CHAPTER ELEVEN

THE GENERAL MISSING FROM THE HIERARCHY*

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1. Semantic Paradox and Generality

As shown by the *semantic paradoxes*, a language with enough *expressive resources* whose background logic is *classical* cannot contain a *truth predicate* validating *every* instance of the *truth* schema:

(T) "**P**" is true iff **P**.[1]

For example, consider the *Liar* sentence λ_0 "λ_0 is not true". The relevant instance of (T) is "'λ_0 is not true' is true iff λ_0 is not true". Since "λ_0 is not true" is identical with λ_0, by indiscernibility of identicals the relevant instance of (T) entails "λ_0 is true iff λ_0 is not true",[2] a classical absurdity.

Keeping henceforth fixed the expressive resources required to achieve the *referential loops* triggering the semantic paradoxes, these then present us with the *choice* between *classical logic* and *unrestricted* (T) (also known as *naive truth*),[3] and solutions to the semantic paradoxes can correspondingly be *classified* according to whether they preserve classical logic and revise naive truth or, instead, preserve naive truth and revise classical logic. It is not an easy choice.[4] Sometimes, solutions that revise classical logic are faced with the charge of *"crippling" (sustained) ordinary and scientific reasoning* with *logical operations* like negation:[5] such solutions are supposed to make trouble for an utterly innocent logical principle like, to take as example a prominent kind of revision of classical logic, "There are infinitely many twin primes or it is not the case that there are infinitely many twin primes" (for that principle has the same form as other principles that such solutions reject). But, just so, solutions that revise naive truth can be faced with the charge of crippling ordinary and scientific reasoning with *semantic properties* like truth: such solutions can

be supposed to make trouble for an utterly innocent semantic principle like, to take as example a prominent kind of revision of naive truth, "If there are infinitely many twin primes, then 'There are infinitely many twin primes' is true" (for that principle has the same form as other principles that such solutions reject).

Often, it is conceded that the clash between classical logic and naive truth can be restricted not only to discourse about truth, but, more specifically, to discourse about truth of sentences that are, in some vague sense that would definitely need to be made more precise, involved in a *referential loop* triggering a semantic paradox (I'll henceforth call such sentences "*ungrounded*"). The concession is crucial first in that it forces the area in which solutions that revise classical logic envisage an *effective* failure of classical logic to be *no larger* (and no smaller) than the area in which solutions that revise naive truth envisage an *effective* failure of naive truth—the initial advantage enjoyed by the latter solutions of only affecting discourse about truth rather than any kind of discourse involving logical operations evaporates. Moreover, the concession is crucial also in that it operates a restriction that is *much less stringent* than it may at first appear to be, since, given the variety of referential loops in discourse about truth, many statements about truth with enough *generality* will be apt to involve ungrounded sentences—within the resulting area, it is far from clear that the restrictions on ordinary and scientific reasoning imposed by solutions that revise naive truth are in any sense less crippling than the restrictions imposed by solutions that revise classical logic. Given such concession, then, solutions that revise classical logic are still faced with the charge of crippling ordinary and scientific reasoning about ungrounded truth: such solutions are still supposed to make trouble for a logical principle like, to stick to one style of example of the last paragraph, "Everything the Sultan says is true or it is not the case that everything the Sultan says is true" (for that principle has the same form as other principles that such solutions reject, *and may well involve an ungrounded sentence*). But, just so, solutions that revise naive truth can still be faced with the charge of crippling ordinary and scientific reasoning about ungrounded truth: such solutions can still be supposed to make trouble for a semantic principle like, to stick to the other style of example of the last paragraph, "If it is not the case that everything the Sultan says is true and 'It is not the case that everything the Sultan says is true' is everything the Grand Vizier says, then everything the Grand Vizier says is true" (for that principle has the same form as other principles that such solutions reject, *and may well involve an ungrounded sentence*).

With these facts in view, it is thus at least far from clear that the restrictions on ordinary and scientific reasoning in effect imposed by solutions that revise naive truth are in any sense more natural or less hampering than the restrictions in effect imposed by solutions that revise classical logic. My two cents is that, in typical cases, *it will actually be solutions that revise naive truth that impose less natural and more hampering restrictions on ordinary and scientific reasoning*, for, in the area of ungrounded truth, while solutions that revise classical logic still preserve *a wealth* of principles for logical operations (e.g. the very useful principles of adjunction, *modus ponens*, De Morgan etc.), solutions that revise naive truth only preserve *precious few* principles for truth (e.g. the very useless principle that a conjunction is true only if both conjuncts are plus a couple of other principles depending on the details of the solution). To make this vivid, notice that, while, given only naive truth and a bunch of other plausible assumptions, it is a *child's play* to establish the complex quantificational claim "If the Grand Vizier contradicts something the Sultan says and everything the Sultan says attributes truth to something the Mother Sultan says, then everything the Grand Vizier says is true only if something the Mother Sultan says is not true" (which, contrary to "Everything the Sultan says is true or it is not the case that everything the Sultan says is true", remains *naturally compelling* in the presence of ungroundedness and is *the typical kind of thing* that ordinary and scientific reasoning about ungrounded truth delivers), it is *arguably impossible* to establish it in the presence of classical logic without falling into triviality (or without rejecting the principle that *what is provable is true*, which remains *naturally compelling* in the presence of ungroundedness and is *a crucial tool* of ordinary and scientific reasoning about ungrounded truth).[6]

2. Hierarchy

Be that as it may with respect to the big debate between solutions to the semantic paradoxes that revise naive truth and solutions that revise classical logic, in this paper I'll focus on whether and how a particular approach falling into the first camp can deal with some of the issues concerning certain uses of truth involving generality. The approach in question, going back to Tarski (1933), is the one consisting in replacing the property of *truth* with a *hierarchy* of properties that are nevertheless, in the sense that will emerge in the next sentence, "*truth-like*". The rough idea (which, for our purposes, we needn't make fully precise) is to introduce a series of *ordinal-indexed truth-like predicates* (and, consequently, of *ranks of sentences*), such that, for every finite ordinal n,[7] the truth-like

predicate "true$_n$", when replaced for "true", makes correct the (T)-schema *as restricted to sentences of rank* $\leq n$.[8] A bit less roughly, we start with a base set of sentences free of truth-like predicates (rank 0); we form rank 1 by adding "true$_0$", which, when replaced for "true", is supposed to make correct the (T)-schema for sentences of rank 0 (let's call the resulting schema "the (T_0)-schema"); we form rank 2 by adding "true$_1$", which, when replaced for "true", is supposed to make correct the (T)-schema for sentences of rank 0 or 1 (let's call the resulting schema "the (T_1)-schema"); we form rank 3 by adding "true$_2$", which, when replaced for "true", is supposed to make correct the (T)-schema for sentences of rank 0, 1 or 2 (let's call the resulting schema "the (T_2)-schema") etc. (we can also add that no truth-like predicate applies correctly to anything that is not a sentence).

Even given a complete evaluation for the sentences of rank 0, these stipulations about the hierarchical theory (henceforth, "the Hierarchy") do not even suffice to fix the correct application of "true$_0$" over sentences of rank 1 and higher (although they do suffice to fix its correct application over sentences of rank 0), and thus they do not even suffice to fix the correct application of "true$_1$" over sentences of rank 1. To fill that gap, I'll assume the version of the Hierarchy according to which "true$_n$" does *not* apply correctly to any sentence of rank $> n$. This assumption has the consequence that the hierarchy is *strictly decreasing in strength*, so that a sentence is true$_n$ only if it is true$_m$ [m: $m > n$][9] whereas the converse needn't hold (we'll see two counterexamples in the next paragraph).

To get a concrete sense of the workings of the resulting hierarchy, let's go through a couple of examples. Since "'Snow is white' is true$_0$" has rank 1, it is not true$_0$. However, since "Snow is white" has rank 0, and since snow is white, by the (T_0)-schema "Snow is white" is true$_0$, and, since "'Snow is white' is true$_0$" has rank 1, by the (T_1)-schema "'Snow is white' is true$_0$" is true$_1$. Or, since the Liar sentence λ_1 "λ_1 is not true$_0$" has rank 1, it is not true$_0$, and so, by the (T_1)-schema, it is true$_1$.

The second example is particularly telling, as it explicitly contradicts a widespread understanding of the Hierarchy according to which it involves some sort of weird syntactic or semantic *prohibition* to apply "true$_n$" to any sentence of rank $> n$ (see e.g. Sainsbury 1995, pp. 118–121). On the alternative understanding of the Hierarchy I'm recommending, it is not as though λ_1 is a syntactically or semantically bad thing to do; quite the contrary, λ_1 is perfectly well-formed and perfectly meaningful. Indeed, λ_1 is actually true$_n$ [n: $n > 0$], which, with a staggering abuse of the concepts available in the Hierarchy, one might take to imply that the Hierarchy thinks that λ_1 is "true" (after all, a valid law like "'Snow is white' is true$_0$

or it is not the case that 'Snow is white' is true$_0$" is also not true$_0$ but true$_n$ [n: $n > 0$], and one might take it that the Hierarchy thinks that such law is "true"). More generally, one might take it that the Hierarchy thinks that Liar sentences are "not true up to their rank" and "true at all higher ranks", and hence that they are "true", each of them in effect amounting to a particular a instance of the general correct point that every sentence of rank n+1 is not true$_n$. One might thus take it that the Hierarchy thinks that Liar sentences offer "true" partial descriptions of an important structural feature of the Hierarchy, and so that Liars speak the "truth" about the Hierarchy. Rather than jumping to the subverting conclusion that the Hierarchy must then be a lie, I emphasise that the particular way I've put things constitutes a "staggering abuse of the concepts available in the Hierarchy" since the Hierarchy *does not envisage any property of truth*, only an infinite series of properties that behave like truth only with respect to a finite number of ranks and do not behave at all like truth on the remaining infinite number of ranks (nor does the Hierarchy envisage the quantification into subscript position implicit in the third last sentence). Obviously, this circumstance does give rise to a cluster of worries concerning the Hierarchy's ability to account for *generalising* uses of truth, worries which will entertain us from section 3 onwards.[10]

A more immediate worry concerns however the Hierarchy's ability to account even for *non-generalising* uses of truth. What we may call "*specifying* non-generalising uses", as, for example, in "'Snow is white' is true' is true", are easy to account for: in the example, the sentence referred to must really be something like "'Snow is white' is true$_n$", and then one can choose a higher rank (say, n+1) guaranteed to do the job of producing a sentence that is correct iff "'Snow is white' is true$_n$" is. What we may call "*blind* non-generalising uses", as, for example, in "The first sentence uttered by a Pole in 1933 is true", are more difficult to account for: in the example, at least *without further information* on which sentence is the first sentence uttered by a Pole in 1933 one can choose no rank guaranteed to do the job of producing a sentence that is correct iff the first sentence uttered by a Pole in 1933 is (whichever rank n one chooses, there is a risk that the first sentence uttered by a Pole in 1933 has rank, say, n+1).[11]

It is this problem with blind non-generalising uses of truth that has at least in part motivated *contextualist* versions of the Hierarchy (a paradigmatic reference is Burge 1979), which, among other things, typically postulate "true" and its likes in natural language to function in such a way as to be guaranteed to pick out in each context an appropriately high rank even if the speaker does *not know* exactly *which* rank that is.

This postulation is however problematic in the light of what seem to be *general* features of the *contextual interpretation of adjectives*. For example, it does not seem that "tall" works in such a way as to be guaranteed to apply correctly to whichever person in a context the speaker tries to denote by uttering "The tall person next door is Polish" (assuming that there is exactly one person next door). But that would be the broad analogue for "tall" of the contextualist postulation for "true", with "true" picking out a rank *n* high enough for the (T_n)-schema to apply to the first sentence uttered by a Pole in 1933 (and so, as *per* the intention of a typical utterance of "The first sentence uttered by a Pole in 1933 is true", say something that is correct iff the first sentence uttered by a Pole in 1933 is correct) comparing to "tall" picking out a height threshold low enough for "the tall person next door" to denote the person next door (and so, as *per* the intention of a typical utterance of "The tall person next door is Polish", say something that is correct iff the person next door is Polish).[12]

Having noted this, I'll henceforth assume for the sake of argument that the kinds of pragmatic mechanisms postulated by the contextualist Hierarchy are acceptable. With this background, we can turn to a well-known worry concerning the Hierarchy's ability to account for generalising uses.

3. Truth Meets Politics

An example due to Kripke (1975, pp. 696–697) is alleged to show that the Hierarchy cannot account for certain generalising uses of truth. We consider Dean and Nixon, each of whom wants to deny everything the other says,[13] by uttering, respectively:

(D) Everything Nixon says is not true;

(N) Everything Dean says is not true.

Let ran(φ) be the rank of φ. Then, by the linear ordering of the ordinals, either ran((D)) < ran((N)) or ran((N)) < ran((D)) or ran((D)) = ran((N)). In the first case, Dean fails to deny at least one sentence said by Nixon, namely (N) itself: for (N) does not satisfy the ($T_{ran((D))-1}$)-schema, and so "(N) is not $true_{ran((D))-1}$" does not entail that it is not the case that everything Dean says is not $true_{ran((N))-1}$ (in fact, for every n [n: $n \geq$ ran((D))], it is $true_n$ purely in virtue of the structural features of the Hierarchy, and so $true_n$ independently of whether everything Dean says is not $true_{ran((N))-1}$). In the second case, it is Nixon who fails to deny at least one sentence said by

Dean, for a reason completely symmetric to the one given in the last case. In the third case, they both fail to do so for reasons similar to those given for the two last cases. Thus, no matter which ranks their truth-like predicates pick out, it seems that not both Dean and Nixon can succeed in denying everything the other says, and so it seems that the Hierarchy cannot account for this generalising use of truth.[14,15]

4. Runners-Up

I'd like first to stave off some natural proposals for "*solving*" the Dean-Nixon controversy (that is, for so understanding Dean's utterance of (D) and Nixon's utterance of (N) so that *Dean and Nixon succeed in denying everything the other says*). The first proposal is to allow for *unrestricted quantification into subscript position*:

> (UQSP) By uttering (D), Dean asserts "For every n, everything Nixon says is not true$_n$" (ditto for Nixon).[16]

Although it might achieve the desired expressive purpose, (UQSP) is off mark, as unrestricted quantification into subscript position is simply not well-behaved in the Hierarchy. For suppose on the contrary that it is, and consider then the sentence λ_2 "For every n, λ_2 is not true$_n$". Suppose that λ_2 is true$_m$. Then, ran(λ_2) $\leq m$, and so, by the (T$_m$)-schema, for every n, λ_2 is not true$_n$, and hence, by universal instantiation, λ_2 is not true$_m$. By *reductio*, λ_2 is not true$_m$, and so, by universal generalisation, for every n, λ_2 is not true$_n$. However, for every l [l: $l \geq$ ran(λ_2)], by the (T$_l$)-schema λ_2 is true$_l$, and so, by universal instantiation and contraposition, it is not the case that, for every n, λ_2 is not true$_n$. Contradiction.[17] Notice that this does not show that no form of *restricted* quantification into subscript position is well-behaved in the Hierarchy (in fact, many such forms are straightforwardly definable in the Hierarchy). Unfortunately, (UQSP) requires *unrestricted* quantification into subscript position, for otherwise not both Dean and Nixon can succeed in denying everything the other says.

When *full-blooded quantification* is unavailable, a usually good idea is to resort to *schemata*. The second proposal is in effect to allow for *unconditional schematic assertion*:

> (USA) By uttering (D), Dean asserts every instance of "Everything Nixon says is not true$_{\mathbf{n}}$" (ditto for Nixon).

In spite of its pertinent label, (USA) does not achieve the desired expressive purpose. For Dean may well assert only sentences whose rank is at least 1 (the only thing we know is that Dean asserts every instance of "Everything Nixon says is not $\text{true}_{\mathbf{n}}$", and the lowest rank such an instance can have is 1, considering the "true_0"-instance), in which case some sentence asserted by Nixon (namely, "Everything Dean says is not true_0") is correct, and so Dean should not after all be willing to deny everything Nixon says. Notice that the problem would simply represent itself one rank higher up if Nixon only asserted every instance of "Everything Dean says is not $\text{true}_{\mathbf{n}}$" except for "Everything Dean says is not true_0": for, by the symmetry between Dean and Nixon, Dean should do the same, but then Dean may well assert only sentences whose rank is at least 2 (the only thing we now know is that Dean asserts every instance of "Everything Nixon says is not $\text{true}_{\mathbf{n}}$" except for "Everything Nixon says is not true_0", and the lowest rank such an instance can have is 2, considering the "true_1"-instance), in which case some sentence asserted by Nixon (namely, "Everything Dean says is not true_1") is correct, and so Dean should not after all be willing to deny everything Nixon says. Notice also that the problem cannot be obviated by having Dean additionally assert some trivially correct sentence of rank 0, for it would then be Nixon that should not after all be willing to deny everything Dean says.[18]

If an assertion is problematic because *it pronounces on unintended cases*, a natural move is to hedge it by *conditionalisation*. The third proposal is in effect to allow for *conditional schematic assertion*:

> (CSA) By uttering (D), Dean asserts every instance of "For everything Nixon says, if it has rank **n**, it is not $\text{true}_{\mathbf{n}}$" (ditto for Nixon).

(CSA) avoids the specific problem, affecting (USA), that Nixon may correctly pronounce all sentences asserted by Dean to be not true_0 (as they may all have at least rank 1), for some sentences asserted by Dean do not have rank 0 and Nixon's conditional schematic assertion does not get to pronounce those sentences to be not true_0. However, in accordance with its anachronistic label, it is quite obvious that (CSA) manages to avoid *that* problem only by opening up *another* possibility (of a different kind) for some of the sentences asserted by Nixon to be correct. For, by the properties of *material implication*, "If it has rank 0, it is not true_0" correctly applies to every sentence asserted by Dean if every such sentence has rank > 0 (which it may well have). The argument can then proceed exactly as in the case of (USA).[19,20]

If a general conditional assertion results in a bunch of *annoyingly vacuous instances*, these can be avoided by opting instead for *carefully*

chosen particular unconditional assertions. The fourth proposal is in effect to allow for this, and has two main components. Firstly, the proposal allows for *schematic restriction*: the asserted instances of a schema may be restricted to those *having a certain property*. It is uncontroversial that we usually employ schematic restriction: for example, "If **n** exists, so does its successor" is usually understood as so restricted that the referent of the instance of "**n**" is a natural number. Secondly, the proposal allows for *schematic dependence*: the asserted instances of a schema containing more than one schematic symbol may be restricted to those *where the instances of the schematic symbols stand in a certain relation*. There is evidence that sometimes schematic restriction comes together with schematic dependence: for example, it is arguable that "If **x** is bald and **y** has less hair than **x**, then **y** is bald" is usually understood as so restricted that the referent of the instance of "**y**" has roughly the same hair properties as the referent of the instance of "**x**". Anyhow, the device of schematic dependence is fully intelligible and may well be introduced in the language to achieve the desired expressive purposes.

The fourth proposal, then, is in effect to allow for *restricted schematic assertion with schematic dependence*:

> (RSASD) By uttering (D), Dean asserts every instance of "'**P**' is not true$_{\mathbf{n}}$" so restricted that:
>
> (RSASD$_1$) The instance of "**P**" is asserted by Nixon;
>
> (RSASD$_2$) The referent of the instance of "**n**" is the rank of the instance of "**P**"
>
> (ditto for Nixon).

Dean can thus finally achieve the desired expressive purpose. By uttering (D) as so understood, for every sentence asserted by Nixon of rank *n* Dean will assert that it is not true$_n$ and will assert nothing more; as wished, he will thereby only assert sentences [that are correct] iff Nixon only asserts sentences that are incorrect.[21,22]

Unfortunately, though it might achieve the desired expressive purpose, (RSASD) is problematic because of its crucial reliance on (RSASD$_1$). Firstly, and less importantly, one might object to such restrictions as imposed by (RSASD$_1$) that, in the light of *general* features of *reference*, they do not seem possible if the speaker does *not know* exactly *which* instances she is restricting to (as Dean might be stipulated to be, since Dean might be stipulated not to know exactly which sentences Nixon

asserts). For example, if the speaker does not know the height of the person next door, it would not seem possible for her to assert every instance of "The height of the person next door is exactly **r**" so restricted that the referent of the instance of "**r**" is exactly the height of the person next door.[23] Secondly, and more importantly, one should object to such restrictions as imposed by ($RSASD_1$) that they make the use of a truth or truth-like predicate *completely superfluous* in our context, since, for example, Dean may then simply assert every instance of "It is not the case that **P**" so restricted that the instance of "**P**" is asserted by Nixon and thereby already achieve the desired expressive purpose (ditto for Nixon).

5. (SCSA) 'n' Politics

My own proposal tries to improve on the deficiencies of the previous proposals. Instead of having context do the unlikely work of picking out *exactly which sentences a speaker asserts*, my proposal appeals to context *only for doing the work that it is supposed to do anyways in the contextualist Hierarchy*, namely that of picking out *appropriately high ranks*. More precisely, my proposal is in effect to allow for *selected conditional schematic assertion*:

> (SCSA) Dean asserts every instance of "For everything Nixon says, if it has rank **n**, it is not true$_{\mathbf{n}}$" such that the referent of the instance of "**n**" is the rank of some sentence asserted by Nixon (ditto for Nixon).

Dean can thus achieve the desired expressive purpose. By uttering (D) as so understood, for every maximal non-empty set of sentences asserted by Nixon of rank *n* Dean will assert something to the effect that every member of it is not true$_n$ and will assert nothing more; as wished, he will thereby only assert sentences [that are correct] iff Nixon only asserts sentences that are incorrect.

I'd like to present and discuss an important objection against (SCSA) (as well as against all the other schematic proposals we've been considering). According to the objection, one significant expressive limit of schemata is that they can only be *used to make assertions*, but cannot be *embedded under logical operations*. This would be detrimental to the generality of (SCSA), since examples analogous to the Dean-Nixon controversy can be devised in which the schema would need to be embedded, only to give two staple examples, under negation or as antecedent of a conditional. It is important to see that the objection is *solid*, in the sense that, in our context, it *cannot be addressed by suggesting that schemata can be embedded under sentence-yielding*

logical operations. For, no matter how appealing such approach may be in other contexts, under plausible assumptions in our context such embedding would suffice to *simulate the effects of unrestricted quantification into subscript position* that we've seen in section 4 to be fatal for (UQSP). For example, consider the sentence λ_3 "It is not the case that it is not the case that λ_3 is not true$_{\mathbf{n}}$" (where the embedded schema is "λ_3 is not true$_{\mathbf{n}}$"): plausibly, the schema-embedding sentence "It is not the case that λ_3 is not true$_{\mathbf{n}}$" is correct iff some instance of "λ_3 is not true$_{\mathbf{n}}$" is not, and so λ_3 itself is correct iff every instance of "λ_3 is not true$_{\mathbf{n}}$" is—that is, iff, for every n, λ_3 is not true$_n$. The argument proceeds then similarly to the case of (UQSP). Suppose that λ_3 is true$_m$. Then, ran(λ_3) $\leq m$, and so, by the (T_m)-schema and the plausible interpretation of λ_3 just sketched, it in effect follows that λ_3 is not true$_m$. By *reductio*, λ_3 is not true$_m$. But, plausibly, if one should assert every instance of a schema, one should also assert the schema-embedding double negation of the schema, and so one should assert λ_3. Then, for every l [l: $l \geq$ ran(λ_3)], by the (T_l)-schema λ_3 is true$_l$. Contradiction with the above conclusion that, for every m, λ_3 is not true$_m$.

Rather than trying to make schemata more similar to sentences, we *should understand logical operations as having a wider domain than the set of sentences, and in particular as extending to (schematic) speech acts*. A rough sketch of a natural way of implementing this plan would go as follows. There are two *fundamental types* of *speech acts*: *assertion* and *denial*. The *atomic* speech act is *atomic (possibly selected) assertion*, which is assertion *of every (possibly selected) instance of a schema* (and which is *sound* iff every (possibly selected) instance is correct). (Assertion of a sentence can be seen as a limit case thereof.) The *molecular* speech acts are *molecular assertion*, which is assertion *of a set of speech acts* (and which is *sound* iff every speech act in the set is sound), and *molecular denial*, which is denial *of a set of speech acts* (and which is *sound* iff some speech act in the set is not sound). Utterances of schema-embedding expressions are then interpreted by assigning, to an *embedded schema*, the relevant atomic assertion; to $\neg\varepsilon$ (if directly or indirectly embedding a schema), the molecular denial of the set of speech acts assigned to ε; to ε_0 & ε_1 (if directly or indirectly embedding a schema), the molecular assertion of the union of the sets of speech acts assigned to ε_0 and to ε_1 (notice that, while molecular assertion and denial are defined over arbitrary sets of speech acts, on the pragmatics just sketched the set of speech acts assigned to a schema-embedding expression is always a singleton).[24] To take an example from a far more sophisticated political arena, an utterance of "If, [for everything Berlusconi says, if it has rank **n**, it is true$_{\mathbf{n}}$], then Dudù is our only hope" (where the embedded schema is

"For everything Berlusconi says, if it has rank **n**, it is true$_{\mathbf{n}}$") is treated—as usual—as tantamount to an utterance of "It is not the case that, [[for everything Berlusconi says, if it has rank **n**, it is true$_{\mathbf{n}}$] and it is not the case that Dudù is our only hope]", and the pragmatics just sketched yields that such utterance is a molecular denial of the molecular assertion of {the atomic selected assertion of "For everything Berlusconi says, if it has rank **n**, it is true$_{\mathbf{n}}$", the atomic assertion of "It is not the case that Dudù is our only hope"}. The compositional clauses for soundness of speech acts just sketched yield then that the original utterance is sound iff either some selected instance of "For everything Berlusconi says, if it has rank **n**, it is true$_{\mathbf{n}}$" is incorrect or "It is not the case that Dudù is our only hope" is incorrect, the intuitively required result.[25]

Formally, this approach may well boil down to a near notational variant of embedding schemata under sentence-yielding logical operations, but, *philosophically*, it is in a completely different ballpark: an utterance of "If, [for everything Berlusconi says, if it has rank **n**, it is true$_{\mathbf{n}}$], then Dudù is our only hope" is not a *simple assertion*—straightforwardly evaluable in the narrow terms of *truth*—involving a *single sentence* whose content features a concept in effect equivalent with the concept of truth (a concept which is unavailable in the Hierarchy), but a *complex speech act*—only evaluable in the broader terms represented by the *soundness* clauses given in the last paragraph—involving *every selected instance* of "For everything Berlusconi says, if it has rank **n**, it is true$_{\mathbf{n}}$" (plus "Dudù is our only hope") whose contents only feature the concepts of truth$_0$, of truth$_1$, of truth$_2$ etc. Because of this, on this approach, the attempt at considering the sentence λ_4 "It is not the case that it is not the case that λ_4 is not true$_{\mathbf{n}}$" fails. For, on this approach, that string of symbols is a schema-embedding expression (where the embedded schema is "λ_4 is not true$_{\mathbf{n}}$") rather than a sentence, and hence cannot be named as such or be sensibly attributed truth$_n$: by uttering such expression, one performs a complex speech act involving as only sentences the instances of "λ_4 is not true$_{\mathbf{n}}$"—there just is no appropriate sentence for "λ_4" to refer to. And, if one insists that "λ_4" is then to refer to the speech act itself, or to the schema-embedding expression itself, since these are not even truth$_n$ bearers every instance of "λ_4 is not true$_{\mathbf{n}}$" is correct, and so the molecular denial performed by uttering "It is not the case that it is not the case that λ_4 is not true$_{\mathbf{n}}$" is sound (even if of course not true$_n$), and that's the end of it.[26]

6. From Politics to Semantics

Having thus defended my proposal for solving the particular Dean-Nixon controversy, I wish to leave for another occasion a more rigorous and systematic development of the idea behind it that the Hierarchy can "scheme out" of many of its alleged expressive limits (for example, see note 25 for a direction for future research). I wish to close instead by developing what I think is a less tractable worry concerning the Hierarchy's ability to account for generalising uses of truth. To locate this less tractable worry in our conceptual territory, I start with conjecturing that, not only does (SCSA) work for the Dean-Nixon controversy, but also variations thereof will indeed allow the Hierarchy to account for all those generalising uses— often, but not only, found in *ordinary* discourse—in which, roughly, one uses predications of "true" as *mere means* to arrive at certain other propositions typically about the non-semantic world (what we may call "*non-semantic* generalising uses", a paradigmatic example of which is a typical assertion of "Every axiom of PA is true", which means to say something about natural numbers rather than about a certain axiom system). But some generalising uses— often, but not only, found in *theoretical* discourse—do not fit into this pattern, as they are uses in which, roughly, one uses predications of "true" as *ends in themselves* that attribute a semantic (i.e. world-relating) property to certain sentences (what we may call "*semantic* generalising uses", a paradigmatic example of which is a typical assertion of "Everything in today's edition of *La Gazzetta dello Sport* is true", which means to say something about the reliability of the newspaper rather than about Italian football).

I'd like to take as running example of semantic generalising use of truth a typical assertion of:

(ALEM) All instances of:

(SLEM) **P** or it is not the case that **P**

are true.

Notice that, as every other suitable sentence, (ALEM) too *also* admits of a *non-semantic* generalising use—a use whose point is to say that snow is white or it is not the case that snow is white, and grass is green or it is not the case that grass is green, and the sky is blue or it is not the case that the sky is blue…—and that *that* can easily be accounted by the Hierarchy, in this case with the selected conditional schematic assertion of "For every instance of (SLEM), if it has rank **n**, it is $\text{true}_{\mathbf{n}}$" (where the referent of the

instance of "**n**" is the rank of some sentence that is an instance of (SLEM), of course a selection that is in this case vacuous). What now interests us is however a *semantic* generalising use of (ALEM)—a use whose point is to say that the *law of excluded middle* (LEM) has a certain nice property (I leave it intentionally open whether LEM itself is to be identified with (ALEM), (SLEM) or something else).

There is no question that, for every instance ι of (SLEM), according to the Hierarchy ι has some "*semantically nice*" property. For "ι is true_n" [n: $n \geq \text{ran}(\iota)$] (and so "'ι is true_n' is true_m" [m: $m > n$]) is something semantically nice that the Hierarchy can say about ι (as *per* section 2 and the last paragraph). However, the battery of truth-like properties that the Hierarchy must envisage makes it also the case that "ι is not true_n" [n: $n < \text{ran}(\iota)$] (and so "'ι is not true_n' is true_m" [m: $m > n$]) is on the contrary something semantically *non*-nice that the Hierarchy must say about ι (as *per* section 2). So, what's the *overall status of* ι in the Hierarchy? A brute answer would be to say that, in the Hierarchy, the overall status of ι is positive because the relevant semantically nice properties that the Hierarchy attributes to it are *infinitely* many while the relevant semantically non-nice properties that the Hierarchy attributes to it are *finitely* many. However, such a *brutely quantitative* observation is in itself totally unilluminating: here as elsewhere, size in itself doesn't matter. A slightly more sophisticated answer would be to say that, in the Hierarchy, the overall status of ι is positive because the truth-like predicates that allow one to attribute semantically non-nice properties to it are *defective* in that each of them, for some n, applies correctly *only* to sentences whose rank is at most n. However, this observation shoots itself in the foot, as, in the Hierarchy, *absolutely every* truth-like predicate—including those that allow one to attribute semantically nice properties to ι—is defective in the sense just specified. A much more sophisticated answer would be to say that, in the Hierarchy, the overall status of ι is positive because the truth-like predicates that allow one to attribute semantically non-nice properties to it are not those that are *really sensitive to whether* ι *is correct*. However, the observation is smoke and mirrors. In the Hierarchy, "correct" can *only* be identified either with "true_0" or with "true_1" or with "true_2" etc. (or with finite Boolean combinations thereof). Suppose now that ι is "'Snow is white' is true_0 or it is not the case that 'Snow is white' is true_0". Then, taking "correct" to be "true_1", it is indeed the case that "true_0" is "not really sensitive to whether ι is correct". But that is so simply in the boring sense that ι is true_1 but is not true_0. For all that has been said by this observation, one can *equally legitimately* take "correct" to be "true_0", and

then it is "true_1" that is "not really sensitive to whether ι is correct" (in the same boring sense that ι is true_1 but is not true_0).[27]

Therefore, while it is the case that, *for every instance of* (SLEM)*, according to the Hierarchy* there is some truth-like predicate that correctly applies to it, it is extremely unclear how significant that is in the overall semantic picture offered by the Hierarchy (to go back to an issue emerged in section 2, it is not just that it is a staggering abuse of the concepts available in the Hierarchy to take it that the Hierarchy thinks that ι is "true", it is even extremely unclear whether there is any good excuse for committing the abuse in the first place).[28,29] It is now time to observe that, in any event, that does not even imply the Barcanian consequence that, *according to the Hierarchy, for every instance of* (SLEM) there is some truth-like predicate that correctly applies to it. For, again, in the Hierarchy "correct" can *only* be identified either with "true_0" or with "true_1" or with "true_2" etc. (or with finite Boolean combinations thereof). But it's easy to see that no instance of "For every instance of (SLEM), there is some truth-like predicate that $\text{truly}_{\mathbf{n}}$ applies to it" is correct. The Hierarchy's thoughts are so blind as to lead to such a stunning failure of the relevant instance of the Barcan formula, and so to a failure of the Hierarchy of accepting an apparent consequence of what it thinks. Although the issue is admittedly somewhat vague, I think that we can safely draw from these and similar supporting considerations also the more general conclusion that *it is not even the case that, according to the Hierarchy, every instance of* (SLEM) *has a semantically nice property*, and so, *a fortiori*, the conclusion that *it is not the case that, according to the Hierarchy, there is a semantically nice property had by all instances of* (SLEM) (where the particular quantifier is now swapped with the universal one).[30] Therefore, there is no semantically nice property which, substituted for truth, the Hierarchy can deploy to vindicate a semantic generalising use of (ALEM).

It is tempting to reply that, although the Hierarchy violates the *letter* of a semantic generalising use of (ALEM), its *spirit* is respected by the Hierarchy in virtue of the already observed fact that, for every instance ι of (SLEM), according to the Hierarchy ι has some semantically nice property. But, on reflection, once that fact has been stripped of its natural implications (as has been done in the last paragraph), it's hard to see that what's left has much to do with even the spirit of (ALEM). For what's basically left is, when all is said and done, the bunch of sundry assertions of every instance of "'**P** or it is not the case that **P**' is $\text{true}_{\mathbf{n}}$" (where the referent of the instance of "**n**" is the rank of the instance of "**P**"), and what, *in themselves*, such assertions achieve is merely to attribute to *different* instances of (SLEM) *different* semantically nice properties, which (as has

been noted in the last paragraph) in this case does not even imply an assertion to the effect that every instance of (SLEM) has some semantically nice property. However, whatever LEM exactly is, clearly to say something semantically nice *about it*, rather than (specifically or non-specifically) about some proper subset of the instances of (SLEM), requires asserting that there is some semantically nice property had by all instances of (SLEM), or at least asserting that every instance of (SLEM) has some semantically nice property—it is clearly not enough, for every instance of (SLEM), to attribute a semantically nice property to it. Don't say that, for example, attributing the property of being true_0 to one instance of (SLEM) and attributing the property of being true_1 to another instance of (SLEM) naturally implies attributing the property of either being true_0 or being true_1 to both. For, while that may be alright as far as it goes, its required generalisation covering all instances of (SLEM) would involve the property of either being true_0 or being true_1 or being true_2…, and that's in a different guise just the infamous property of, for some *n*, being true_n which has been discussed in connection with (UQSP) in section 4. Therefore, *in themselves*, the bunch of sundry assertions of every instance of "'**P** or it is not the case that **P**' is $\text{true}_\mathbf{n}$" do not achieve to say anything semantically nice *about* LEM.[31] If such instances are then the semantically nicest thing that the Hierarchy can offer about all the instances of (SLEM) (and they are), the Hierarchy certainly goes against not only the letter, but also the spirit of (ALEM), and, more generally, against the *minimal requirement on a semantic theory to say something semantically nice about* LEM. Thus, by being unable to account for the relevant semantic generalising use, *the Hierarchy does not really say anything about* LEM[32]—in a sense, as far as the Hierarchy is concerned it is as though as LEM did not exist.[33]

The point obviously generalises to many other general principles: by being unable to account for the relevant semantic generalising uses of truth, the Hierarchy does not really say anything about them—as far as the Hierarchy is concerned, it is as though as such principles did not exist. *Many general principles can only be detected by hierarchy-transcending semantic generalising uses*. And the point is even amplified in the case of general principles whose very formulation involves a semantic generalising use (which may not be the case for LEM). Consider, for example, the general principle about *negation*:

(NEG) The negation of a sentence is true iff the sentence is not true.

Again, what's basically left of (NEG) in the Hierarchy is, when all is said and done, the bunch of sundry assertions of every instance of "'It is

not the case that **P**' is true$_{\mathbf{n}}$ iff '**P**' is not true$_{\mathbf{n}}$" (where the referent of the instance of "**n**" is the rank of the instance of "**P**"). However, (NEG) is a principle about negation, and clearly to say something semantically interesting *about negation*, rather than (specifically or non-specifically) about some proper subset of the instances of "It is not the case that **P**", requires asserting that there is some semantically interesting property had by all instances of "It is not the case that **P**", or at least asserting that every instance of "It is not the case that **P**" has some semantically interesting property—it is clearly not enough, for every instance of "It is not the case that **P**", to attribute a semantically interesting property to it. Therefore, *in themselves*, the bunch of sundry assertions of every instance of "'It is not the case that **P**' is true$_{\mathbf{n}}$ iff '**P**' is not true$_{\mathbf{n}}$" do not achieve to say anything semantically interesting *about negation*. If such instances are then the semantically most interesting thing that the Hierarchy can offer about all the instances of "It is not the case that **P**" (and they are), the Hierarchy certainly goes against not only the letter, but also the spirit of (NEG),[34] and, more generally, against the *minimal requirement on a semantic theory to say something semantically interesting about negation*. Thus, by being unable to account for the relevant semantic generalising use of truth, *the Hierarchy does not really say anything about negation*—in a sense, as far as the Hierarchy is concerned it is as though as negation did not exist.[35]

Many general principles and notions can only be detected by hierarchy-transcending semantic generalising uses of truth. By appealing to selected conditional schematic assertion and its scheming likes, the Hierarchy may be able to account for the kind of generality required by non-semantic generalising uses, and so able to account for political controversies; but the Hierarchy is in any case unable to account for the kind of generality required by semantic generalising uses, and so unable to account for LEM or negation.

I like to think that, in a good sense, for a semantic theory to preserve classical logic involves, for example, saying something semantically nice about LEM. In that sense, the Hierarchy does not after all preserve classical logic.

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Notes

[*] I've drastically changed my mind a couple of times on the topics of this paper, and as a consequence I've been working on it for a long while (or is it *vice versa*?). Earlier versions of the material in the paper have been presented in 2006 at the 5th Conference of the Spanish Society of Logic, Methodology and Philosophy of Science (University of Granada); in 2007, at the Arché Reading Party in Raasay (University of St Andrews); in 2011, in a course on *Formal Theories of Truth* at UNAM; in 2012, in a lecture at the University of Barcelona; in 2013, in a course on *Philosophy of Logic* (co-taught with Sven Rosenkranz) at the University of Barcelona, in a lecture at the Institute of Catalan Studies and in a course on *Semantic Paradoxes* at the University of the Azores; in 2014, at the LOGOS

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[1] Throughout, I use bold-faced symbols for schematic expressions. I'll assume that a *schema* is *uttered*, but the *sentences* that are thereby *said*, *asserted*, *denied* etc. (but *not* themselves uttered) are the schema's relevant *instances* (I'll introduce new objects of assertion and denial in section 5). Even so, throughout I consider the *collective sayings* achieved by utterances of schemata as *speech acts* in their own right (and, for simplicity's sake, I sometimes conflate these and similar *illocutory* acts with the *locutory* acts that utterances strictly speaking are).

[2] In the following, analogous indiscernibility-of-identicals steps will be left implicit (see Skyrms 1984 for a problematisation thereof).

[3] Some solutions to the semantic paradoxes include as crucial component a shift from *sentences* to *utterances* as the operative *truth bearers* (see e.g. Goldstein 1992), and they are sometimes presented as having the virtue of preserving *both* classical logic *and* naive truth (see e.g. Sorensen 2001, pp. 181–182). Let's set aside the grave problem for such utterance-based solutions consisting in the fact that they are subject to *revenge paradoxes of utterance truth* (see Zardini 2008, pp. 561–566). And let's also set aside the other grave problem for utterance-based solutions consisting in the fact that, in the case of the semantic paradoxes, we clearly have *notions of truth* (maybe not the fundamental or central ones) *applying to coarse-grained entities* like sentences and propositions, and that one can develop semantic paradoxes employing those notions which offer considerable *resistance* to the strategies pursued by utterance-based solutions (see Zardini 2014b). What should be stressed here instead is that, for some *meaningful* and *non-context-dependent* sentence "*P*" (like e.g. "The Liar utterance is not true"), standard utterance-based solutions accept that *P* but also accept of a certain

utterance of "*P*" that it is not true. Willy-nilly, that is a *violation of the natural analogue of* (T) *for utterance truth*, and so utterance-based solutions do not after all preserve naive (utterance) truth. (This having been noted, throughout I assume that the operative truth bearers are sentences.)

[4] I myself have chosen to go the second way (see e.g. Zardini 2011), although I am no friend of extreme understandings of naive truth (see Zardini 2008, pp. 550–561; 2012; 2014a).

[5] Stylistically, it may be interesting to observe that such charges typically come rife with grisly anatomical metaphors (for example, "mutilate" and its relatives are another favourite). I'm not sure it's particularly illuminating to think of reasoning or logic as having, say, feet, and to think of revisions of classical logic as damaging or chopping off such feet, and I suspect that the abundance of rhetoric stands in for the scarcity of something else.

[6] Reason for the impossibility: to establish the complex quantificational claim in the text, one will arguably need to appeal in effect to the general principle that, *if* φ *contradicts* ψ *and* ψ *attributes truth to* χ, φ *is true only if* χ *is not*. But, considering a sentence like λ_0 (whose contradictory is "λ_0 is true"), it's easy to see that that principle together with the further general principle that *what is provable is true* are inconsistent in classical logic. It is interesting to observe that, while, under plausible assumptions, these principles are entailed by naive truth, even together they fall very much short of entailing anything like naive truth (see Zardini 2014a for further details and discussion of these issues).

[7] The construction can be extended into the *transfinite*, but, for our purposes, that would only bring unnecessary complications (see also note 17).

[8] I'll often help myself to something like the concept of *truth* (typically expressing it with "correct" and its relatives to avoid confusion with truth-like predicates), although this is strictly speaking *unavailable* in the hierarchical theory. I do this for ease of expression; whether or not the same point can *always* be made using a construction available in the theory is however *moot*, and in fact I'll argue in section 6 that sometimes this is *not* the case.

[9] I'll often help myself to *unrestricted quantification into subscript position*, although this is strictly speaking *unavailable* in the Hierarchy (see the discussion of (UQSP) in section 4). I do this for ease of expression; whether or not the same point can *always* be made using a construction available in the Hierarchy is however *moot*, and in fact I'll argue in section 6 that sometimes this is *not* the case.

[10] Thanks to Pilar Terrés for pushing me on the issues discussed in this paragraph.

[11] The distinction between non-generalising and generalising uses of truth is really meant to be the distinction between, very roughly, uses only involving attributions of truth such that all the relevant sentences can be assumed to have, for some rank n, rank $\leq n$, and uses involving attributions of truth for which this is not the case. So understood, the *non-generalising/generalising* distinction is orthogonal to the *referential/quantificational* distinction (as examples of the not totally obvious combinations, the *non-generalising* "Every sentence uttered by John Sobieski on 12/09/1683 is true" is *quantificational*, and the *generalising* "These are true"—pointing at the infinite series of sentences "Snow is white", "'Snow is white' is

$true_0$", "'Snow is white' is $true_1$"…—is *referential*). Thanks to Sergi Oms for urging this clarification.

[12] Burge (1979, p. 193) tries to make the postulation of the contextualist Hierarchy plausible by pointing to the example of a road sign saying "(You) slow down". Setting aside whether one is really doing a favour to the contextualist Hierarchy by resorting to such atypical and puzzling uses of language in explaining typical and non-puzzling blind non-generalising uses of truth, the example would seem anyways to be scarcely relevant, as it is an example in which, on Burge's preferred (and controversial) interpretation, it is *an element of the context* (i.e. the audience) that supplies the materials for interpreting "you". But, in the case of an utterance of "The first sentence uttered by a Pole in 1933 is true", there is *no element of the context* that supplies the materials for interpreting "true": for what supplies such materials is the first sentence uttered by a Pole in 1933, but not everything that is *quantified over in a context* is an *element of that context* (assuming standard understandings of definite descriptions and of contexts). More relevantly, there is at least one kind of case in which it would seem that "tall" is guaranteed to apply correctly to whichever person in a context the speaker tries to denote by uttering "The tall person next door is Polish": namely, the case of *contrastive* use in which the speaker, knowing that there are exactly two people of substantially different heights next door, intends to denote the taller one. However, the postulation of the contextualist Hierarchy obviously needs to concern many cases that are *not* cases of contrastive use. Indeed, in the case of a contrastive use in which the speaker, knowing that there are exactly two correct sentences of substantially different ranks uttered next door, tries to denote the one of higher rank by uttering "the true sentence uttered next door", she clearly *fails* to do so. Thanks to Carlos Romero Castillo for conversations on some of these issues.

[13] Throughout, by "*deny*" and its relatives, I generally mean something along the lines of "*rule out*", so that a denial of φ is something along the lines of ruling out φ. Fortunately, in a *classical* framework, that can be taken to be equivalent with *asserting* "It is *not* the case that φ" (the connection between denial and assertion of the negation becomes looser in many *non-classical* frameworks, see Restall 2005).

[14] In fact, the same point could have been made using "true" rather than "not true" in the Dean-Nixon "controversy" (I'm not quite sure why Kripke chose to use "not true", but I'll follow him in this choice to show that it does not create additional problems).

[15] Burge (1979, p. 194; 1982, pp. 360–361) does consider the example but oddly seems to rest content with showing how, given various ways of filling in the details of the Dean-Nixon controversy, his versions of the Hierarchy "deftly" provide allegedly "intuitively sound" evaluations in terms of their truth-like predicates. He thereby seems to overlook completely the crucial point of Kripke's example that I've insisted on in the text (and that Kripke 1975, p. 696 himself takes pains to emphasise), which rather concerns the apparent fact that, by the Hierarchy's lights, not both Dean and Nixon can succeed in denying everything the other says. Curiously, Burge (1979, pp. 191–192) does mention the possibility of interpreting

certain utterances of certain sentences containing "true" as performing schematic speech acts, but does not apply this insight to the case at hand.

[16] Throughout, I only consider proposals for solving the Dean-Nixon controversy that treat Dean and Nixon *symmetrically*.

[17] The contradiction could be avoided by introducing *new ranks beyond the finite ordinals*, and postulating that ran(λ_2) is one such. However, to achieve the desired expressive purpose, (UQSP) should then understand the relevant quantification into subscript position to *encompass* such ranks, and an analogous argument would show that *that* quantification is simply not well-behaved in the Hierarchy.

[18] Thanks to Philipp Blum for suggesting to consider this last move.

[19] A variation on (CSA) would rephrase it in terms of a *binary* universal quantifier taking the pairs of the properties expressed by coordinated instances of "It has rank **n**" and "It is not true$_{\mathbf{n}}$" respectively (rather than, as in (CSA), a *unary* universal quantifier taking the properties expressed by instances of "If it has rank **n**, it is not true$_{\mathbf{n}}$"). The argument would then appeal to the usual feature of the binary universal quantifier of always yielding a *correct* sentence when taking as first property one that is not had by anything (if such feature is denied, it still is at least very unclear that Dean should be willing to assert every instance of the resulting schema—certainly, he should *not* be so willing if, as is sometimes suggested (see e.g. Strawson 1952, pp. 173–179), the binary universal quantifier always yields an *incorrect* sentence when taking as first property one that is not had by anything).

[20] It is the possibility of entangling a blind non-generalising use of truth in, for example, a Dean-Nixon-like controversy that convinces me that such use cannot be understood in terms of an unrestricted or of a conditional schematic assertion doing away with the postulation of the contextualist Hierarchy discussed in section 2 (for such understanding would be affected by problems analogous to those affecting (USA) and (CSA)). Indeed, my own schematic proposal in section 5 for solving the Dean-Nixon controversy will crucially appeal to that postulation. Thanks to José Martínez for raising this issue.

[21] Throughout, I use square brackets (among other things) to disambiguate constituent structure.

[22] It's easy to see that, contrary to the previous proposals, (RSASD) has the plausible consequence that neither Dean nor Nixon assert in fact *anything* if the only utterances made by them are those of (D) and (N) respectively. An analogous comment applies to (SCSA) in section 5.

[23] One might grant the intuition that it is not possible to impose such restriction in the example just considered in the text, but still think that the example is *relevantly different* from the Dean-Nixon controversy, since the restriction that (RSASD$_1$) imposes in that case does not refer to the *semantic* properties of the target instances (such as those referred to by "The referent of the instance of '**r**' is exactly the height of the person next door"), but only to *non-semantic* ones (such as pragmatic ones as those referred to by "The instance of '**P**' is asserted by Nixon"). However, the relevance of such *semantic/non-semantic* distinction would seem to be undermined by the very examples I've used to support the ideas of schematic restriction and schematic dependence. Those examples, as well as many other

considerations, would rather seem to point to the relevance of some sort of *particular/general* distinction (with respect to which ($RSASD_1$) would seem to fall on the wrong side).

[24] A variation on this idea would also envisage as *atomic* speech act *atomic (possibly selected) denial*, which is denial *of some (possibly selected) instance of a schema* (and which is *sound* iff some (possibly selected) instance is incorrect). Utterances of schema-embedding expressions would then be interpreted by assigning, to an *embedded schema or its negation*, the relevant atomic assertion or atomic denial (it may then be natural to modify along similar lines also the rest of the pragmatics just sketched in the text). The differences, if any, between these two approaches do not matter much in our context (but see note 26).

[25] This indicates how, on this approach, it is also possible to assert or deny "what is said" by utterances of schema-embedding expressions (scare quotes being used since such utterances are not simple assertions). For the compositional clauses for soundness of speech acts provide in effect a definition of what it is, for every such well-behaved expression ε, to be an "*ε-good*" set of sentences—that is, a set of sentences whose correctness makes an utterance of ε sound. One can then *assert* "what is said" by an utterance of ε by uttering "For some ε-good set X, for every φ in X, φ is $\text{true}_{\mathbf{n}}$" (where the embedded schema is "φ is $\text{true}_{\mathbf{n}}$"). (This requires a natural extension of the approach to cover quantification while consequently allowing for the selection of the instances of a schema to depend on the value of the variable bound by the schema-embedding quantifier; interestingly, such extension would also lead to a simplification of (SCSA).) And, given that such schema-embedding expression is *fully further embeddable*, one can also use it to deny, suppose, disjoin etc. "what is said" by an utterance of ε. Indeed, although the details lie beyond the scope of this paper, I expect that, under plausible assumptions (including natural clauses dealing with referentially looping schema-embedding expressions), the approach can be so developed as to offer *a systematic account of the kind of generalising use of truth focussed on in sections 3–5 in terms of the schema-embedding expression just introduced* (an account that would basically reduce to (SCSA) in the case of the Dean-Nixon controversy). Thanks to José Martínez for pressing me on this.

[26] It might be felt that my reply to the objection requires too sharp a distinction between truth-like properties of sentences (and of assertions thereof) and soundness of speech acts. Haven't I myself just ended up giving—with my compositional clauses for soundness of speech acts—a perfectly natural explanation of truth of speech acts? I think that such explanation would actually be very far from natural. For, arguably, a molecular assertion of a set reduces to the *conjunctive* fact that one performs all the members of the set (with an atomic assertion of a schema reducing to the *conjunctive* fact that one asserts every instance of the schema), and a molecular denial of a set reduces to the *conditional* fact that, if one performs all the members of the set but one assertion (denial), one performs the opposite denial (assertion). (This requires the notion of atomic denial as *per* note 24, with an atomic denial of a schema reducing to the *conditional* fact

that, if one asserts every instance of the schema but one instance, one denies that instance.) It is very far from natural to attribute truth or truth-like properties to such facts.

[27] It might be protested that what is meant by saying that "$true_0$" is "not really sensitive to whether ι is correct" is simply [that ["Snow is white" is $true_0$ or it is not the case that "Snow is white" is $true_0$] but ι is not $true_0$]. But then, since *it is completely trivial* (at least from section 2) that ["Snow is white" is $true_0$ or it is not the case that "Snow is white" is $true_0$], and since anyways *that in itself involves no mention of* ι (let alone of its correctness), what is meant is just, in effect, that ι is not $true_0$—hardly something that tells against the significance of the fact that ι is not $true_0$!

[28] In this dialectic, I freely switch, according to what is expositorily more convenient, between talk of properties and their being had, on the one hand, and talk of predicates and their correctly applying, on the other hand.

[29] This point affects also the ability of the Hierarchy to account for *semantic* (specifying or blind) *non-generalising* uses of truth (notice that, in my discussion of non-generalising uses in section 2, I was implicitly focussing on non-semantic ones).

[30] For example, a similar supporting consideration is to the effect that, although, according to the Hierarchy, the semantic property of being, say, [$true_{79}$ if of rank 79] is had by all instances of (SLEM), that is not a semantically *nice* property, as it is had also, for example, by the semantically totally *ugly* "'Snow is white' is $true_{79}$ and it is not the case that 'Snow is white' is $true_{79}$" (I insist that the latter is semantically *totally* ugly: the fact that is not of rank 79 does not make it semantically any better than "Snow is white and it is not the case that snow is white").

[31] This situation is usefully compared with the one envisaged by a common position on the *preface paradox* (see Makinson 1965 for the introduction of the paradox), according to which, roughly, although the author of a book believes of *each statement* in the book that it is correct, she does not believe that the book *as a whole* is correct. But, while that common position is attractive, the one taken by the Hierarchy on LEM is much less so, since LEM, contrary to typical empirical theories, can be assumed to be *certain*. Notice that the author might still believe that the book as a whole contains *some*, or even *a lot of*, correct statements. But the analogous beliefs in our context (for example, the belief that (SLEM) has some $true_{79}$ instances) are semantically nice beliefs (non-specifically) about some proper subset of the instances of (SLEM) rather than about LEM (*by the standards of our context*, it is not enough for a principle to have a semantically nice property that some, or even a lot, of its instances have some such property, for in that sense *affirming the consequent*, or *enumerative induction*, would have just as semantically nice a property as LEM).

[32] "Really" because, as *per* note 30, the Hierarchy does say, for example, that the semantic property of being, say, [$true_{79}$ if of rank 79] is had by all instances of (SLEM); however, for the reason pointed out in that note, that is not something

semantically nice (nor something semantically non-nice), and so not really something *relevant* to say.

[33] One might picture the present dialectic thus. EZ: "Hierarchy, what do you think about LEM?"; Hierarchy (taking a big breath): "'Snow is white or it is not the case that snow is white' is true_0; ''Snow is white' is true_0 or it is not the case that 'Snow is white' is true_0' is true_1; ''Snow is white' is true_1 or it is not the case that 'Snow is white' is true_1' is true_2—"; EZ: "Stop, stop—I meant *about* LEM *itself*"; Hierarchy (taking time): "Well, in that case, I would say that every instance of (SLEM) is, hum,"—silence.

[34] Ironically, Tarski (1933, p. 197) famously took to task certain theories of truth for failing to prove a close kin of (NEG). *Tu quoque Alfrede*!

[35] Points similar to those made in notes 30–33 for LEM apply for negation.