathcal{K}_{s,t}(P\wedge\neg\exists st\mathcal{K}_{s,t}P)$	7 $\Box\varphi\Rightarrow\blacksquare\varphi$
	(9)	$\neg\blacklozenge\exists st\mathcal{K}_{s,t}(P\wedge\neg\exists st\mathcal{K}_{s,t}P)$	8 $\blacksquare\neg\varphi\Rightarrow\neg\blacklozenge\varphi$
	(10)	$\neg T`P\wedge\neg\exists st\mathcal{K}_{s,t}P'$	9 (TFPK)
	(11)	$\neg(P\wedge\neg\exists st\mathcal{K}_{s,t}P)$	10 $\subset$-direction of (DT)
	(12)	$\neg(T`P'\wedge\neg\exists st\mathcal{K}_{s,t}P)$	11 $\supset$-direction of (DT)
	(13)	$\Pi P\neg(T`P'\wedge\neg\exists st\mathcal{K}_{s,t}P)$	12 substitutional universal generalization
	(14)	$\neg\Sigma P(T`P'\wedge\neg\exists st\mathcal{K}_{s,t}P)$	13 $\Pi\varepsilon\neg\varphi\Rightarrow\neg\Sigma\varepsilon\varphi$,

where $\mathcal{K}_{\tau_0,\tau_1}\varphi$ formalizes 'τ_0 knows at τ_1 that φ', and $\blacksquare$ and $\blacklozenge$ are operators of feasible-necessity and feasible-possibility respectively.

[10] Williamson [1982] would have an anti-realist swallow the pill and accept that that there are no unknown truths. Setting aside more obvious problems, I argue in Zardini [2015e] that Williamson's proposal is unstable.

[11] What follows is obviously very much inspired by Dummett's works (see e.g. Dummett [1975]), in turn broadly influenced by the later Wittgenstein's reflections on meaning and use (see e.g. Wittgenstein [1953]), although I certainly do not wish to claim that the detailed way in which I put the argument is completely faithful to Dummett's own thinking, let alone Wittgenstein's. To make sure you don't quote me on that, I stress that I myself remain neutral about the soundness of the argument in its generality, although I do find it attractive at least as far as certain kinds of expressions are concerned (for example, observational predicates like 'red', 'heap', 'feels cold' etc.). I am not neutral however about its interest, and that's why I wrote this paper.

[12] The important and widespread phenomenon of *semantic deference* can safely be ignored in this context, as we can restrict our attention to the non-deferring speakers competent with the expression. Clearly, whatever conclusion we reach concerning the relevant language will also hold for the language spoken by the deferring speakers, as the "languages" in question are one and the same, at least under

utterly *mysterious* capacity for beings like us to have. For what else could its *basis* be, since there clearly must be one? How else would it be possible to *assess* whether it is present or not, as we clearly do? How else would it be possible for it to be *transmissible*, as it clearly is? Although an adequate development of these considerations lies beyond the scope of this paper, let us then assume a *manifestation constraint on understanding*: whatever understanding of an expression a speaker has, she must manifest it in her use of the expression.[13]

Now, *one* use linguistic expressions are typically put to is *assertion* (whatever that exactly is): so we may ask what can be manifested by a participant in such use. On the one hand, we should all agree that assertion is a *norm-governed* practice in at least the following, rather minimal, sense: assertions are *warranted by* and in turn *warrant* certain things. Henceforth, focus on the input side of assertion,[14] and, in this section, focus on the *atomic sentences* of the language, henceforth understanding these *not* to have

a semantic-deference thesis strong enough to call in the first place for a restriction of the argument in the text (see Burge [1979] for a seminal paper on semantic deference and Williamson [2003] for a recent emphasis broadly related to some of the issues discussed in this paper; I do not see that much remains of Williamson's semantic-deference-based attack on inferentialism once attention is restricted to non-deferring speakers, as it is certainly legitimate to do for inferentialism at least as motivated by the theory of understanding).

[13]Thanks to an anonymous referee for feedback that led to improvements in this paragraph.

[14]It is essential to distinguish two different kinds of warrant for an assertion: *objective* warrant—that which makes an assertion at least partially correct *independently of the speaker's point of view*—and *subjective* warrant—that which makes an assertion at least partially correct *from the speaker's point of view*. For example, on the one hand, the world's being such that, say, under certain conditions glass would break is an objective warrant for an assertion of 'Glass is fragile': independently of the speaker's point of view, such assertion is at least partially correct in a way that it would not be if it were not the case that, under certain conditions, glass would break. On the other hand, the world's being such that the speaker has some evidence that, under certain conditions, glass would break is a subjective warrant for an assertion of 'Glass is fragile': from her point of view, such assertion is at least partially correct in a way that it would not be if she did not have such evidence. To go back to an issue emerged in fn 11, the same contrast occurs with more observational predicates like 'red'. If, as we are about to see in the text, the practice of assertion is so constituted that nothing over and above a speaker's taking the world's being in a certain way to be a necessary or sufficient condition for an assertion of a certain sentence to be warranted can be manifested in it, and if, by the manifestation constraint on understanding, a speaker only understands what she manifests, if the necessary and sufficient condition for an assertion of 'Tomato sauce is red' to be warranted were that tomato sauce *looks red to the speaker*, it would very implausibly follow that the speaker understands 'Tomato sauce is red' as saying that tomato sauce looks red to her, whereas she very plausibly has a conception of the difference between tomato sauce's *being red* and tomato sauce's *looking red to her*. Observational predicates like 'red' still draw at least this kind of distinction between *appearance* and *reality*, and, on the view developed in this paper, to account for this it is sufficient that the world's being such that, say, under certain conditions tomato sauce would look red to the speaker is an objective warrant for an assertion of 'Tomato sauce is red' (the world's being such that the speaker has some evidence that, under certain conditions, tomato sauce would look red—for example, by being such that tomato sauce looks red to her—would then be a subjective warrant for an assertion of 'Tomato sauce is red'). (I emphasize once and for all that I am not committed to such "counterfactual analyses": I use them simply because they are easy to work with and go in the right direction as far as the relevant issues about objectivity are concerned.) Looking red to the speaker can play a crucial role in the meaning of 'red' without the latter being reduced to the former (see section 6 for more on objectivity). Henceforth, focus on objective warrant (henceforth simply 'warrant').

semantically significant constituents (and so to be broadly akin to 'It's raining' rather than to 'João walks'). A speaker manifests her taking the world's being W to warrant an assertion of an atomic sentence α by and only by being disposed ([under *extension*[15] and *presentation*] concerning the world's being W)[16] to [assert α *if* the world is W].[17] Conversely, a speaker manifests her taking an assertion of α to be warranted only by the world's being W by and only by being disposed (under ideal conditions)[18] to [assert α *only if* the world is W]. This much she *can* do. But, crucially, this much is also *all* she can do: the practice of assertion is so constituted that nothing over and above a speaker's taking the world's being in a certain way to be a necessary or sufficient condition for an assertion of a certain sentence to be warranted can be manifested in it.

To take the world's being W to be the necessary and sufficient condition for an assertion of an atomic sentence α to be warranted thus requires being disposed (under ideal conditions) to [assert α iff the world is W]. But that disposition in turn requires the disposition (under ideal conditions) to [believe that the world is W iff the world is W]—otherwise, either there would be a possible situation (under ideal conditions) in which the world is W but the speaker does not believe it to be W (*and so* does not assert α) or a possible situation (under ideal conditions) in which the world is not W but she believes it to be W (*and so* asserts α). I assume that the latter disposition suffices for the disposition (under ideal conditions) to *know* whether the world is W.[19] Given the further assumption that ideal conditions are *compatible* with the world's being W, it follows that, if the world is W, it is *feasibly knowable* that the world is W, since the antecedent of this conditional satisfies the factivity constraint, the envisaged kind of extensions satisfy the finitude constraint (see fn 15) and the envisaged kind of presentations satisfy the accessibility constraint (see fn 16).[20]

[15]The notion of extension in question relates to the one explicated in relation to the finitude constraint and additionally allows for the relevant finite extension to *perform the relevant computations*.

[16]The notion of presentation in question is such that a presentation is made to a speaker concerning an object's being F iff there is *no better epistemic position* the speaker could be in in order to determine of the object whether it is F, where the range of candidate epistemic positions is subject to the accessibility constraint. I assume suitable constraints on the *de re* attitude expressed by 'determine of the object whether it is F', so that, for example, accepting that the saliently F object, if it exists, is F is not a way of holding that attitude. Thanks to Gabriel Uzquiano for pressing me to be more explicit about this.

[17]Throughout, what I mean by this and similar constructions is, splitting my infinitives and italicizing square brackets, that the speaker *actually and presently* is disposed to, *[*under certain conditions, do certain things*]*.

[18]When it is contextually clear what is presented, I will henceforth say 'under ideal conditions' instead of '[under extension and presentation] concerning . . . '.

[19]The assumption boils down to the assumption that *tracking* whether P (in the sense made explicit in the text) is *sufficient* for knowledge whether P. The assumption is far more plausible than the converse assumption that tracking whether P is *necessary* for knowledge whether P. The assumption is most likely still subject to boring counterexamples relating to the possible presence of defeaters and the like, but it is extremely plausible that, while these are counterexamples to the current claim of *disposition to know*, they are not counterexamples to the claim of *feasible knowability* that I am about to draw from it in the text: in other words, it is extremely plausible that the counterexamples are, as it were, *wiped out* in the move from the *stronger* claim of disposition to know to the *weaker* claim of feasible knowability.

[20]The argument in the text can be represented thus:

On the other hand (the first one going back to the second sentence of the second last paragraph!), we should all also agree that assertion is a *world-directed* practice: assertions *represent* the world as being in a certain way and are evaluated accordingly. For example, whether snow is white or not is in this sense always relevant to the evaluation of an assertion of 'Snow is white'. And it would seem that this is so because *that* snow is white is what 'Snow is white' *says*. If 'Snow is white' did not say that snow is white (or something related), it would be hard to see how whether snow is white or not could still be in this sense always relevant for the evaluation of an assertion of 'Snow is white'. The dimension of evaluation in question, connecting the status of an asserted sentence with the way the world is and the way the sentence says it is, is best identified with the one of *truth*, given the compellingness of the principle about *truth and saying*:

(TS) For every P, 'P' is true iff the world is the way 'P' says it is.

But what a sentence 'P' *says* cannot outrun what a competent speaker *understands* it to say. And, by the manifestation constraint on understanding, what a competent speaker understands 'P' to say is something she can manifest in the practice of assertion. Since, as we have seen, all a speaker can manifest in the practice of assertion is what she takes to warrant an assertion of 'P' (say, the world's being W), the world is the way 'P' says it is iff it is W.[21,22] But, by the above argument, henceforth assuming at the relevant places that 'P' is atomic, if the world is W, it is feasibly knowable that the world is W,

(i) A speaker manifests her taking the world's being W to be the necessary and sufficient condition for an assertion of an atomic sentence α to be warranted by and only by being disposed (under ideal conditions) to [assert α iff the world is W]. (What can be manifested in the practice of assertion.)

(ii) That disposition ultimately requires the disposition (under ideal conditions) to know whether the world is W. (Tracking suffices for knowledge.)

(iii) And that disposition implies that there is a possible situation (under ideal conditions) in which the speaker knows that the world is W. (Ideal conditions are compatible with the world's being W.)

(iv) Therefore, provided that the speaker manifests her taking the world's being W to be the necessary and sufficient condition for an assertion of α to be warranted, if the world is W it is feasibly knowable that the world is W. (From (i), (ii) and (iii) given that the factivity, finitude and accessibility constraints are satisfied.)

[21] Equivalence in 'say that'-contexts is notoriously a highly controversial matter. But, even if it turns out that 'P' does not say that the world is W, what it says is something which is in some sense *analytically equivalent* with the world's being W (the sense relating what a sentence says with what it is understood to say by a competent speaker), and that is sufficient to validate the claim in the text that the world is the way 'P' says it is iff it is W.

[22] Notice that we have just concluded that the world's being W is in a very strong sense (the one mentioned in fn 21) a sufficient condition for its being the way 'P' says it is, and so that the world's being W is a *conclusive* warrant for an assertion of 'P'. Thus, when motivated along the lines of the argument in this paper (with its assumption of very tight saying-understanding-manifestation-warrant connections), anti-realism requires conclusive warrants in *every* discourse to which it applies (rather than, contrary to a common way of thinking, requiring conclusive warrants only in *mathematical* discourse and its like and merely defeasible warrants in *empirical* discourses). (Fn 14 in effect exploits the same connections to motivate an *objective* notion of warrant.)

and, given a highly plausible principle of *closure of feasible knowability under analytic equivalence* (see fn 21), if it is feasibly knowable that the world is W what 'P' says is feasibly knowably the case. Therefore, if what 'P' says is the case, it is feasibly knowable that it is the case. Given (TS) and the principle of *disquotation for sentences' saying*:

(DSS) What 'P' says is that P,[23]

we finally have that, if 'P' is true, it is feasibly knowable that P—that is, (TFPK) restricted to atomic sentences.[24]

4 Compositionality and Demonstration

You may think that, given what we have seen in section 2, something must have gone terribly wrong somewhere in section 3. Not so. The argument in section 3 is fine as far as it goes, as it only establishes (TFPK) restricted to *atomic sentences*, while it is arguably crucial for the paradox of knowability that (TFPK) includes in its scope *semantically complex sentences*. Granted, one might try to stipulate a *complex meaning* for a *syntactically simple*—i.e. atomic—*sentence*. In particular, one might try to stipulate, for some (without loss of generality) semantically simple P, that a certain atomic sentence β says that [P and it is not known that P]. But this attempt would be both *problematic* and *futile*. The attempt would be problematic because, under ideal conditions, the necessary and sufficient condition for an assertion of β to be warranted would not hold (since, essentially by the argument in section 3, under ideal conditions, if P, the speaker knows that P). But, assuming that it is possible that the world is W,[25] it is plausible to maintain that a speaker manifests her taking the world's being W to be the necessary and sufficient condition for an assertion of an atomic sentence α to be warranted only if, under ideal conditions, her assertions *genuinely track* the world's being W in the sense that, in *some*

[23] Notice that (TS) and (DSS) together entail (and arguably ground) (DT).

[24] The argument in the text can be represented thus:

(I) What a sentence 'P' says cannot outrun what a competent speaker understands it to say. (Connection between saying und understanding.)

(II) Hence, since all a speaker can manifest in the practice of assertion is what she takes to warrant an assertion of 'P' (say, the world's being W), the world is the way 'P' says it is iff it is W. (From (I) and the manifestation constraint on understanding.)

(III) If the world is W, it is feasibly knowable that the world is W. (From (iv) in fn 20.)

(IV) Hence, if what 'P' says is the case, it is feasibly knowable that it is the case. (From (II) and (III) by closure of feasible knowability under analytic equivalence.)

(V) Therefore, if 'P' is true, it is feasibly knowable that P. (From (IV), (TS) and (DSS).)

[25] As, in the offending cases, we evidently may, on pain of ending up independently validating the intermediate conclusion of the Church-Fitch reasoning to the effect that, necessarily, it is not known that [P and it is not known that P].

possible situation (under ideal conditions), it is the case that [she asserts α iff the world is W] because *both* she asserts α *and* the world is W (rather than its being the case that, in *every* possible situation (under ideal conditions), it is the case that [she asserts α iff the world is W] because *neither* does she assert α *nor* is the world W, as it would happen in the case of β). The attempt would be futile because, in any event, the argument in section 3 *crucially relied on the assumption that ideal conditions are compatible with the world's being* W, an assumption that, while compelling for semantically simple sentences, as we have just observed fails in the case of the necessary and sufficient condition for an assertion of β to be warranted.

Moving on to *compound sentences*, the general idea is that, by the principle of *compositionality*, understanding a compound sentence, contrary to understanding an atomic one, simply consists in understanding its components (plus, of course, their modes of composition). There should be no immediate requirement that understanding a compound sentence requires being disposed (under ideal conditions) to [assert the sentence iff the world is in a certain way], for that would amount to treating compound sentences as though they were atomic, with no semantically significant parts on which understanding can build. That a speaker takes the world's being W to be the necessary and sufficient condition for an assertion of a compound sentence φ to be warranted is determined by her understanding of φ's component expressions, not by her being disposed (under ideal conditions) to [assert φ iff the world is W].

For our purposes, we can focus on a simple standard first-order epistemic language without identity and with only general terms of arbitrary arity as its non-logical constants.[26] From the point of view which sees assertion as a *norm-governed* practice, the basic notion will now be the notion of an *application* of a *formula*[27] to certain *objects* being warranted by certain facts. The practice of assertion is so constituted that nothing over and above a speaker's disposition to take certain objects' being in a certain way to be the necessary or sufficient condition for an application of a certain formula to them to be warranted can be manifested in it.

Start with *atomic formulae*. A speaker manifests her taking a sequence of objects[28] being W to warrant application of an atomic formula $\varphi(\xi)$ to it (where $\varphi(\xi)$ and its like are formulae that may be open in several variables) by and only by being disposed ([under extension and presentation] concerning the sequence's being W) to [apply $\varphi(\xi)$ to the sequence *if* the sequence is W]. Conversely, a speaker manifests her taking an application of an atomic formula $\varphi(\xi)$ to a sequence to be warranted only by the sequence's

[26]I do not consider usual logical expressive resources going beyond such language simply because they do not pose additional problems in relation to the issues raised by the paradox of knowability. It should be clear from what follows how to treat, for example, identity predicates, absurdity constants or possibility operators; as for singular terms, their introduction would be unproblematic and would go along the main lines of what follows, but would require bringing into play new, dedicated notions at the level of speech acts, warrant, meaning and semantic evaluation. Thanks to Gabriel Uzquiano for urging these clarifications.

[27]I understand *formulae* so that they differ from *sentences* in that they might contain *free variables*.

[28]Throughout, I assume that the language has denumerably many variables and focus on ω-long sequences (you can think of each such sequence as an assignment of objects to the variables of the language).

being W by and only by being disposed (under ideal conditions) to [apply $\varphi(\xi)$ to the sequence *only if* the sequence is W]. The considerations in section 3 linking manifesting one's taking something to warrant something, dispositions and knowledge require the disposition (under ideal conditions) to *know* whether the sequence is W. Given the further assumption that ideal conditions are *compatible* with the sequence's being W,[29] it follows that, if the sequence is W, it is *feasibly knowable* that the sequence is W, since the antecedent of this conditional satisfies the factivity constraint, the envisaged kind of extensions satisfy the finitude constraint (see fn 15) and the envisaged kind of presentations satisfy the accessibility constraint (see fn 16).[30]

Say that a sequence *permonstrates* a formula $\varphi(\xi)$ iff the application of $\varphi(\xi)$ to it is warranted. Enter then *logical operators*. They form more complex formulae out of simpler ones, the permonstration condition of the compound formula being a *function* of the permonstration conditions of its components (I will briefly discuss and criticize in section 7 a view on which there is no such functionality). A speaker's taking it that the operation characteristic of an $n$ary logical operator Ω is a certain function will (very plausibly) be manifested by and only by her being disposed ([under extension and presentation] concerning the permonstration conditions of $\varphi_0(\xi_0), \varphi_1(\xi_1), \ldots, \varphi_{n-1}(\xi_{n-1})$ by a sequence) to [apply $\Omega\varphi_0(\xi_0), \varphi_1(\xi_1), \ldots, \varphi_{n-1}(\xi_{n-1})$ to the sequence so that such application is an *isomorphic* function of her applications of $\varphi_0(\xi_0), \varphi_1(\xi_1), \ldots, \varphi_{n-1}(\xi_{n-1})$ to it].

Let us see how this plays out in detail for the usual logical operators. A speaker may be disposed ([under extension and presentation] concerning the permonstration conditions of $\varphi(\xi_0)$ and $\psi(\xi_1)$ by a sequence) to [apply $\varphi(\xi_0) \wedge \psi(\xi_1)$ to the sequence iff she applies

[29]For the reason brought out by the paradox of knowability, given *rich enough ontology and ideology* the assumption will not always be satisfied. For example, ideal conditions are not compatible with the *fact* that [there are 1,963 trees in St Andrews and it is not known that there 1,963 trees in St Andrews] *existing*, with the *sentence* 'There are 1,963 trees in St Andrews and it is not known that there 1,963 trees in St Andrews' being *true* or with 1,963 *belonging* to the *set* $\{n :$ There are n trees in St Andrews and it is not known that there n trees in St Andrews$\}$. Having noted this, since the existence of these and similar cases does not alter the substance of the view developed in this paper I will henceforth assume such compatibility. Thanks to Gabriel Uzquiano for questions that prompted this fn.

[30]The argument in the text can be represented thus:

(i′) A speaker manifests her taking a sequence's being W to be the necessary and sufficient condition for an application of an atomic formula $\varphi(\xi)$ to it to be warranted by and only by being disposed (under ideal conditions) to [apply $\varphi(\xi)$ to the sequence iff the sequence is W]. (What can be manifested in the practice of assertion.)

(ii′) That disposition ultimately requires the disposition (under ideal conditions) to know whether the sequence is W. (Tracking suffices for knowledge.)

(iii′) And that disposition implies that there is a possible situation (under ideal conditions) in which the speaker knows that the sequence is W. (Ideal conditions are compatible with the sequence's being W.)

(iv′) Therefore, provided that the speaker manifests her taking the sequence's being W to be the necessary and sufficient condition for an application of $\varphi(\xi)$ to it to be warranted, if the sequence is W it is feasibly knowable that the sequence is W. (From (i′), (ii′) and (iii′) given that the factivity, finitude and accessibility constraints are satisfied.)

$\varphi(\xi_0)$ to it *and* applies $\psi(\xi_1)$ to it]. She thereby manifests her taking a sequence permonstrating $\varphi(\xi_0)$ and permonstrating $\psi(\xi_1)$ to be the necessary and sufficient condition for the sequence to permonstrate $\varphi(\xi_0) \wedge \psi(\xi_1)$.

A speaker may be disposed (under ideal conditions) to [apply $\varphi(\xi_0) \vee \psi(\xi_1)$ to a sequence iff she applies $\varphi(\xi_0)$ to it *or* applies $\psi(\xi_1)$ to it].[31] She thereby manifests her taking a sequence permonstrating $\varphi(\xi_0)$ or permonstrating $\psi(\xi_1)$ to be the necessary and sufficient condition for the sequence to permonstrate $\varphi(\xi_0) \vee \psi(\xi_1)$.

A speaker may be disposed (under ideal conditions) to [apply $\varphi(\xi_0) \supset \psi(\xi_1)$ to a sequence iff, *if* she applies $\varphi(\xi_0)$ to it, *then* she applies $\psi(\xi_1)$ to it].[32] She thereby manifests her taking a sequence permonstrating $\psi(\xi_1)$ if it permonstrates $\varphi(\xi_0)$ to be the necessary and sufficient condition for the sequence to permonstrate $\varphi(\xi_0) \supset \psi(\xi_1)$.

A speaker may be disposed (under ideal conditions) to [apply $\neg\varphi(\xi)$ to a sequence iff she does not apply $\varphi(\xi)$ to it].[33] She thereby manifests her taking a sequence not permonstrating $\varphi(\xi)$ to be the necessary and sufficient condition for the sequence to permonstrate $\neg\varphi(\xi)$.

A speaker may be disposed ([under extension and presentation] concerning the permonstration conditions of $\varphi(\xi_1)$ by a sequence and by every relevant sequence differing from it at most at its ξ_0-corresponding coordinate) to [apply $\forall\xi_0\varphi(\xi_1)$ to the original sequence iff, for *every* relevant sequence differing from the original sequence at most at its ξ_0-corresponding coordinate, she applies $\varphi(\xi_1)$ to it].[34] She thereby manifests her taking a sequence to be such that every relevant sequence differing from it at most at

[31]It is uncontroversial that, under *non-ideal* conditions, a speaker might apply $\varphi(\xi_0) \vee \psi(\xi_1)$ to a sequence without either applying $\varphi(\xi_0)$ to it or applying $\psi(\xi_1)$ to it (for example, she might apply 'x is either divisible by 2 or the successor of a number divisible by 2' to 1963 merely on the ground of an inductive proof of 'Every number is either divisible by 2 or the successor of a number divisible by 2'). But the possible situations (under non-ideal conditions) that uncontroversially witness such pattern of application are situations in which she nevertheless possesses a procedure whose implementation either would allow her to apply $\varphi(\xi_0)$ to the sequence or would allow her to apply $\psi(\xi_1)$ to it. Under *ideal* conditions, such procedure would be implemented.

[32]By the properties of material 'If..., then...', provided that a speaker does not apply $\varphi(\xi_0)$ to a sequence it follows that, *if* she applies $\varphi(\xi_0)$ to it, *then* she applies $\psi(\xi_1)$ to it. Of course, under *non-ideal* conditions, it would be crazy for her to apply $\varphi(\xi_0) \supset \psi(\xi_1)$ to it simply because she does not apply $\varphi(\xi_0)$ to it. But what would be epistemic hubris under *non-ideal* conditions is not such under *ideal* conditions.

[33]Similarly to what was observed in fn 32, of course, under *non-ideal* conditions, it would be crazy for a speaker to apply $\neg\varphi(\xi)$ to a sequence simply because she does not apply $\varphi(\xi)$ to it. But what would be epistemic hubris under *non-ideal* conditions is not such under *ideal* conditions.

[34]Throughout, I use 'apply' and its relatives in a suitably *dispositional* sense, so that a finite being, even though not capable of infinitely many *occurring judgements* at the same time, is indeed capable of infinitely many *standing applications* at the same time, and also so that such being can apply a formula to an object *without being able to single out that object* (see Zardini [2015a], p. 41, fn 9, p. 43, fn 13; [2015b] for some more relevant discussion). Such dispositions, when present, are typically grounded in possession of some sort of "proof", for every relevant object, that it is in a certain way (in the very weak sense that one possesses a form of ground that one in principle knows how, and so is disposed, to apply occurrently to each object to produce a specific ground for that object). Thus, for example, I do apply 'x is either divisible by 2 or the successor of a number divisible by 2' to all the infinitely many numbers, and I do so because I possess a proof, for every number, that it is either divisible by 2 or the successor of a number divisible by 2. Indeed, to fix ideas, I will henceforth assume that, in the infinite

its ξ_0-corresponding coordinate permonstrates $\varphi(\xi_1)$ to be the necessary and sufficient condition for the sequence to permonstrate $\forall\xi_0\varphi(\xi_1)$.

A speaker may be disposed (under ideal conditions) to [apply $\exists\xi_0\varphi(\xi_1)$ to a sequence iff, for *some* relevant sequence differing from the original sequence at most at its ξ_0-corresponding coordinate, she applies $\varphi(\xi_1)$ to it]. She thereby manifests her taking a sequence to be such that some relevant sequence differing from it at most at its ξ_0-corresponding coordinate permonstrates $\varphi(\xi_1)$ to be the necessary and sufficient condition for the sequence to permonstrate $\exists\xi_0\varphi(\xi_1)$.

As for $\mathcal{K}$, it is more natural to treat it as a *non-logical* operator in the sense of treating $\mathcal{K}$-initial formulae similarly to how atomic formulae are treated. Thus, a speaker may be disposed ([under extension and presentation] concerning whether a sequence's value for ξ_0 at the sequence's value for ξ_1 knows $\varphi(\xi_2)$ relative to the sequence)[35] to [apply $\mathcal{K}_{\xi_0,\xi_1}\varphi(\xi_2)$ to the sequence iff the sequence's value for ξ_0 at the sequence's value for ξ_1 *knows* $\varphi(\xi_2)$]. She thereby manifests her taking a sequence's value for ξ_0 at the sequence's value for ξ_1 knowing $\varphi(\xi_2)$ to be the necessary and sufficient condition for the sequence to permonstrate $\mathcal{K}_{\xi_0,\xi_1}\varphi(\xi_2)$.

Finally, say that there is a *demonstration* of a *sentence* φ (see fn 27) iff φ is permonstrated by some (every) sequence. It follows that there is a demonstration of $\varphi \wedge \psi$ iff there is a demonstration of φ and a demonstration of ψ; that there is a demonstration of $\varphi \vee \psi$ iff there is either a demonstration of φ or a demonstration of ψ; that there is a demonstration of $\varphi \supset \psi$ iff, if there is a demonstration of φ, then there is a demonstration of ψ; that there is a demonstration of $\neg\varphi$ iff there is no demonstration of φ; that there is a

case, the relevant dispositions must be grounded in possession of some sort of "proof". Given this, *the mathematical platonist's dream* that, for some formula $\varphi(\xi)$, *every relevant sequence permonstrates* $\varphi(\xi)$ *without there being any sort of "proof" of* $\forall\xi\varphi(\xi)$ implies that, in every such case, conditions cannot be ideal for applications of $\varphi(\xi)$ to every relevant sequence, as that would require performing *infinitely many* computations, thereby leading to a violation of the *finitude* constraint. On the view developed in this paper, the fact that conditions cannot be ideal for applications of $\varphi(\xi)$ to every relevant sequence is no more problematic for understanding $\forall\xi\varphi(\xi)$ than the fact that conditions cannot be ideal for assertions of both conjuncts of a Church-Fitch sentence is for understanding that sentence (as I will discuss in more detail in section 6): in both cases, a speaker can non-vacuously manifest her understanding of the relevant logical operator in relation to many other component formulae (indeed, in the case of $\forall\xi\varphi(\xi)$, contrary to the case of a Church-Fitch sentence she can non-vacuously manifest her understanding of $\forall$ by applying formulae of the same form as $\varphi(\xi)$ to every relevant sequence!); moreover, since she can unproblematically manifest her understanding of the component formulae, it follows by compositionality that she does understand the relevant sentence. I should add that I find it extremely odd to envisage as candidate conditions for some formula only conditions such that in each of them *a speaker fails to perform a certain relevant (finite) computation.* I find it extremely attractive to postulate instead that, for every formula, there are conditions in which *a speaker does not fail to perform the relevant computations* (wasn't one of the point of idealization to screen off boring issues arising from failures to perform relevant computations?). As I have observed, in the cases in which the mathematical platonist's dream comes true, such extremely attractive postulation leads to a violation of the finitude constraint. From the point of view of idealization, finitude can thus be nothing less than a *defect*: removing defects, idealization removes then finitude. But, in this paper (fn 4), I have imposed myself not to follow further this train of thought.

[35] I will henceforth leave implicit the relativization to sequences of knowledge of formulae.

demonstration of $\forall\xi\varphi(\xi)$ iff there is a permonstration of $\varphi(\xi)$ by every relevant sequence; that there is a demonstration of $\exists\xi\varphi(\xi)$ iff there is a permonstration of $\varphi(\xi)$ by some relevant sequence.[36]

5 Demonstration and Truth

From the point of view which sees assertion as a *world-directed* practice, applications *represent* their objects as being in a certain way and are evaluated accordingly. For example, whether snow is white or not is in this sense always relevant to the evaluation of an application of 'x is white' to it. And it would seem that this is so because, in such application, *that* it is white is what 'x is white' *says* of snow. If the formula did not say of snow that it is white (or something related), it would be hard to see how whether snow is white or not could still be in this sense always relevant for the evaluation of an application of 'x is white' to it. The dimension of evaluation in question, connecting the status of an applied formula with the way the objects it is applied to are and the way the formula says they are, is best identified with that of *satisfaction*,[37] given the compellingness of the principle about *satisfaction and saying*:

(SS) For every formula $P(x)$, a sequence $\langle x_0, x_1, \ldots\rangle$ satisfies '$P(x)$' iff x_0, $x_1, \ldots$ are the way '$P(x)$' says they are.

But what a formula '$P(x)$' *says* cannot outrun what a competent speaker *understands* it to say. And, by the manifestation constraint on understanding, what a competent speaker understands '$P(x)$' to say is something she can manifest in the practice of assertion. Since, as we have seen, all a speaker can manifest in the practice of assertion is what she takes to be a permonstration of '$P(x)$' by a sequence, $x_0, x_1, \ldots$ are the way '$P(x)$' says they are iff $\langle x_0, x_1, \ldots\rangle$ permonstrates '$P(x)$'.[38] By (SS), a sequence satisfies '$P(x)$'

[36] If the language is enriched with constants for every object of the domain and the definition of demonstration is expanded in a suitable way so as to cover the new set of atomic sentences, the *demonstration functionality* of the quantifiers can be restored as follows: there is a demonstration of $\forall\xi\varphi(\xi)$ iff, for every constant κ, there is a demonstration of $\varphi(\kappa/\xi)$ (where $\varphi(\tau_0/\tau_1)$ is the result of, under the usual proviso, substituting τ_0 for the free occurrences of τ_1 in $\varphi(\tau_1)$); there is a demonstration of $\exists\xi\varphi(\xi)$ iff, for some constant κ, there is a demonstration of $\varphi(\kappa/\xi)$.

[37] The intuitive semantic notion here is the one of *being true of*, whose arity seems to be variable ('x is a football team' is true of Sporting Lisbon, 'x is better than y' is true of Sporting Lisbon and FC Porto in this order). The notion of satisfaction can then be extracted by first forming a 2ary predicate 'x_0 is true* of x_1' true of formulae and sequences and then taking its converse.

[38] And so, given the principle of *disquotation for formulae's saying*:

(DFS) '$P(x)$' says of $x_0, x_1, \ldots$ that they are P,

the corresponding principles of *disquotation for permonstration* and *disquotation for demonstration* follow:

(DP) There is a permonstration of '$P(x)$' by $\langle x_0, x_1, \ldots\rangle$ iff $x_0, x_1, \ldots$ are P;

(DD) There is a demonstration of 'P' iff P.

iff it permonstrates '$P(x)$'.

Now, a formula $\varphi(\xi)$ is true iff it is a sentence satisfied by some (every) sequence. By the above argument, we can then conclude that a formula $\varphi(\xi)$ is true iff it is a sentence permonstrated by some (every) sequence—that is, iff there is a demonstration of it.[39] What the proposed reconstruction of the anti-realist argument from understanding yields is thus the principle of *demonstration of truth*:

(TD) If 'P' is true, there is a demonstration of 'P'.

6 Demonstration and Knowledge

Crucially, the argument in sections 4 and 5 does not discriminate between different kinds of sentences, and so (TD) holds *unrestrictedly* (contrary to other prominent anti-realist reactions to the paradox of knowability, which abandon unrestricted anti-realism).[40] Does the (TD)-version of unrestricted anti-realism fall prey to the paradox of knowability?

To see that this is not the case, return first to the basic notion of a permonstration. A speaker manifests her taking a sequence's being W to permonstrate an atomic formula $\varphi(\xi)$ by and only by being disposed (under ideal conditions) to [apply $\varphi(\xi)$ to the sequence iff the sequence is W]. But a sequence permonstrates $\varphi(\xi)$ iff speakers take it to do so. Therefore, $\varphi(\xi)$ is permonstrated by a sequence iff the sequence *is* W (rather than

[39]The argument in the text can be represented thus:

(I′) What a formula '$P(x)$' says cannot outrun what a competent speaker understands it to say. (Connection between saying and understanding.)

(II′) Hence, since all a speaker can manifest in the practice of assertion is what she takes to be a permonstration of '$P(x)$' by a sequence, $x_0, x_1, \ldots$ are the way '$P(x)$' says they are iff $\langle x_0, x_1, \ldots\rangle$ permonstrates '$P(x)$'. (From (I′) and the manifestation constraint on understanding.)

(III′) Hence, a sequence satisfies '$P(x)$' iff it permonstrates '$P(x)$'. (From (II′) and (SS).)

(IV′) Therefore, if a sentence is true, there is a demonstration of it. (From (III′) and the definitions of truth and demonstration.)

[40]Cases in point are Tennant [1997], pp. 245–279 and, at least at a first glance, Dummett [2001]. Tennant [2002] criticizes Dummett for in effect restricting (TFPK) to *atomic* formulae, pointing out that (TFPK) is supposed to get much of its bite for, say, arithmetical discourse by including in its scope *compound* (in particular, *quantified*) formulae. That might seem like a fair criticism of Dummett's proposal (at least as the proposal was originally stated), but it would also seem that Tennant's own restriction is subject to a similar problem. For Tennant restricts (TFPK) to sentences that it is *broadly logically possible to know*, and so leaves out of its scope sentences like 'Everyone is cognitively impaired', thus preventing (TFPK) to have its supposed bite for *certain* quantifications over infinite empirical domains. Worse, if it is conceded that sentences like 'Everyone is cognitively impaired' can be determined to be either true or false independently of our recognition of that, it is unclear on exactly what grounds it can still be maintained that *other* quantifications over infinite domains cannot be determined to be either true or false independently of our recognition of that. As for Dummett's own proposal, I will comment a bit on it in fn 59, after introducing the relevant elements of the dialectic.

"*seems* to be W")—where a sequence's being W *need not imply anything about a speaker's situation* (see fn 14).[41] Nothing more is required from a permonstration of $\varphi(\xi)$ by a sequence. In particular, nothing more is required in terms of a speaker's *epistemic position*.

Analogously, a speaker manifests her taking the permonstration condition of a compound formula $\Omega\varphi_0(\xi_0), \varphi_1(\xi_1), \ldots, \varphi_{n-1}(\xi_{n-1})$ (with Ω's being an $n$ary logical operator) to be a certain function of the permonstration conditions of $\varphi_0(\xi_0), \varphi_1(\xi_1), \ldots, \varphi_{n-1}(\xi_{n-1})$ (so that, say, things' being W as regards the permonstration conditions of $\varphi_0(\xi_0), \varphi_1(\xi_1), \ldots, \varphi_{n-1}(\xi_{n-1})$ by a sequence is necessary and sufficient for the sequence's permonstrating $\Omega\varphi_0(\xi_0), \varphi_1(\xi_1), \ldots, \varphi_{n-1}(\xi_{n-1})$) by and only by, for every formulae $\psi_0(\xi_0), \psi_1(\xi_1), \ldots, \psi_{n-1}(\xi_{n-1})$, being disposed (under ideal conditions) to [apply $\Omega\psi_0(\xi_0), \psi_1(\xi_1), \ldots, \psi_{n-1}(\xi_{n-1})$ to a sequence iff things are W as regards the permonstration conditions of $\psi_0(\xi_0), \psi_1(\xi_1), \ldots, \psi_{n-1}(\xi_{n-1})$ by the sequence]. But a sequence permonstrates $\Omega\varphi_0(\xi_0), \varphi_1(\xi_1), \ldots, \varphi_{n-1}(\xi_{n-1})$ iff speakers take it to do so. Therefore, $\Omega\varphi_0(\xi_0), \varphi_1(\xi_1), \ldots, \varphi_{n-1}(\xi_{n-1})$ is permonstrated by a sequence iff things *are* W as regards the permonstration conditions of $\varphi_0(\xi_0), \varphi_1(\xi_1), \ldots, \varphi_{n-1}(\xi_{n-1})$ by the sequence (rather than "*seem* to be W")—where things' being W as regards the permonstration conditions of $\varphi_0(\xi_0), \varphi_1(\xi_1), \ldots, \varphi_{n-1}(\xi_{n-1})$ by a sequence *need not imply anything about a speaker's situation* (see fn 14). Nothing more is required from a permonstration of $\Omega\varphi_0(\xi_0), \varphi_1(\xi_1), \ldots, \varphi_{n-1}(\xi_{n-1})$ by a sequence. In particular, nothing more is required in terms of a speaker's *epistemic position*.

I submit that this is a basic fact about the *logic* of manifesting one's understanding of a rule in general. Consider the manifestation of a player's understanding of chess rules. All a player can do is, under certain conditions, to accept moves made in accordance with those rules and reject any other move, but this is crucially *not* taken to manifest the player's understanding that, say, two chess situations s_0 and s_1 are such that s_1 is a permissible development of s_0 *only if* the player judges them to be so; what the player's behaviour is taken to manifest is, rather, her understanding that s_0 and s_1 are such that s_1 *simply is* a permissible development of s_0, no matter, say, whether anyone judges it to be so or not. The logic of manifesting one's understanding is thus peculiarly *opaque*: what is understood is what is manifested, but what is manifested does not include the (unavoidable) fact that it is manifested.

A permonstration must thus be accurately distinguished from its *knowledge-conferring implementation*. It suffices for a sequence to permonstrate a formula $\varphi(\xi)$ that it be, say, W, where, as we have seen in the third and second last paragraphs, a sequence's being W need not imply anything about a speaker's situation. But for a speaker to acquire

[41] To go back to an example in fn 14, a speaker manifests her taking an object's being such that, under ideal conditions, it would look red to the speaker to permonstrate 'x is red' by and only by being disposed (under ideal conditions) to [apply 'x is red' to the object iff the object is such that, under ideal conditions, it would look red to the speaker] (which is tantamount to being disposed (under ideal conditions) to [apply 'x is red' to the object iff the object looks red to the speaker]). And an object's being such that, under ideal conditions, it would look red to the speaker implies neither that the object looks red to the speaker nor that ideal conditions hold.

knowledge that the objects of the sequence are the way $\varphi(\xi)$ says they are the relevant extension and presentation might need to occur. There is no general guarantee that these additional conditions are *compatible* with the way $\varphi(\xi)$ says objects are, since the way $\varphi(\xi)$ says objects are may exactly be the way they are only if either of these conditions does not hold (respective toy counterexamples: 'Everyone is drunk' and 'Everyone is in the dark').[42]

This distinction between a permonstration and its knowledge-conferring implementation is essential also to avoid an all too easy validation of the object-language conditional $\varphi(\xi_0) \supset \exists\xi_1\xi_2\mathcal{K}_{\xi_1,\xi_2}\varphi(\xi_0)$ by an anti-realist semantics, validation which does not even go through the detour of the paradox of knowability (see Hart [1979], p. 165, fn 3; the insightful reply in Williamson [1982], pp. 206–207—hinging upon the distinction between a proof *type* and a proof *token*—has been a major source of inspiration for this paper). If the basic notion of an anti-realist semantics were the one of the possibility of a knowledge-conferring implementation of a permonstration (where, of course, there would be a lot of room for manoeuvre in understanding exactly how tight the operative notion of possibility is), there could presumably be a knowledge-conferring implementation of a permonstration of $\varphi(\xi_0) \supset \exists\xi_1\xi_2\mathcal{K}_{\xi_1,\xi_2}\varphi(\xi_0)$ by a sequence iff, if there could be a knowledge-conferring implementation of a permonstration of $\varphi(\xi_0)$ by the sequence, then there could be a knowledge-conferring implementation of a permonstration of $\exists\xi_1\xi_2\mathcal{K}_{\xi_1,\xi_2}\varphi(\xi_0)$ by the sequence. And, necessarily, if there is a knowledge-conferring implementation of a permonstration by a sequence S of 'There is a knowledge-conferring implementation of a permonstration of '$\varphi(\xi_0)$' by S', there is also a knowledge-conferring implementation of a permonstration of $\exists\xi_1\xi_2\mathcal{K}_{\xi_1,\xi_2}\varphi(\xi_0)$ by S. The absurd $\varphi(\xi_0) \supset \exists\xi_1\xi_2\mathcal{K}_{\xi_1,\xi_2}\varphi(\xi_0)$ would then be validated by the not absurd and indeed traditional epistemological view according to which, if there could be a knowledge-conferring implementation of a permonstration of $\varphi(\xi)$ by a sequence S, there could also be a knowledge-conferring implementation of a permonstration of 'There is a knowledge-conferring implementation of a permonstration of '$\varphi(\xi)$' by S' by S.

The absurd $\varphi(\xi_0) \supset \exists\xi_1\xi_2\mathcal{K}_{\xi_1,\xi_2}\varphi(\xi_0)$ is *not* so validated if the basic notion of an anti-realist semantics is the one of a permonstration rather than the one of the possibility of a knowledge-conferring implementation of a permonstration: for it is consistent, at least for a non-$\mathcal{K}$-initial formula $\varphi(\xi_0)$, that there is a permonstration of $\varphi(\xi_0)$ by a sequence without there being a knowledge-conferring implementation of it by the sequence, and so *a fortiori* without there being a permonstration of $\exists\xi_1\xi_2\mathcal{K}_{\xi_1,\xi_2}\varphi(\xi_0)$ by the sequence, even in the extreme theoretical scenario in which there could not be a

[42]Assuming that, as is the case with many other sentences, one can only perform the computations relevant for deciding 'Everyone is drunk' if one is not drunk, the *extension* concerning the permonstration condition of 'Everyone is drunk' is incompatible with everyone's being drunk (see fn 15); assuming that, as is the case with many other sentences, a good epistemic position for deciding 'Everyone is in the dark' involves some decent lighting, the *presentation* concerning the permonstration condition of 'Everyone is in the dark' is incompatible with everyone's being in the dark (see fn 16). Thus, in the case of both sentences, conditions cannot be ideal for assertion of the sentence. I will discuss below in the text whether, on the view developed in this paper, this circumstance affects understanding of such sentences (see also fn 34). Thanks to an anonymous referee for discussion of these examples.

knowledge-conferring implementation of a permonstration of $\varphi(\xi_0)$ by a sequence S without there being a knowledge-conferring implementation of a permonstration of 'There is a knowledge-conferring implementation of a permonstration of '$\varphi(\xi_0)$' by S' by S.

The distinction between a permonstration and its knowledge-conferring implementation entails an analogous distinction between a demonstration and its knowledge-conferring implementation. We can then see how, contrary to the (TFPK)-version of unrestricted anti-realism, the (TD)-version—the version of unrestricted anti-realism that *is* the conclusion of the proposed reconstruction of the anti-realist argument from understanding—does not fall prey to the paradox of knowability. Mimicking the original Church-Fitch reasoning, we can go from 'There is a demonstration of 'P and it is not known that P'' to 'There is a demonstration of 'P' and there is a demonstration of 'It is not known that P'' (by demonstration functionality of conjunction). But this is *not* absurd (as, on the contrary, 'It is known that P and it is known that it is not known that P' is), since, as we have seen, the existence of a demonstration of 'P' does not imply the existence or even the possibility of a knowledge-conferring implementation of the demonstration (even though it does imply the feasible possibility that it is known that P if 'P' is an atomic formula or the $n$ple negation of an atomic formula, as should be clear from putting together sections 4 and 5). The Church-Fitch reasoning thus breaks down: it is *consistent* that there is both a demonstration of 'P' and a demonstration of 'It is not known that P', and therefore consistent (by demonstration functionality of conjunction) that there is a demonstration of 'P and it is not known that P'.[43]

But how does a specific Church-Fitch sentence like 'There are 1,963 trees in St Andrews and it is not known that there 1,963 trees in St Andrews' exactly interact with the view developed in this paper? There is no problem in supposing that there is a possible situation ([under extension and presentation] concerning the demonstration condition of 'There are 1,963 trees in St Andrews') in which a speaker asserts 'There are 1,963 trees in St Andrews'; equally, there is no problem in additionally supposing that there is a possible situation ([under extension and presentation] concerning the demonstration condition of 'It is not known that there are 1,963 trees in St Andrews') in which she asserts 'It is not known that there are 1,963 trees in St Andrews'. But it does not follow from all this that there is a possible situation ([under extension and presentation] concerning the demonstration conditions of 'There are 1,963 trees in St Andrews' and 'It is not known that there are 1,963 trees in St Andrews') in which she asserts 'There are 1,963 trees in St Andrews' and asserts 'It is not known that there are 1,963 trees in St Andrews'. For, essentially by the argument in section 4, any possible situation ([under extension and presentation] concerning the demonstration condition of 'There are 1,963 trees in St Andrews') in which she asserts 'There are 1,963 trees in St Andrews' is a situation in which she knows that there are 1,963 trees in St Andrews; however, essentially by the argument in section 4, any possible situation ([under extension and presentation] concerning the demonstration condition of 'It is not known that there are 1,963 trees in St Andrews') in

[43] Of course, given demonstration functionality of conjunction and factivity of demonstration (see (DD) in fn 38), one can successfully mimic the original Church-Fitch reasoning with respect to 'P and there is no demonstration of 'P'', but that only yields (TD) all over again.

which she asserts 'It is not known that there are 1,963 trees in St Andrews' is a situation in which the demonstration condition of 'It is not known that there are 1,963 trees in St Andrews' holds, and so, by factivity of demonstration (see (DD) in fn 38), a situation in which it is not known that there are 1,963 trees in St Andrews. Therefore, there is no possible situation ([under extension and presentation] concerning the demonstration conditions of 'There are 1,963 trees in St Andrews' and 'It is not the case that there are 1,963 trees in St Andrews') in which she asserts 'There are 1,963 trees in St Andrews' and asserts 'It is not known that there are 1,963 trees in St Andrews'. Conditions cannot be ideal for assertions of both sentences. And, by a similar argument, conditions cannot be ideal for assertions of the negations of both sentences, so that conditions can only be ideal for assertion of one sentence and assertion of the negation of the other sentence (see fn 45 for more details).

Letting φ_0 be 'There are 1,963 trees in St Andrews' and ψ_1 be 'It is not known that there are 1,963 trees in St Andrews', in this case it is thus *vacuously* true that a speaker is disposed ([under extension and presentation] concerning the permonstration conditions of $\varphi(\xi_0)$ and $\psi(\xi_1)$ by a sequence) to [apply $\varphi(\xi_0) \wedge \psi(\xi_1)$ to the sequence iff she applies $\varphi(\xi_0)$ to it *and* applies $\psi(\xi_1)$ to it] (in the sense that both sides of the embedded biconditional are always untrue). Still, going back to an issue emerged in section 4, a speaker can *non-vacuously* manifest her understanding of $\wedge$ in relation to many other pairs of sentences (as we have observed in the last paragraph, even in relation to 'There are 1,963 trees in St Andrews' and 'It is known that there are 1,963 trees in St Andrews' or in relation to 'It is not the case that there are 1,963 trees in St Andrews' and 'It is not known that there are 1,963 trees in St Andrews'!); moreover, since she can unproblematically manifest her understanding of the two component sentences of 'There are 1,963 trees in St Andrews and it is not known that there are 1,963 trees in St Andrews', as *per* section 4 it follows by compositionality that she does understand that sentence.

Obviously, the view developed in this paper raises a host of issues, of which I would like to discuss a particularly salient one. It might rightly be observed that my talk of demonstrations is rather *unsubstantial*: keeping in mind the telling case of negation, I have basically stipulated that the non-existence of a demonstration of φ suffices for the existence of a demonstration of $\neg\varphi$. That prevents demonstrations from being always guaranteed to be "*constructive objects*"—that is, roughly, objects consisting of a *structure of procedures*. And doesn't that in turn take any interesting epistemic bite out of the existence of a demonstration? It doesn't. Although, as I have been stressing in this section, the existence of a demonstration has an *objective* component that makes it *non-reducible to facts about possible knowledge*, the view developed in this paper still has *very clear and substantial epistemic consequences*.

Firstly, it's easy to see that the existence of a demonstration of φ still entails, by permonstration functionality, a certain pattern of permonstrations or lack thereof for the atomic formulae occurring in φ; as I have mentioned above in this section, *for such formulae and their negations* permonstrations can always be implemented in a knowledge-conferring way, and the view developed in this paper does nothing to cast into doubt the extremely plausible principle that logical operators are "*scrutable*" in the sense that, if the truth value of each component is *known*, the truth value of the compound formula is *feasibly knowable* (quite the contrary, under extremely plausible assumptions the view developed in this paper actually allows one to prove such principle). In other, looser words, the existence of a demonstration of φ entails that the basic facts making φ true are feasibly knowable, and that these compose the fact described by φ *via* successive applications of feasibly knowable operations.

Secondly, a simple induction shows that the view developed in this paper entails the principle of *knowledge of truth under ideal conditions*:

(ICTK) Under ideal conditions, if 'P' is true, it is known that P[44]

(it's just that, in the case of a Church-Fitch sentence, under ideal conditions the sentence is always untrue and (so) not known and (indeed) such that its negation is known).[45]

[44]*Proof.* We prove that, under ideal conditions, a speaker tracks whether P, which, as *per* fn 19, is throughout assumed to suffice for the fact that, under ideal conditions, if 'P' is true, it is known that P. As said, the proof is by induction. For simplicity and without loss of generality, we assume the enriched language and expanded definition mentioned in fn 36.

- *Base case.* If 'P' is atomic, by the argument in sections 4 and 5, under ideal conditions, a speaker asserts 'P' iff P.
- *Inductive step.*
  - If 'P' is of the form 'Q_0 and Q_1', by the clause for $\wedge$, under ideal conditions, a speaker asserts 'Q_0 and Q_1' iff she asserts 'Q_0' and asserts 'Q_1', and so, by the induction hypothesis, iff Q_0 and Q_1.
  - If 'P' is of the form 'Q_0 or Q_1', by the clause for $\vee$, under ideal conditions, a speaker asserts 'Q_0 or Q_1' iff she asserts 'Q_0' or asserts 'Q_1', and so, by the induction hypothesis, iff Q_0 or Q_1.
  - If 'P' is of the form 'If Q_0, then Q_1', by the clause for $\supset$, under ideal conditions, a speaker asserts 'If Q_0, then Q_1' iff, if she asserts 'Q_0', then she asserts 'Q_1', and so, by the induction hypothesis, iff, if Q_0, then Q_1.
  - If 'P' is of the form 'It is not the case that Q', by the clause for $\neg$, under ideal conditions, a speaker asserts 'It is not the case that Q' iff she does not assert 'Q', and so, by the induction hypothesis, iff it is not the case that Q.
  - If 'P' is of the form 'For every x, $Q(x)$', by the clause for $\forall$, under ideal conditions, a speaker asserts 'For every x, $Q(x)$' iff, for every a (substitutional quantification), she asserts '$Q(a)$', and so, by the induction hypothesis, iff, for every a, $Q(a)$, and so iff, for every x (objectual quantification), $Q(x)$.
  - If 'P' is of the form 'For some x, $Q(x)$', by the clause for $\exists$, under ideal conditions, a speaker asserts 'For some x, $Q(x)$' iff, for some a (substitutional quantification), she asserts '$Q(a)$', and so, by the induction hypothesis, iff, for some a, $Q(a)$, and so iff, for some x (objectual quantification), $Q(x)$. QED.

[45]More in detail, taking again as Church-Fitch sentence 'There are 1,963 trees in St Andrews and it is not known that there 1,963 trees in St Andrews', under ideal conditions either there are 1,963 trees in St Andrews or it is not the case that there are 1,963 trees in St Andrews, and, by (ICTK), the speaker knows which. If she knows that there are 1,963 trees in St Andrews, by (ICTK) she knows that it is known that there are 1,963 trees in St Andrews, and so knows that it is not the case that [there are 1,963

In fact, for several reasons independent of the specific view developed in this paper, (ICTK) rather than (TFPK) would seem to be the *from-truth-to-knowledge* principle an anti-realist should go for.[46] To start with, *the most fundamental from-truth-to-knowledge principle* is the principle of *knowledge of truth*:

(TK) If 'P' is true, it is known that P.

An anti-realist retreats from (TK) to weaker from-truth-to-knowledge principles such as (TFPK) or (ICTK) not because (TK) is fundamentally wrong, but simply because of the annoying fact that, while fundamentally right, (TK) suffers from boring counterexamples basically due to failures to perform relevant computations or to be at the right place at the right time. If so, an anti-realist should simply screen off such complications, by maintaining that (TK) *holds as long as they are absent*—that is, as long as conditions are ideal. (ICTK) is such version of (TK), whereas (TFPK), far from being such, is an extraneous principle oddly requiring the compatibility of these complications with conditions' being ideal. Moreover, focussing on anti-realism *as motivated by the manifestation constraint on understanding*, the essential idea is that *the link between meaning and use is forged under ideal conditions*, and so it should just be expected that, similarly, the resulting link between truth and knowledge also holds under ideal conditions. (ICTK) places the link exactly there, whereas (TFPK), far from doing so, postulates a weird, "transconditional" link between truth under non-ideal conditions and knowledge under some ideal conditions. (BTW, why only "some"?!) Finally, from a more *eschatological* point of view, the natural idea of "*epistemic doomsday*" is the ameliorative one of a condition in which *all our present defects are eliminated so that everything that is then the case is revealed* (see e.g. *1 Corinthians*, 13: 8–12). (ICTK) is a version of that idea, whereas (TFPK), far from being such, is a version of the heterodox, conciliatory idea of epistemic doomsday as of a condition in which all our present defects are among the things that are revealed. An anti-realist has ample reasons to shift her focus from *what can feasibly be known* to *what is known under ideal conditions* (possibly even under an unusually broad sense of 'ideal', see fn 34).[47]

trees in St Andrews and it is not known that there are 1,963 trees in St Andrews]; if she knows that it is not the case that there are 1,963 trees in St Andrews, she knows that it is not the case that [there are 1,963 trees in St Andrews and it is not known that there are 1,963 trees in St Andrews] (and, by (ICTK), she knows that it is not known that there are 1,963 trees in St Andrews).

[46] I should stress however that, on the view developed in this paper, *the most fundamental epistemic constraint*, the one that is *the direct outcome of the argument from understanding* is (TD). (Since, as I have mentioned in the text, the existence of a demonstration has an objective component that makes it non-reducible to facts about possible knowledge, (TD) is *not* a from-truth-to-knowledge principle, but, since, as I have argued in the first point in the text concerning the epistemic consequences of the view developed in this paper, the existence of a demonstration does have very clear and substantial epistemic consequences, (TD) *is* an epistemic constraint.) (ICTK) is rather a *by-product of the machinery used in the argument from understanding*. But it is a crucial such by-product all the same, on the one hand, because it further highlights that that machinery has *very clear and substantial epistemic consequences* and, on the other hand, because its *congeniality to general anti-realist thinking* (which I am about to argue for in the text) can be taken as a further confirmation that the machinery is on the right track.

[47] The move from (TFPK) to (ICTK) broadly correlates to the move, in the debate on *response depen-*

7 Consequences for Intuitionism

Having observed all this, the view developed in this paper can in principle be so modified as to yield constructive demonstrations along the lines of the famous, so-called *BHK semantics* (from Brouwer [1907]; Kolmogorov [1932]; Heyting [1934], in an order that is to me unclear) which typically accompanies adoption of *intuitionist logic*. This can be done by modifying the clauses for $\supset$, $\neg$ and $\forall$ bringing into play the further speech acts of *conditional application* and *rejection of application*. More in detail, as for $\supset$ a speaker may be disposed (under ideal conditions) to [apply $\varphi(\xi_0) \supset \psi(\xi_1)$ to a sequence iff she *applies* $\psi(\xi_1)$ to it *conditionally* on $\varphi(\xi_0)$].[48,49] She thereby (plausibly) manifests her taking the existence of a procedure taking any permonstration of $\varphi(\xi_0)$ by a sequence into a permonstration of $\psi(\xi_1)$ by it to be the necessary and sufficient condition for the sequence to permonstrate $\varphi(\xi_0) \supset \psi(\xi_1)$. As for $\neg$, a speaker may be disposed (under ideal conditions) to [apply $\neg\varphi(\xi)$ to a sequence iff she *rejects* applying $\varphi(\xi)$ to it].[50,51] She thereby (plausibly) manifests her taking the existence of a procedure taking any permonstration of $\varphi(\xi_0)$ by a sequence into a permonstration of the absurdity by it to be the necessary and sufficient condition for the sequence to permonstrate $\neg\varphi(\xi)$. As for $\forall$, a

dence, from what Wright [1992], pp. 108–139 calls '*basic equations*' to what he calls '*provisional equations*'. The limitation that Wright brings out concerning the move to provisional equations in the case of response dependence—that *they leave truth under non-ideal conditions epistemically unconstrained*—would *not* seem to have a correlate applying to the overall view developed in this paper, since, even under non-ideal conditions, by (TD) the truth of φ requires the existence of a demonstration of φ, and, as *per* the first point in the text concerning the view's epistemic consequences, even under non-ideal conditions the existence of a demonstration of φ entails that the basic facts making φ true are feasibly knowable, and that these compose the fact described by φ *via* successive applications of feasibly knowable operations.

[48]Such *conditional application* is stronger than the *material-implicational fact about application* to the effect that, if the speaker applies $\varphi(\xi_0)$ to the sequence, she applies $\psi(\xi_1)$ to it (analogously to how the *conditional belief* that, given P, Q is stronger than the *material-implicational fact about belief* to the effect that, if one believes that P, one believes that Q). For example, since I do not apply 'x is Spanish' to the person I have just seen in the Rossio, by the properties of material implication it follows that, if I apply 'x is Spanish' to her, I apply 'x is French' to her, even though, of course, it is not the case that I apply 'x is French' to her conditionally on 'x is Spanish' (conversely, by *closure of application under conditional application*, conditional application does entail the corresponding material-implicational fact about application).

[49]To go back to the issue discussed in fn 32, it is not the case that, by the properties of conditional application, provided that a speaker does *not* apply $\varphi(\xi_0)$ to a sequence it follows that she applies $\psi(\xi_1)$ to it *conditionally* on $\varphi(\xi_0)$. It is indeed the case that, by the properties of conditional application, provided that a speaker *rejects* applying $\varphi(\xi_0)$ to a sequence it follows that she applies $\psi(\xi_1)$ to it conditionally on $\varphi(\xi_0)$. And, whether or not conditions are ideal, it is not crazy for a speaker to apply $\varphi(\xi_0) \supset \psi(\xi_1)$ to a sequence simply because she rejects applying $\varphi(\xi_0)$ to it.

[50]Such *rejection of application* is stronger than the *negative fact about application* to the effect that the speaker does not apply $\varphi(\xi)$ to the sequence (analogously to how the *rejection* that P is stronger than the *negative fact about belief* to the effect that one does not believe that P). For example, I do not apply 'x is Spanish' to the person I have just seen in the Rossio, even though, of course, it is not the case that I reject applying 'x is Spanish' to her (conversely, by *exclusivity of rejection of application and application*, rejection of application does entail the corresponding negative fact about application).

[51]To go back to the issue discussed in fn 33, whether or not conditions are ideal, it is not crazy for a speaker to apply $\neg\varphi(\xi)$ to a sequence simply because she rejects applying $\varphi(\xi)$ to it.

speaker may be disposed (under ideal conditions) to [apply $\forall\xi_0\varphi(\xi_1)$ to a sequence iff, for *every* sequence differing from the original sequence at most at its ξ_0-corresponding coordinate, she *applies* $\varphi(\xi_1)$ to it *conditionally* on the object at its ξ_0-corresponding coordinate being in the domain].[52] She thereby (plausibly) manifests her taking the existence of a procedure taking any permonstration that an object at the ξ_0-corresponding coordinate of a sequence differing from a sequence at most at its ξ_0-corresponding coordinate is in the domain into a permonstration of $\varphi(\xi_1)$ by the former sequence to be the necessary and sufficient condition for the latter sequence to permonstrate $\forall\xi_0\varphi(\xi_1)$.

However, in the present context such constructive demonstrations are arguably the realist's Trojan horse. For, given that, for some P, P and it is not known that P, it follows that *there is a (broadly) mathematical object constituting the constructive demonstration of the corresponding sentence which is not feasibly (or metaphysically) knowable.*[53] And, if an anti-realist is happy to admit such objects, what's the fuss about usual mathematical objects having to be feasibly knowable?[54]

[52]Similarly to how a speaker's *application* of $\psi(\xi_1)$ to a sequence *conditional* on $\varphi(\xi_0)$ is stronger than the *material-implicational fact about application* to the effect that, if she applies $\varphi(\xi_0)$ to it, she applies $\psi(\xi_1)$ to it (see fn 48), so a speaker's *application* of $\varphi(\xi_1)$ to a sequence and to every sequence differing from the original sequence at most at its ξ_0-corresponding coordinate *conditional* on the object at its ξ_0-corresponding coordinate being in the domain is stronger than the *universal fact about application* to the effect that, for every relevant sequence differing from the original sequence at most at its ξ_0-corresponding coordinate, she applies $\varphi(\xi_1)$ to it. For example, if the domain is the set of students of my class (these being Afonso, Bento, ..., and not including António), since I apply 'x is either Afonso, or Bento, ...' to each of Afonso, Bento,..., it follows that, for every object in the domain, I apply 'x is either Afonso, or Bento,...' to her, even though, of course, it is not the case that I apply 'x is either Afonso, or Bento,...' to António conditionally on António's being a student of my class (conversely, by *closure of application under conditional application*, and assuming that the speaker knows of the objects in the domain that they are in the domain, conditional application does entail the corresponding universal fact about application).

[53]Throughout, by 'An object is knowable' and its like, I mean that it is knowable *what its characterizing properties are.*

[54]Within the anti-realist tradition, there is a long-standing approach—which could aptly be called "*the Stockholm approach*"—that treats constructive demonstrations as *self-standing objects* and deploys this conception to address the paradox of knowability by accepting (TD) under the understanding that such self-standing constructive demonstrations are in some cases *feasibly unknowable* (see Prawitz [1987] for an early work containing a general statement of such conception of constructive demonstrations and Cozzo [1994]; Pagin [1994] for two seminal works deploying the conception to address the paradox of knowability; *pace* Prawitz [1998b], p. 48, Prawitz [1998a], pp. 302–303 explicitly endorses such application, whereas, although Martin-Löf is sometimes mentioned in connection with the Stockholm approach, Martin-Löf [2013], pp. 12–13 apparently accepts an unrestricted principle implying that, if it is not feasibly knowable that P, it is not the case that P, which basically contradicts the Stockholm approach). (More specifically, Cozzo [1994] adopts a positive stance, but fills in the details of his account in such a way as actually not to deliver the target view (thus highlighting just how unnatural the postulation of feasibly unknowable constructive demonstrations is). For Cozzo claims that, if φ is true, there is an "*ideal argument*" for φ, which is supposed to be an argument for φ that would be accepted in an "*ideal epistemic situation*" for φ, which is in turn supposed to be a situation that would be reached in the long run if an inquiry concerning φ were to be pursued in the best way etc. Since in every ideal epistemic situation for a Church-Fitch sentence a speaker knows its negation (see fn 45), and so *a fortiori* does not accept any argument for the sentence, it follows that there is no ideal argument for any Church-Fitch sentence, and so, contraposing on Cozzo's version of (TD), that every Church-Fitch sentence is untrue, thereby

Crucially, no such realist attack can be mounted against the non-constructive demonstrations envisaged by the view developed in this paper. The existence of a non-constructive demonstration of an *atomic* sentence reduces to things' being in a certain way couchable in a basic vocabulary that does not involve talk of demonstrations and procedures (as it *also* happens in the case of constructive demonstrations); *by non-constructive-permonstration functionality*, so does the existence of a non-constructive demonstration

falling prey to the paradox of knowability. Pagin [1994] adopts a more negative stance, containing a perceptive discussion of some of the *problems incurred by a traditional anti-realist* if she accepts feasibly unknowable constructive demonstrations (not the one I have insisted on in the text, see fn 58) and of the *relevance of compositionality* for some of the issues raised by the paradox of knowability. Prawitz [1998a], pp. 302–303 also adopts a positive stance, but does so at the expense of jeopardizing a series of other traditional anti-realist tenets (in addition to the one of the feasible knowability of mathematical objects). For Prawitz claims that, for every Church-Fitch sentence, the *bare collection* of the demonstrations of *its two conjuncts* is a demonstration of the sentence. But, by the same token, for every universal quantification the *bare collection* of the demonstrations of *all its instances* should be a demonstration of the quantification, a consequence which is however multiply unacceptable for a traditional anti-realist. To begin with, that consequence contradicts the *BHK definition of what a demonstration of a universal quantification is.* Moreover, it would seem to validate a *generalized version of the ω-rule*, thus leading to accepting deductive systems that are extraordinarily strong—indeed, typically negation complete (unless one goes in for some wacky non-classical metatheory of deductive systems). Finally, it does *no longer support the relevant versions of one of the most natural arguments for a broadly intuitionist revision of classical logic* that we will consider in the second next paragraph in the text (for we have reason to doubt 'Either there is a demonstration of 'Everything is F' or there is a demonstration of 'It is not the case that everything is F' ' only if we have reasons to doubt 'Either there is a demonstration of 'Everything is F' or there is a demonstration of 'Something is not F' ', but, by the consequence under consideration and the clause for $\exists$, the latter now boils down to 'Either there is a demonstration of every instance of 'Everything is F' or there is a demonstration of some instance 'Everything is not F' ', which, given that, in typical versions of this revisionary argument, being F is decidable and so 'There is no demonstration of 'a is F' ' is equivalent with 'There is a demonstration of 'a is not F' ', in turn boils down to simply another instance of the classical law 'Either everything is F or something is not F' ('Either there is a demonstration of every instance of 'Everything is F' or there is no demonstration of some instance of 'Everything is F' '), and no reason has been given by the revisionary argument to doubt *that*). Notice that, although unacceptable for these reasons, such take on demonstrations of universal quantifications seems actually forced not only by Prawitz' take on demonstrations of Church-Fitch sentences, but also by his *reduction of truth to the existence of a demonstration*, given that the truth (and so, by Prawitz' reduction, the existence of a demonstration) of every instance extremely plausibly constitutes the truth (and so, by Prawitz' reduction, the existence of a demonstration) of the corresponding universal quantification.) While sharing some features of the view developed in this paper, the Stockholm approach is *subject to the problem raised in the text* (Dummett [1982] offers an early statement of the problem when considering the hypothesis that there are feasibly unknowable constructive demonstrations of mathematical sentences, although he does not bring the point to bear on the paradox of knowability). Something like the Stockholm approach has recently been adopted by a few other authors, in my view without significant improvements as far as the problem raised in the text is concerned (for example, Hand [2010] imposes a ban on any consideration that is not meaning-theoretic—without hinting at which finely discriminating meaning-theoretic argument is supposed to establish the knowability of choice functions but not of constructive demonstrations—while Dean and Kurokawa [2010] go for a language in which the contents of Church-Fitch sentences are inexpressible—thus making one wonder what is so bad about the (TFPK)-version of unrestricted anti-realism). A glaring gap in all these works is that they simply *assume their favoured version of (TD) without explaining what argument is supposed to yield it in the first place* (a point pressed e.g. by Murzi [2012], pp. 24–25), contrary to the attempt I have made on behalf of my favoured version of (TD) in sections 4 and 5. Thanks to an anonymous referee for encouraging a discussion of this literature.

of a *compound* sentence (as it does *not* happen in the case of constructive demonstrations, at least for $\supset$, $\neg$ and $\forall$, since the clauses given in the second last paragraph *introduce objects whose existence is not reducible to things' being in any way couchable in a basic vocabulary that does not involve talk of demonstrations and procedures*).[55] And, while we have seen in sections 4, 5 and 6 that it is not a consequence of the anti-realist argument from understanding that *every way things are* is feasibly knowable, it's hard to see how an anti-realist can maintain that [*a mathematical object* like, say, a choice function for a particular family of sets can only be admitted if it is feasibly knowable whereas a mathematical object like a certain constructive demonstration should be admitted although it is not feasibly knowable]. Therefore, although the introduction of constructive demonstrations is in principle possible, it is not open *precisely to an anti-realist. The paradox of knowability provides a hitherto unnoticed reason for an anti-realist to reject the BHK semantics.*[56]

There is more bad news for intuitionism. Not only is one of its most congenial semantics foreclosed by the paradox of knowability; contrary to the BHK semantics, the related but alternative semantics coming with the view developed in this paper (which, in view of its simple functionality, might be called 'EZ semantics', US pronunciation) *no longer supports one of the most natural arguments for a broadly intuitionist revision of classical logic*. For example, if the (TFPK)-version of unrestricted anti-realism holds, 'Either P or it is not the case that P' implies 'Either it is feasibly knowable that P or it is feasibly knowable that it is not the case that P'. Yet, one of the most natural arguments for a broadly intuitionist revision of classical logic claims that, even if the (TFPK)-version of unrestricted anti-realism holds, we do not have sufficient reasons for accepting every instance of 'Either it is feasibly knowable that P or it is feasibly knowable that it is not the

[55]This reducibility implies that we are really associating truth with *facts* rather than with *objects*. That is not only more *natural*, but also avoids *cardinality worries* that I will not go into in this paper.

[56]If the anti-realistically offending non-reducible demonstrations are those for certain *conjunctions*, could an anti-realist uphold *non-reducible* demonstrations for at least the intuitionistically central cases of *conditionals*, *negations* and *universal quantifications*, and go for some sort of *reductive* account as the one I myself have advocated in the case of (some?) conjunctions? Would that avoid commitment to feasibly unknowable objects while preserving enough features of the BHK semantics as to still support the relevant versions of one of the most natural arguments for a broadly intuitionist revision of classical logic that we will consider in the next paragraph in the text? Without even in passing remarking on the *adhocness* of such manoeuvre, and also setting completely aside the *definability of conjunctions in terms of higher-order universal quantifications and conditionals* (see Prawitz [1965], pp. 67–68), the answer is negative. As for universal quantifications, some non-reducible demonstrations of them would be feasibly unknowable (given the equivalence between 'P and Q' and 'Every sentence that is either 'P' or 'Q' is true'). As for conditionals and negations, I presume that we only want to envisage procedures taking any permonstration of a certain kind into a permonstration of the absurdity (i.e. non-reducible demonstrations of negations) if we are ready more generally to envisage procedures taking any permonstration of a certain kind into a permonstration of a certain kind (i.e. non-reducible demonstrations of conditionals). But then some non-reducible demonstrations of negations would be feasibly unknowable (given that it is just as plausible that, [for some P, it is not the case that, if it is not the case that P, it is known that it is not the case that P] as it is that, [for some P, P and it is not known that P], and given that every witness for the former claim entails both that it is not the case that P and that it is not known that it is not the case that P, thus being metaphysically unknowable by the Church-Fitch reasoning). Thanks to Sven Rosenkranz for discussion of this move.

case that P',[57] from which it plausibly follows that we do not have sufficient reasons for accepting every instance of 'Either P or it is not the case that P'. Similarly, if the (TD)-version of unrestricted anti-realism based on constructive demonstrations holds, 'Either P or it is not the case that P' implies 'Either there is a constructive demonstration of 'P' or there is a constructive demonstration of 'It is not the case that P' '. Yet, essentially the same revisionary argument claims that, even if the (TD)-version of unrestricted anti-realism based on constructive demonstrations holds, we do not have sufficient reasons for accepting every instance of 'Either there is a constructive demonstration of 'P' or there is a constructive demonstration of 'It is not the case that P' ',[58] from which it

[57] You might think that the claim is clearly correct because it is clearly the case that it is possible that, for some P, it is neither feasibly knowable that P nor feasibly knowable that it is not the case that P. However, *as soon as (TFPK) includes 'P' and its negation in its scope*, that possibility is actually *ruled out* by (contraposed) (TFPK) and the law of non-contradiction. Someone might suppose that one can preserve the possibility of feasibly necessary ignorance *at the cost of the law of non-contradiction*: if that supposition were correct, the revisionary argument would arguably turn into an argument for a broadly *dual-intuitionist* rather than broadly *intuitionist* revision of classical logic (Incurvati and Murzi [2008], p. 308, fn 9, who endorse the supposition, suggest that what the revisionary argument could turn into is an argument for Nelson's logic N3, which is doubly odd since, by (TFPK), the possibility of feasibly necessary ignorance whether P implies the *possibility of a contradiction* ('It is not the case that P and it is not the case that it is not the case that P') which, far from implying the possibility that the law of excluded middle fails, implies the *possibility of its relevant instance* ('Either P or it is not the case that P'), whereas N3 *is not paraconsistent* and *does not validate the law of excluded middle*). But the supposition *does violence to the natural understanding of the possibility of feasibly necessary ignorance whether P*, since (TFPK) also implies that, if it is not the case P, it is feasibly knowable that it is not the case that P, which certainly wasn't what you thought when you thought that it is clearly the case that it is possible that there is feasibly necessary ignorance whether P. Ignorance *excludes* knowledge. (Someone else might suppose that one can preserve the possibility of feasibly necessary ignorance by postulating that the operative conditional operator in (TFPK) *is not contraposable*, and preserve the validity of the revisionary argument by postulating at the same time that it is *detachable in disjunctive environments*. But that supposition *is incoherent*, since, if the operative conditional operator in (TFPK) is detachable in disjunctive environments, 'Either P or it is not the case that P' implies 'Either it is feasibly knowable that P or it is not the case that P', which in turn intuitionistically entails 'If it is not feasibly knowable that P, it is not the case that P'; if a finite collection of those conditionals leads to a falsehood, so would then the finite collection of the corresponding instances of the law of excluded middle, which is however intuitionistically absurd.) But, if the possibility of feasibly necessary ignorance which would typically be taken to *ground* the alleged fact that we do not have sufficient reasons for accepting every instance of 'Either it is feasibly knowable that P or it is feasibly knowable that it is not the case that P' is ruled out, what else is left to ground that alleged fact? As far as I can see, pretty much nothing. If so, given that, as I have just argued, the clash between (TFPK) and feasibly necessary ignorance is irremediable, keeping fixed the relevant instances of (TFPK) we do have sufficient reasons for accepting the relevant instances of 'Either it is feasibly knowable that P or it is feasibly knowable that it is not the case that P'. Wherever (TFPK) rules, not only there is no *ignorabimus*, but also, one way or the other, we will know. (Anticlimax: having noted all this, in the text I leave it to proponents of the revisionary argument to fix these problems, and turn instead to an even more straightforward problem that would be caused by adoption of the anti-realistically acceptable EZ semantics.)

[58] I would here side with Cozzo [1994], p. 77 against Pagin [1994], p. 99 in considering this to be a plausible claim: *given what specific kind of objects constructive demonstrations are (whether feasibly knowable or not)*, I do not see that we have sufficient reasons for accepting every instance of 'Either there is a constructive demonstration of 'P' or there is a constructive demonstration of 'It is not the case that P' '. In my view, the problem with feasibly unknowable demonstrations is not that they spoil the

plausibly follows that we do not have sufficient reasons for accepting every instance of 'Either P or it is not the case that P'. Be as it may with respect to the revisionary argument *qua* relying on the (TFPK)-version of unrestricted anti-realism or *qua* relying on the (TD)-version of unrestricted anti-realism based on constructive demonstrations, it is clear that the revisionary argument *qua* relying on the (TD)-version of unrestricted anti-realism based on *non-constructive* demonstrations is a non-starter. For 'Either there is a demonstration of 'P' or there is a demonstration of 'It is not the case that P' ' now boils down to simply another instance of the law of excluded middle ('Either there is a demonstration of 'P' or there is no demonstration of 'P' '), and no reason has been given by the revisionary argument to doubt *that*.[59]

In conclusion, the view developed in this paper vindicates a moderate version of *Gödelian optimism* (see Tennant [1997], pp. 166–167): by (ICTK) (and the unscathed law of excluded middle), for every P, under ideal conditions, either it is known that P or it is known that it is not the case that P (see fn 57 for further support for Gödelian optimism, and fn 34 for an additional line of thought that, extending the range of ideal conditions, would strengthen the import of (ICTK)). The usual objection to such optimism—the worry that, for some P, there is no reason for thinking that, even under ideal conditions, either there is a demonstration of 'P' or there is a demonstration of 'It is not the case that P'—rests on the *constructive* assumption that the existence of a demonstration of 'It is not the case that P' is something that goes beyond the non-existence of a demonstration of 'P'. *Precisely from an anti-realist point of view*, that assumption is mistaken because it relies on a conception of a range of objects—constructive demonstrations—that is shown

revisionary argument *qua* relying on the (TD)-version of unrestricted anti-realism based on constructive demonstrations, but that they contradict the traditional anti-realist idea that mathematical objects are feasibly knowable.

[59]Dummett [2001]'s proposal, which basically consists in letting the truth of atomic sentences be epistemically constrained by (TFPK) and then giving the standard characterization of truth for compound sentences, has much to recommend it. Unfortunately, contrary to the view developed in this paper, Dummett strangely does not ground his proposal in the anti-realist argument from understanding (nor in any other argument for anti-realism). In addition to making his proposal rather *unprincipled*, that makes it less clear than it could be that his proposal *too* can actually be seen as endorsing the (TD)-version of unrestricted anti-realism based on a certain kind of demonstrations (with the clauses for compound formulae being orthographically identical with mine but understood in the different way I will mention below), and that, on his proposal *too*, the existence of a demonstration of φ entails that the basic facts making φ true are feasibly knowable, and that these compose the fact described by φ *via* successive applications of feasibly knowable operations. More negatively, the lack of grounding in the anti-realist argument from understanding precludes Dummett's proposal from supporting an *unrestricted from-truth-to-knowledge* principle like (ICTK). Interestingly, in order to save intuitionism Dummett is forced to stipulate by brute force that the logical operators used in the standard characterization of truth for compound sentences obey the principles of *intuitionist* logic. While such move may make Dummett's overall views *consistent* in the relevant respects, it obviously makes the resulting epistemic constraint utterly useless *as a basis for an argument* for a broadly intuitionist revision of classical logic, thus highlighting just how difficult it is to produce a decent anti-realist revisionary argument that does not fall prey in one way or another to the paradox of knowability (notice that Dummett eschews using a more neutral language and giving a characterization of compound sentences in terms of their constructive-demonstration conditions probably because of the issue discussed above in this section, see especially fn 54).

to be untenable by the paradox of knowability. On the only notion of negation that this paper has argued to be anti-realistically acceptable, under ideal conditions any remaining ignorance that P is transfigured into knowledge that it is not the case that P.

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