Context and Consequence. An Intercontextual Substructural Logic*

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> If it were resolved that such transformations were in general inadmissible, that would prevent any deeper logical investigation (Frege [1918], p. 64, my translation)

*I let the core idea of this paper already surface in a (fittingly enough) fleeting fashion in Moruzzi and Zardini [2007], p. 180. Earlier versions of the material in the paper have then been presented in 2010 at the 3rd Arché Foundations of Logical Consequence Workshop on *Propositions, Context and Consequence* (University of St Andrews); in 2011, at the COGITO Philosophy of Language Seminar (University of Bologna) and at the PETAF Mid-Term Conference (University of Aberdeen); in 2012, at the GAP 8 on *What may we believe? What ought we to do?* (University of Konstanz). I would like to thank all these audiences for very stimulating comments and discussions. Special thanks go to Fabrice Correia, Peter Fritz, Dan López de Sa, Sebastiano Moruzzi, Julien Murzi, Eugenio Orlandelli, Peter Pagin, Jim Pryor, Steve Read, François Recanati, Sven Rosenkranz, Isidora Stojanović and several anonymous referees. At different stages during the writing of the paper, I have benefitted from an AHRC Postdoctoral Research Fellowship, from a UNAM Postdoctoral Research Fellowship and from the FP7 Marie Curie Intra-European Research Fellowship 301493 on *A Non-Contractive Theory of Naive Semantic Properties: Logical Developments and Metaphysical Foundations* (NTNSP), as well as from partial funds from the project CONSOLIDER-INGENIO 2010 CSD2009-00056 of the Spanish Ministry of Science and Innovation on *Philosophy of Perspectival Thoughts and Facts* (PERSP), from the FP7 Marie Curie Initial Training Network 238128 on *Perspectival Thoughts and Facts* (PETAF), from the project FFI2011-25626 of the Spanish Ministry of Science and Innovation on *Reference, Self-Reference and Empirical Data* and from the project FFI2012-35026 of the Spanish Ministry of Economy and Competition on *The Makings of Truth: Nature, Extent, and Applications of Truthmaking.*

Abstract

Some apparently valid arguments crucially rely on context change. To take a kind of example first discussed by Frege, 'Tomorrow, it'll be sunny' taken on a day seems to entail 'Today, it's sunny' taken on the next day, but the first sentence taken on a day sadly does not seem to entail the second sentence taken on the second next day. Mid-argument context change has not been accounted for by the tradition that has extensively studied the distinctive logical properties of context-dependent languages, for that tradition has exclusively focussed on arguments whose premises and conclusions are taken at the same context. I first argue for the desiderability of having a logic that accounts for mid-argument context change and I explain how one can informally understand such context change in a standard framework in which the relation of logical consequence holds among sentences. I then propose a family of simple temporal "intercontextual" logics that adequately model the validity of certain arguments in which the context changes. In particular, such logics validate the apparently valid argument in the Fregean example. The logics lack many traditional structural properties (reflexivity, contraction, commutativity etc.) as a consequence of the logical significance acquired by the sequence structure of premises and conclusions. The logics are however strong enough to capture in the form of logical truths all the valid arguments of both classical logic and Kaplan-style "intracontextual" logic. Finally, I extend the framework by introducing new operations into the object language, such as intercontextual conjunction, disjunction and implication, which, contrary to intracontextual conjunction, disjunction and implication, perfectly match the metalinguistic, intercontextual notions of premise combination, conclusion combination and logical consequence by representing their respective two operands as taken at different contexts.

1 Mid-Argument Context Change

Natural language typically contains *context-dependent* expressions. Context dependence is widely acknowledged to have deep repercussions on *formal logic* and *semantics*, and in the few last decades both these disciplines have in fact seen enormous advances in the study of this subject. However, certain particularly *dynamic* features of context dependence have remained typically outwith the purview of logical and semantic theories of context-dependent languages. What I have in mind are certain *changes of context* that happen *mid-argument* (and even mid-sentence). There is no denying that, in real life, such changes do occur (indeed, that, in real life, they are the norm). And there is also no denying that we do manage to infer[1] no less effectively when they occur (indeed, that we manage to exploit their occurrence in our inferences): for example, if on a day you told me 'Tomorrow, it'll be sunny', on the next day I can exploit that fact by taking the sentence[2] you uttered as a premise for an effective inference to the conclusion 'Today, it's

[1]Throughout, by 'infer' and its relatives I mean the *activity of drawing conclusions from premises*.

[2]Throughout, by 'sentence' I mean a *meaningful abstract syntactic structure* (of the usual type)—i.e., roughly, an abstract syntactic structure (of the usual type) whose basic elements are taken together with their meaning.

sunny'.[3]

Yet, this pervasive and important phenomenon has typically defied rigorous logical treatment, with some logicians and semanticists even sceptically suggesting that formal logic, *standardly understood* as the study of the relation of *logical consequence* holding among *sentences*, cannot possibly account for mid-argument context change. This negative attitude is often epitomised by the observation that, given such context change, even the property of *reflexivity* (according to which a sentence logically follows from itself) would fail (since, in the example of the last paragraph, on the next day I cannot take the sentence you uttered as a premise to infer the conclusion 'Tomorrow, it'll be sunny'). And the observation implicitly takes failure of reflexivity to signify the end of *anything* worth calling 'formal logic'. For example, David Kaplan in effect argues, rather cursorily, against the idea of developing a logic that accounts for mid-argument context change:

> [...] this will not do for the analysis of validity. By the time an agent finished uttering a very, very long true premise and began uttering the conclusion, the premise may have gone false. Thus even the most trivial of inferences, P therefore P, may appear invalid [...] With no idealizations, the rules of repetition and double negation become invalid. This seems hopeless. (Kaplan [1989b], pp. 584–585)

And Kaplan's rather sketchy scepticism has influenced much of the subsequent literature on the logic of context-dependent expressions:

> [...] results of logical truth are obtainable within a type-oriented approach only on the basis of a definition which *guarantees* that various expressions be evaluated at one fixed index [...] (Predelli [2005], p. 88)

In this paper, I wish to use insights from the burgeoning field of *substructural* logics in order to argue that, contrary to the sceptical line of thought mentioned in the last paragraph, *formal logic as standardly understood can account for mid-argument context change*, providing, with regard to this pervasive and important phenomenon, the systematisation and ensuing understanding to be expected from formal modelling. In section 2, I'll argue for the desiderability of having a logic that accounts for mid-argument context change and I'll explain how one can informally understand such context change in a standard framework in which the relation of logical consequence holds among sentences. In sections 3–5, I'll then substantiate my main contention by developing certain families of logics that, while keeping fixed the assumption that the relation of logical consequence holds among sentences, allow for mid-argument (and even mid-sentence) context change in virtue of the fine structure into which they combine premises and conclusions.

[3] Henceforth, by '*mid-argument context change*' and its like I mean the kind of mid-argument context change instantiated in the example just presented in the text.

2 Entailments among Sentences at Contexts

In this section, I'll lay down the groundworks of the philosophical picture that motivates the formal systems of sections 3–5 (see also Moruzzi and Zardini [2007]; Zardini [2014a] for more relevant background on logical consequence). The example of mid-argument context change introduced in section 1 is naturally conceptualised as a case in which a sentence φ at a context c_0 *entails*[4] a sentence ψ at a context c_1. More specifically, 'Tomorrow, it'll be sunny' at a context c_0 with time t would seem to entail 'Today, it's sunny' at a context c_1 with a time located at the day after the day at which t is located and with the values of all the other relevant parametres identical to those of c_0 (let's henceforth label this entailment 'the *Fregean Entailment*' and use 'c_0^{FE}' and 'c_1^{FE}' as names for two contexts with the features just described).[5] Such conceptualisation as a genuine entailment is intuitively warranted by the fact that, given the specifics of c_0^{FE} and c_1^{FE}, the *meanings themselves* of 'today' and 'tomorrow' *guarantee* that, if the premise is true in c_0^{FE}, the conclusion is true in c_1^{FE}, and do so with the same sense of "*semantic necessity*" as, for example, the meaning itself of 'actually' guarantees that, if 'Snow is white' is true in c_0^{FE}, 'Actually, snow is white' is true in c_0^{FE}.[6]

Now, if, equipped with the awareness that entailment can hold among sentences at contexts, one looks at traditional accounts of logical consequence for a context-dependent language (such as paradigmatically Kaplan [1989a], pp. 541–553), one is struck by the fact that it is implicitly assumed that the relation only holds between premises and con-

[4]Throughout, I use 'entail' and its relatives to denote the relation that is the converse of the relation denoted by 'logically follow' and its relatives.

[5]Notoriously, Frege [1918], p. 64 went so far as to claim that in similar situations exactly the same thought—in Frege's technical sense of 'thought'—is expressed in c_0^{FE} and c_1^{FE}. Our discussion will be neutral with respect to this stronger claim.

[6]Notice that, strictly speaking, while useful in first introducing the idea, it is actually rather misleading to say that 'Tomorrow, it'll be sunny' at (precisely) c_0^{FE} entails 'Today, it's sunny' at (precisely) c_1^{FE}, for c_0^{FE} and c_1^{FE} contain *a lot of information* that is utterly irrelevant to the issue whether the Fregean Entailment holds (since, as the notion is used in formal semantics, contexts must be individuated in an extremely fine-grained way)—indeed, they contain so much information as to determine, for every sentence φ, whether φ is true or false in them, and so rob of any real modal force the claim in the text that "the *meanings themselves* of 'today' and 'tomorrow' *guarantee* that, if the premise is true in c_0^{FE}, the conclusion is true in c_1^{FE}" (at least if, in that claim, 'if' expresses material implication and if contexts are guaranteed to have the parametre values which they happen to have). The 'at'-modifications that are crucial for the Fregean Entailment and similar entailments to hold consist not in taking the relevant sentences at *maximally specific individual contexts*, but in taking them at *very abstract types of contexts*, in particular at *relationally specified* types of contexts (for example, in the case of the Fregean Entailment, the crucial 'at'-modifications are that 'Tomorrow, it'll be sunny' entails 'Today, it's sunny' with the latter taken at any context whose time is located at the day after the day at which the time of the context at which the former is taken is located, and whose values of all the other relevant parametres are identical to those of the context at which the former is taken). A virtue of the families of logics developed in sections 3–5 is that they account for the Fregean Entailment and similar entailments at such level of abstraction (see also the second challenge raised in this section for a logic of utterances). Having noticed this, in order to avoid unnecessary verbosity, in the following 'context' will be used in a suitably flexible way, applying on an occasion to those types of contexts that are at the level of abstraction appropriate for that occasion.

clusion *at the same context*. This implicit assumption is evidenced by the fact that logical consequence is defined, roughly, as necessary preservation of truth in the same context from the premises to the conclusion and by the ensuing fact that the resulting logics only make sense if the premises and the conclusion are at the same context (since, for example, 'Tomorrow, it'll be sunny' at c_0^{FE} should entail 'Tomorrow, it'll be sunny' at c_0^{FE} and 'Tomorrow, it'll be sunny' at c_0^{FE} should not entail 'Tomorrow, it'll be sunny' at c_1^{FE}, but, on the resulting logics, 'Tomorrow, it'll be sunny' entails 'Tomorrow, it'll be sunny' full stop).

Entailment at the same context is certainly a very interesting case of entailing, especially for studying the logical behaviour of non-context-dependent operators (such as 'and', at least in one of its senses) as well as that of a few context-dependent operators (such as 'actually'), so that the assumption made by traditional accounts is quite warranted for those purposes. However, there are many other cases of entailing involving more than one context of which we seem to have a *clear concept*, as witnessed by the Fregean Entailment. Moreover, some of these cases actually seem to be *essential for studying the logical behaviour of certain context-dependent operators* (such as 'today' and 'tomorrow') in its full extent—an account that does not vindicate the Fregean Entailment is arguably missing out an important bit of the logic of 'today' and 'tomorrow'. Finally, if logical consequence is to have something to do with *real-life inference*, as I've already observed in section 1 it seems that cases like the Fregean Entailment are the norm rather than the exception: since, as I've already remarked in fn 6, for the purposes of semantic analysis contexts must be individuated in an extremely fine-grained way, extremely few real-life inferences turn out to be *intra-* rather than *inter*contextual, and in many cases of the latter sort linguistic adjustments similar to those occurring in the Fregean Entailment are necessary in order to keep track of context change. For all these reasons, it would be desirable to have an *intercontextual logic* that accounts for the validity of the Fregean Entailment and other arguments in which the context changes.

Before proceeding to the task of developing such a logic, I need to address a foundational issue that will be crucial for some of the modelling choices that I'll make. As I've already anticipated in section 1, in this paper I'll rely on work done elsewhere (Moruzzi and Zardini [2007], pp. 179–180; Zardini [2014d]) in assuming the influential view (represented for example by Kaplan [1989a]) that, even in the presence of context dependence, the standard focus of formal logic on a relation of logical consequence holding among *sentences* should be maintained. In other words, letting a *logical-consequence bearer* be, roughly, a thing that can logically follow from or entail other things (see Zardini [2014c] for an indication of one dimension in which this characterisation is rough), I'll assume that, even in the presence of context dependence, the (primary) logical-consequence bearers are sentences (rather than, say, propositions, utterances[7] or sentence-context pairs).[8]

[7]Throughout, by 'utterances' I mean *particular speech acts* (i.e. particular events of asserting, asking, commanding etc.).

[8]Gumb [1979] introduces an *informal notion of diachronic inconsistency among tensed statements that is sensitive to the times at which the statements are made* (without noting that it is merely a particular case of the general phenomenon of mid-argument context change). He then provides a *formal* framework

This assumption, even granting that it is defensible in the presence of the less dynamic features of context dependence on which the logical tradition has typically focussed, seems however to come under pressure precisely from the phenomenon of mid-argument context change whose importance I've emphasised in the second last paragraph. I myself have granted that, in the Fregean Entailment, it is intuitively 'Tomorrow, it'll be sunny' *at* c_0^{FE} that entails 'Today, it's sunny' *at* c_1^{FE}. The emphasised prepositional phrases are crucial: 'Tomorrow, it'll be sunny' at c_0^{FE} does not intuitively entail 'Today, it's sunny' at a context whose time is the same as the time of c_0^{FE}. More generally, a sentence φ at a context c_2 can entail a sentence ψ at a context c_3, without φ at a context c_4 entailing ψ at a context c_5. Now, the need to append such prepositional phrases may suggest that, in the Fregean Entailment, the relation of entailment is not one between the *sentences* 'Tomorrow, it'll be sunny' and 'Today, it's sunny', but one that, somehow involving the contexts in which the sentences are uttered, is rather a relation between the two *propositions* expressed at c_0^{FE} and c_1^{FE} by those two sentences respectively, or a relation between an *utterance* made in c_0^{FE} and one made in c_1^{FE}, or a relation between the *sentence-context pairs* $\langle$'Tomorrow, it'll be sunny', $c_0^{\mathrm{FE}}\rangle$ and $\langle$'Today, it's sunny', $c_1^{\mathrm{FE}}\rangle$.

Although an extended discussion of all these options lies beyond the scope of this paper (see for example Moruzzi and Zardini [2007], pp. 179–180; Zardini [2014d] for attempts at a critical treatment), let me pause here to mention briefly some reasons particularly relevant to our discussion for why I think that the suggestion that mid-argument context change involves logical-consequence bearers other than sentences should be resisted.[9] To begin with, consider the alternative of taking propositions as logical-consequence bearers. First, in certain phenomena of context dependence like *reference failure* of demonstratives, it is doubtful that any proposition is expressed, but it is not doubtful that there could *still* be valid or not valid arguments to be made, like, taking for example a case in which one is hallucinating a dagger, the argument from 'That dagger is on the table' to 'That dagger is on something'. Second, even in those cases in which propositions are expressed, they seem anyways ill-suited to tracking *entailments generated by context dependence* like the one from 'Actually, snow is white' to 'Snow is white': it is implausible to think that the proposition itself that, in @, snow is white entails the proposition that snow is white, since

that in effect *takes sentence-context pairs as logical-consequence bearers*. Gumb does *not* consider a language with context-dependent expressions like 'today', and actually the most natural extension of his framework to include such expressions does *not* validate the Fregean Entailment. Gumb also provides an *adequate tableaux-style deductive system* for the resulting logic (while this paper leaves an analogous task for future work). After presenting an earlier version of the material in this paper at the 3rd Arché Foundations of Logical Consequence Workshop on *Propositions, Context and Consequence* (University of St Andrews), I was informed that Geoff Georgi and Alexandru Radulescu both have work-in-progress that also in effect takes sentence-context pairs as logical-consequence bearers. Both Georgi and Radulescu do consider a language with context-dependent expressions like 'today'. All of Gumb's, Georgi's and Radulescu's discussions are largely motivated by the same broad kind of considerations that I'm offering in this section. Given limitations of space (plus, as for Georgi's and Radulescu's work, the fact that it is still unpublished), a detailed critical comparison of these works with the present one is however better reserved for a future occasion (although see the challenges raised in this section and in fn 32 for a logic of sentence-context pairs). Thanks to an anonymous referee for alerting me to the existence and relevance of Gumb's work.

[9]Thanks to an anonymous referee for urging me to be more explicit about these issues.

things *could* have been such that the former proposition is *true* while the latter is *false*. Third, on many views of propositions (see for example fn 5), the proposition expressed at c_0^{FE} by 'Tomorrow, it'll be sunny' is the *same* as the proposition expressed at c_1^{FE} by 'Today, it's sunny', and so, on such views, even if the Fregean Entailment were somehow tracked at the level of propositions taking propositions as logical-consequence bearers would implausibly turn that entailment into the *trivial* entailment from a proposition to itself. Fourth, taking propositions as logical-consequence bearers may in any event not completely avoid that situations *analogous* to the Fregean Entailment arise, and so may still need the broad kind of intercontextual logic that I'll develop for sentences (for example, if there are *temporalist* propositions, the temporalist proposition [that it will always be the case that it's sunny][10] at c_0^{FE} would seem to entail the temporalist proposition [that it's sunny] at c_1^{FE}).

Consider then the alternative of taking utterances or sentence-context pairs as logical-consequence bearers. First, utterances (conceived of as in fn 7) are *too few*: many intuitively valid arguments have at least some premises or some conclusions that have never been and will never be uttered. Second, utterances are also *too many*: a natural utterance-based account (as the one sketched in fn 32) would fragment the Fregean Entailment into a myriad of different arguments (like the argument from your utterance on Sunday of 'Tomorrow, it'll be sunny' to my utterance on Monday of 'Today, it's sunny', the argument from your utterance on Monday of 'Tomorrow, it'll be sunny' to my utterance on Tuesday of 'Today, it's sunny', the argument from your utterance on Tuesday of 'Tomorrow, it'll be sunny' to my utterance on Wednesday of 'Today, it's sunny' etc.), thus missing out the apparent underlying *unity* of the phenomenon (see also fn 6). Third, since it is neither *necessary* nor *a priori* in which context an utterance is made, and since, on a natural utterance-based account (as the one sketched in fn 32), the context in which an utterance is made is nevertheless crucial for the logical properties of an utterance, logical consequence would become a *contingent* and *a posteriori* business. Nor would it help if it were on the contrary *necessary in which context an utterance is made* (possibly *modulo* the utterance's existence), for then it would arguably also be *necessary whether the utterance is true* (again, possibly *modulo* the utterance's existence), and so the "necessary" truth preservation characteristic of logical consequence would be *robbed of any real modal force* (see Zardini [2012] for an in-depth discussion of how to understand such force in the presence of context dependence). Notice that only the first problem, but not the remaining ones (nor the additional, more specific problem to be introduced in fn 32), can to some extent be addressed by shifting from utterances to *sentence-context pairs*.

Although I don't take these difficulties to doom the very interesting project of accounting for mid-argument context change by appealing to logical-consequence bearers other than sentences, I think that they do provide some motivation for sticking to the assumption of sentences as logical-consequence bearers. Now, in order to explain how that assumption is compatible with mid-argument context change, and so complete the groundworks of the philosophical picture that motivates the formal systems of sections

[10]Throughout, I use square brackets to disambiguate constituent structure in natural language.

3–5, I first need to undertake a brief metaphysical digression. Most properties[11] are such that they can be variously *modified*. For example, Joan *in his capacity as president of the local wine club* may collaborate with Juan *in his capacity as president of the local cheese club* without Joan *in his capacity as president of the local Barcelona club* collaborating with Juan *in his capacity as president of the local Real Madrid club*. Now, we should all agree that, if a property may hold or fail to hold depending on how it is modified, this does not imply that the property has *additional bearers* corresponding to such modifications: collaborating is merely a relation between people (or other suitable entities),[12] not among people and capacities—it's people (or other suitable entities) that collaborate with one another, although they may do so in particular capacities.[13] Notice that this important

[11]Throughout, I use 'properties' in a general sense, including relations with arbitrarily high arity.

[12]Other suitable entities prominently include (to remain within the *human* realm) all kinds of broadly *social* entities, like clubs, teams, countries etc.: for example, Spain may collaborate with Argentina. Also for these other entities we can observe the circumstance that they may collaborate or fail to collaborate in particular capacities: for example, Spain in its capacity as current holder of the Presidency of the EU Council may collaborate with Argentina in its capacity as current holder of the Presidency of the Mercosur Council without Spain in its capacity as current holder of the Presidency of the Directive Council of the Organisation of Ibero-American States collaborating with Argentina in its capacity as current holder of the Presidency Pro Tempore of UNASUR. And, again, we should all agree that, in such cases, collaborating is merely a relation between Spain and Argentina, not among Spain, Argentina and the relevant capacities. Thanks to an anonymous referee for reminding me of such cases.

[13](Warning: what follows goes into rather subtle issues about *collaborating*, to an extent most likely to be unprecedented by any previous discussion of *logical consequence*. However, as far as I can tell, analogous considerations would eventually be required by *any* other natural example that could be used instead to make the point that I'm making in the text. Thus, plausibly, the *structure* of the following discussion of collaborating is actually an essential component of the argument about logical consequence that I'm developing in the text.) It might be claimed that, at least in other cases, collaborating does hold between capacities. For example, there is arguably a reading of 'In this town, the president of the local wine club collaborates with the president of the local cheese club' meaning, roughly, that the collaboration in question is a matter of tradition going beyond the individual holders of the relevant positions, and it might be conjectured that an adequate analysis of such reading will have to posit that collaborating holds between capacities. It might then be further suggested that, if so, this somehow speaks in favour of the thesis that, in the example considered in the text, collaborating is after all either a relation among people and capacities or a relation between capacities, in either case spoiling the intended analogy. In reply, I'll make two points. First, granting the conjecture, that would in effect simply amount to recognising capacities as being among the other suitable entities between which collaborating can hold (see fn 12). Now, the fact that collaborating can hold *2arily* between capacities does not seem to speak much in favour of the idea that, in the example considered in the text, it holds *4arily* among people and capacities (the first disjunct of the suggestion); at best, it could speak in favour of the idea that, in the example considered in the text, it still holds 2arily between capacities (the second disjunct of the suggestion). But the latter claim is extremely problematic given that it is extremely plausible that, in the example considered in the text, *people are among the bearers* (for example, assuming that both Joan and Juan are tall, the discourse in the text can be glossed as 'Two tall people collaborate, one in his capacity as president of the local wine club and the other in his capacity as president of the local cheese club'). Second, similar glosses also show the conjecture itself to be extremely problematic (for example, assuming that, as a matter of tradition, both the president of the local wine club and the president of the local cheese club must be tall, the original discourse in this fn can be glossed as 'In this town, two tall people collaborate, the president of the local wine club and the president of the local cheese club'). In my view, all such discourses are better analysed as involving an *implicit modal element*, so that, for example, our original discourse is roughly synonymous with 'In this town, as a matter of tradition, the president of the

distinction made by our ordinary metaphysics between *bearers* and *modifications* of a property, although typically absent from the syntax or semantics of *formal* languages, is intriguingly intimated in *natural* languages already at the *syntactic* level in the distinction between *syntactic arguments* and *adjuncts*,[14] and vividly reflected at the *semantic* level in the distinction between *semantic arguments* and *semantic modifiers*.

Returning to our main thread, the bearer/modification distinction is crucial for understanding how the assumption of sentences as logical-consequence bearers can be compatible with mid-argument context change. For while it is true that entailments among sentences hold or fail to hold depending on at which contexts the sentences are taken, this does not imply that the relation of entailing has bearers other than (or in addition to) sentences, just like the fact that collaborations among people hold or fail to hold depending on in which capacities the people are taken does not imply that the relation of collaborating has bearers other than (or in addition to) people. We can thus acknowledge mid-argument context change while upholding the assumption of sentences as logical-consequence bearers if we think of the crucial role played by contexts in that phenomenon in terms of modifications rather than (additional) bearers.[15] In fact, the picture of sentences as the only bearers of the relation of logical consequence with contexts confined to

local wine club collaborates with the president of the local cheese club' (whose analysis arguably does not require positing that collaborating holds between capacities, just as the analysis of 'As a matter of tradition, the bridegroom kisses the bride' arguably does not require positing that kissing holds between wedding roles). Thanks to an anonymous referee for putting forth to me this kind of example.

[14]The correlation is *not* perfect: sometimes bearers are not expressed by syntactic arguments (for example, albeit an adjunct, the second prepositional phrase in 'Mexico sold California to the US for $15,000,000' plausibly expresses a bearer of the relation of selling), sometimes syntactic arguments do not express bearers (for example, albeit a syntactic argument, the expletive pronoun in 'It is raining' does not plausibly express a bearer of the property of raining), sometimes modifications are not expressed by adjuncts (for example, albeit a syntactic argument, the adverbial phrase in 'Juana worded this line beautifully' plausibly expresses a modification of the relation of wording) and sometimes adjuncts do not express modifications (as *per* the first example mentioned in this fn). These points being noted, the correlation, imperfect as it may be, is very suggestive all the same.

[15]Distinguish two *degrees of modifiability* of a property: the *first* degree of modifiability is such that the presence of one modification and the absence of another modification *entails* the relevant unmodified claim, the *second* degree of modifiability is such that the presence of one modification and the absence of another modification entails that the relevant unmodified claim is (in a natural sense) *ill-defined*. For example, the modifiability by cooking methods of the relation of cooking is of first degree (for instance, the modified 'Ampar cooked the rice in the pan' and 'Ampar did not cook the rice in the oven' entail the unmodified 'Ampar cooked the rice'), while the modifiability by capacities of the relation of collaborating is of second degree (for instance, the modified 'Joan in his capacity as president of the local wine club collaborates with Juan in his capacity as president of the local cheese club' and 'Joan in his capacity as president of the local Barcelona club does not collaborate with Juan in his capacity as president of the local Real Madrid club' entail that the unmodified 'Joan collaborates with Juan' is ill-defined). Notice that the second degree of modifiability is in itself compatible with *unmodified claims being well-defined (indeed, either true or false)*, and even with *the presence of one modification defeasibly entailing the relevant unmodified claim* (for example, [if the presidency of the local wine club and the presidency of the local cheese club are respectively Joan's and Juan's only capacities, the unmodified 'Joan collaborates with Juan' is well-defined (indeed, in the example in the text, true)], and the modified positive 'Joan in his capacity as president of the local wine club collaborates with Juan in his capacity as president of the local cheese club' defeasibly entails the unmodified 'Joan collaborates with Juan'). Now, the point of using as running example in the text an example with modifiability of second degree is that *the modifiability by*

being merely modifications of that relation is not only *available*, but it is also arguably *preferable* in certain respects to alternative pictures in which contexts are involved in the logical-consequence bearers themselves either in the form of utterances or in the form of sentence-context pairs (see the challenges raised in this section and in fn 32 for a logic of utterances or sentence-context pairs).

Having thus sketched an appealing (at least for me) *philosophical* picture, the task is now to implement it *formally*—to represent formally a relation of logical consequence that holds among sentences but that, being sensitive to the contexts at which the sentences are taken, is able to account for the validity of the Fregean Entailment and other arguments in which the context changes.

3 Intercontextual Logic

In this paper, I'll limit the construction of intercontextual logic to the particular fragment concerning 'yesterday', 'today' and 'tomorrow' (see fn 35 for a hint about how to extend the construction to other fragments). The development of the fragment will however be instructive enough to appreciate the kind of framework within which I envisage that the more general study of intercontextual logic could profitably take place. I'll start in this section with the basic construction.

As for *syntax*, let's work with a standard *multimodal* 0*th-order* language $\mathscr{L}$:

Definition 1. The set $AS_{\mathscr{L}}$ of the *atomic symbols* of $\mathscr{L}$ is defined by the enumeration:

- The denumerable set $A_{\mathscr{L}}$ of *atoms* $P, P_0, P_1 \ldots, Q, Q_0, Q_1 \ldots, R, R_0, R_1 \ldots$ is a subset of $AS_{\mathscr{L}}$;
- The extensional 1ary connective $\neg$ ("not") belongs to $AS_{\mathscr{L}}$;
- The extensional 2ary connective $\wedge$ ("and") belongs to $AS_{\mathscr{L}}$;
- The intensional and context-dependent 1ary connectives $\mathcal{Y}$ ("yesterday"), $\mathcal{T}$ ("today") and $\mathcal{M}$ ("tomorrow") belong to $AS_{\mathscr{L}}$;
- The punctuation marks (and) belong to $AS_{\mathscr{L}}$.

Definition 2. *Well-formedness in* $\mathscr{L}$ is defined by the recursion: $A_{\mathscr{L}}|(\neg\varphi)|(\varphi \wedge \psi)|(\mathcal{Y}\varphi)|(\mathcal{T}\varphi)|(\mathcal{M}\varphi)$.

Definition 3. *Disjunction* and *material implication* are defined as $(\varphi \vee \psi) := (\neg((\neg\varphi) \wedge (\neg\psi)))$ and $(\varphi \supset \psi) := (\neg(\varphi \wedge (\neg\psi)))$ respectively.

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contexts of the relation of logical consequence is itself arguably of second degree (for example, the modified ' 'Tomorrow, it'll be sunny' at c_0^{FE} entails 'Today, it's sunny' at c_1^{FE}' and ' 'Tomorrow, it'll be sunny' at c_0^{FE} does not entail 'Today, it's sunny' at c_0^{FE}' arguably entail that the unmodified ' 'Tomorrow, it'll be sunny' entails 'Today, it's sunny' ' is ill-defined).

The *intensional context-dependent semantics* for $\mathscr{L}$ is also completely standard:

Definition 4. An *$\mathscr{L}$-structure* $\mathfrak{S}$ is a pair $\langle D_{\mathfrak{S}}, \lhd_{\mathfrak{S}}\rangle$ where $D_{\mathfrak{S}}$ is a non-empty set of objects ("days") and $\lhd_{\mathfrak{S}}$ is a strict linear ordering on $D_{\mathfrak{S}}$ (the "earlier-than relation").[16]

Definition 5.

- For every $d_0 \in D_{\mathfrak{S}}$, $\mathsf{pre}_{\mathfrak{S}}(d_0)$ is the day $d_1 \lhd_{\mathfrak{S}} d_0$ (if it exists) such that, for every $d_2 \lhd_{\mathfrak{S}} d_0$, either $d_2 \lhd_{\mathfrak{S}} d_1$ or $d_2 = d_1$ (the "predecessor");
- Letting $\rhd_{\mathfrak{S}}$ be the converse of $\lhd_{\mathfrak{S}}$, for every $d_0 \in D_{\mathfrak{S}}$, $\mathsf{suc}_{\mathfrak{S}}(d_0)$ is the day $d_1 \rhd_{\mathfrak{S}} d_0$ (if it exists) such that, for every $d_2 \rhd_{\mathfrak{S}} d_0$, either $d_2 \rhd_{\mathfrak{S}} d_1$ or $d_2 = d_1$ (the "successor").

Definition 6. An *$\mathscr{L}$-model $\mathfrak{M}$ on an $\mathscr{L}$-structure* $\mathfrak{S}$ is a pair $\langle \mathfrak{S}, \mathsf{int}_{\mathfrak{M}}\rangle$ where $\mathfrak{S}$ is as in definition 4 and $\mathsf{int}_{\mathfrak{M}} : A_{\mathscr{L}} \times D_{\mathfrak{S}} \mapsto \{0, 1\}$ (the "interpretation function" to "truth" and "falsity").

Definition 7. *Truth in a context d_0 at a circumstance d_1*[17] *in an $\mathscr{L}$-model $\mathfrak{M}$ on an $\mathscr{L}$-structure* $\mathfrak{S}$ is defined by the recursion:

- If $\varphi \in A_{\mathscr{L}}$, then $[\![\varphi]\!]_{d_0,d_1,\mathfrak{M}} = \mathsf{int}_{\mathfrak{M}}(\varphi, d_1)$;
- $[\![\neg\varphi]\!]_{d_0,d_1,\mathfrak{M}} = 1 - [\![\varphi]\!]_{d_0,d_1,\mathfrak{M}}$;[18]
- $[\![\varphi \wedge \psi]\!]_{d_0,d_1,\mathfrak{M}} = \mathsf{min}([\![\varphi]\!]_{d_0,d_1,\mathfrak{M}}, [\![\psi]\!]_{d_0,d_1,\mathfrak{M}})$;[19]
- $[\![\mathcal{Y}\varphi]\!]_{d_0,d_1,\mathfrak{M}} = 1$ iff $[\![\varphi]\!]_{d_0,\mathsf{pre}_{\mathfrak{S}}(d_0),\mathfrak{M}} = 1$, 0 otherwise (including if $\mathsf{pre}_{\mathfrak{S}}(d_0)$ does not exist);
- $[\![\mathcal{T}\varphi]\!]_{d_0,d_1,\mathfrak{M}} = [\![\varphi]\!]_{d_0,d_0,\mathfrak{M}}$;
- $[\![\mathcal{M}\varphi]\!]_{d_0,d_1,\mathfrak{M}} = 1$ iff $[\![\varphi]\!]_{d_0,\mathsf{suc}_{\mathfrak{S}}(d_0),\mathfrak{M}} = 1$, 0 otherwise (including if $\mathsf{suc}_{\mathfrak{S}}(d_0)$ does not exist).

Definition 8. *Truth in a context d in an $\mathscr{L}$-model $\mathfrak{M}$* is defined by the diagonalisation: $[\![\varphi]\!]_{d,\mathfrak{M}} = [\![\varphi]\!]_{d,d,\mathfrak{M}}$.

[16] I thought it interesting to show how some basic features of the framework can already be developed under this relatively weak assumption about the structure of days. In the next section, I'll strengthen the assumption to the more usual one of a *discrete* strict linear ordering. In the other direction, I'm still requiring that the structure of days be *linear*: that property is notoriously controversial in view of the problem of the *open future*, but relinquishing it would lead to complications that would be quite unnecessary for the purposes of this paper (see fn 35).

[17] Notice that, given the very limited expressive resources of $\mathscr{L}$, both contexts and circumstances can be identified with days.

[18] I'll henceforth adopt standard scope conventions to save on brackets, assuming right-associativity for 2ary connectives.

[19] $\mathsf{min}(n, m)$ is the smaller natural number between n and m.

With this *semantics* in place, how should we define *logical consequence*? As I've already mentioned in section 2, if one looks at traditional accounts of logical consequence for a context-dependent language, they are examples of *intracontextual* logics, since they define logical consequence as necessary preservation of the truth of the premises *in the same context* by the truth of the conclusion *in the same context as the premises* (all of course in the same model),[20] and so implicitly assume that *all the premises and the conclusion are at the same context.* In view of the development of intercontextual logic below, it'll be helpful so to extend the notion of logical consequence as to cover *multiple conclusions* (including the limit case of *no* conclusions at all) and to think of both premises and conclusions as combined into *sequences* (rather than, as more usual, into *sets*), where such sequences are in turn understood as functions from an initial (possibly transfinite) segment of the *ordinals* into the set of sentences of $\mathscr{L}$ (and where ordinals are understood as *von Neumann ordinals*). Under these assumptions, the intracontextual logic of 'yesterday', 'today' and 'tomorrow' $\mathbf{Intra}_{\mathcal{YTM}}$ would thus amount to:

Definition 9. $\Gamma \vdash_{\mathbf{Intra}_{\mathcal{YTM}}} \Delta$ holds iff, for every $\mathscr{L}$-structure $\mathfrak{S}$, for every $\mathscr{L}$-model $\mathfrak{M}$ on $\mathfrak{S}$, for every context $d \in D_{\mathfrak{S}}$, if, for every $\varphi \in \mathsf{ran}(\Gamma)$,[21] $[\![\varphi]\!]_{d,\mathfrak{M}} = 1$, then, for some $\psi \in \mathsf{ran}(\Delta)$, $[\![\psi]\!]_{d,\mathfrak{M}} = 1$.

Our intercontextual logic of 'yesterday', 'today' and 'tomorrow' $\mathbf{Inter}_{\mathcal{YTM}}$ will of course lift this crippling restriction to the same context: in the sense which I've explained in section 2, each sentence in an argument will be seen as being at a certain context, *with possibly different contexts for different sentences.* Indeed, since we seem to have a clear concept also of cases like the one in which a sentence at a context entails itself at a different context (for example, 'If snow is white, snow is white' at c_0^{FE} entails 'If snow is white, snow is white' at c_1^{FE}), we should allow for a sentence being at *more than one* context. What is at *exactly one* context is not the sentence in itself, but the sentence *as it occurs in a particular position in an argument* (in the above example, it is 'If snow is white, snow is white' as it occurs as the premise that is at c_0^{FE} and 'If snow is white, snow is white' as it occurs as the conclusion that is at c_1^{FE}).[22]

[20] As *per* Zardini [2012], I actually think that precisely context dependence shows that there are *substantial gaps between logical consequence and truth preservation.* However, without going into the nitty-gritty details, I note that those gaps are compatible with what this paper assumes about logical consequence and truth preservation.

[21] $\mathsf{ran}(\Gamma)$ is the range of the function Γ.

[22] Humberstone [1988] studies logics whose general abstract feature is that *premises and conclusions are thought of as evaluated by two different valuations.* Humberstone [1988], p. 403 explains that one particular instantiation of this general abstract feature results if, with a tensed language including 'yesterday' and 'tomorrow', one in effect thinks of the premises as uttered at a day immediately before the day at which the conclusions are uttered, and notes that, on this scheme, reflexivity fails whereas close relatives of the Fregean Entailment hold (of the other particular instantiations that Humberstone discusses, the only one that seems to lend itself naturally to an account along the broad lines proposed in this paper is the one that results if, with two different but overlapping languages, one thinks of the premises as being in one language and of the conclusions as being in the other language). While a detailed critical comparison of Humberstone's work with the present one lies beyond the scope of this paper, it is in order to comment on what, for our purposes, the main difference arguably is between the two. Whereas Humberstone's

But how can we implement formally this informal idea of a sentence [as it occurs in a particular position in an argument] being at a context? Fortunately, *sequences* give us already the materials to represent the association of sentences [as they occur in particular positions in an argument] with contexts. Given an argument with premises Γ and conclusions Δ, we can put these together into a single sequence Γ, Δ; each ordinal α in the domain of the sequence represents in effect a sentence φ as φ occurs in the αth position of the argument from Γ to Δ. We can then assign to each such ordinal α a context d in an $\mathscr{L}$-structure, thus representing the fact that φ as it occurs in the αth position of the argument from Γ to Δ is at the context represented by d. More formally:

Definition 10. Let κ be an initial segment of the ordinals suitable for representing our sequences and let $\mathbb{S}$ be the set of $\mathscr{L}$-structures.

Definition 11. Let an indexed function $\mathsf{ass}^{\mathsf{i}} : \kappa \times \mathbb{S} \mapsto \bigcup_{\mathfrak{S} \in \mathbb{S}} (D_{\mathfrak{S}})$ be such that, for every $\alpha < \kappa$ and $\mathfrak{S} \in \mathbb{S}$, $\mathsf{ass}^{\mathsf{i}}(\alpha, \mathfrak{S}) \in D_{\mathfrak{S}}$.

Thus, each $\mathsf{ass}^{\mathsf{i}}$ represents, for every argument and $\mathscr{L}$-structure, a particular way of specifying which sentences as they occur in a particular position in that argument are at which contexts of that $\mathscr{L}$-structure.

Now, while intracontextual logic defines logical consequence as necessary preservation of the truth of the premises *in the same context* by the truth of the conclusion *in the same context as the premises* (all of course in the same model), the generalisation made by intercontextual logic consists informally in defining logical consequence as necessary preservation of the truth of the premises *each in a certain context* by the truth of the conclusions *each in a certain context* (all of course in the same model). In such informal characterisation, 'certain' is parametric, but we can now use each $\mathsf{ass}^{\mathsf{i}}$ in order to give it a particular, complete specification. Thus, each $\mathsf{ass}^{\mathsf{i}}$ generates in effect its own intercontextual logic $\mathbf{Inter}^{\mathsf{i}}_{\mathcal{YTM}}$, which, for every $\mathscr{L}$-structure, looks at whether there is necessary preservation of the truth of the premises *each in the context assigned to them by* $\mathsf{ass}^{\mathsf{i}}$ by the truth of the conclusions *each in the context assigned to them by* $\mathsf{ass}^{\mathsf{i}}$ (all of course in the same model). More formally, we have the *family* of logics:

Definition 12. $\Gamma \vdash^{\mathsf{i}}_{\mathbf{Inter}_{\mathcal{YTM}}} \Delta$ holds iff, for every $\mathscr{L}$-structure $\mathfrak{S}$ and $\mathscr{L}$-model $\mathfrak{M}$ on $\mathfrak{S}$, if, for every $\gamma \in \mathsf{dom}(\Gamma)$,[23] $[\![\Gamma(\gamma)]\!]_{\mathsf{ass}^{\mathsf{i}}(\gamma,\mathfrak{S}),\mathfrak{M}} = 1$, then, for some δ such that

logics only account for a *single* context change which occurs *between the premises and the conclusions*, the intercontextual logic of this paper accounts for *arbitrarily many* context changes which occur *with each new premise or conclusion* (indeed, in the extension of the basic construction to be developed in section 5, *with each new relevant component of a premise or conclusion*). (It should be mentioned though that both developments are anticipated as possible avenues of further inquiry by Humberstone [1988], p. 432, n. 6 and Humberstone [1988], p. 402 respectively.) It is a straightforward consequence of this difference that, while the *substructurality* of Humberstone's logics only involves failures of *reflexivity* and *transitivity*, as will become apparent in theorem 7 the substructurality of the intercontextual logic of this paper also extends to failures of *monotonicity*, and even allows for failures of properties like *contraction* and *commutativity*, with the latter circumstance forcing premises and conclusions to be combined into objects that are *finer-grained* than *sets* (as *sequences* are). Thanks to an anonymous referee for alerting me to the existence and relevance of Humberstone's work.

[23] $\mathsf{dom}(\Gamma)$ is the domain of the function Γ.

$\delta \in \mathsf{dom}(\Gamma, \Delta)$ and $\delta > \mathsf{max}(\mathsf{dom}(\Gamma))$[24] ($\delta \geq \mathsf{lub}(\mathsf{dom}(\Gamma))$ if $\mathsf{dom}(\Gamma)$ has no maximum element),[25] $[\![\Gamma, \Delta(\delta)]\!]_{\mathsf{ass}^{\mathfrak{i}}(\delta,\mathfrak{S}),\mathfrak{M}} = 1$.

4 Properties

Having thus defined the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}}$s, I'll look in this section at some of their salient properties. Quite generally, intercontextual logic is a *generalisation* of intracontextual logic, and so, in particular, the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}}$s is a *generalisation* of $\mathbf{Intra}_{\mathcal{YTM}}$. For consider the *constancy* constraint for an index $\mathfrak{i}$:

(C) For every $\mathscr{L}$-structure $\mathfrak{S}$, for some context $d \in D_{\mathfrak{S}}$, for every $\alpha < \kappa$, $\mathsf{ass}^{\mathfrak{i}}(\alpha, \mathfrak{S}) = d$.

(C) in effect forces $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}}$, given an $\mathscr{L}$-structure, to look for necessary truth preservation in the same context. Unsurprisingly, it can thus be established that $\mathbf{Intra}_{\mathcal{YTM}}$ is a particular member of the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}}$s:

Theorem 1. *For some* $\mathfrak{i}$ *satisfying* (C)*,* $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}} = \mathbf{Intra}_{\mathcal{YTM}}$.[26]

As I've suggested in section 2, the logic generated by indices satisfying (C) is especially appropriate for studying the logical behaviour of non-context-dependent operators as well as that of a few context-dependent operators, but it arguably misses out an important bit of the logic of $\mathcal{Y}$, $\mathcal{T}$ and $\mathcal{M}$, which is rather captured by the logics generated by some indices that do not satisfy (C). Consider for example the alternative *immediate-monotonicity* constraint for an index $\mathfrak{i}$:

(IM) For every $\mathscr{L}$-structure $\mathfrak{S}$, for every $\alpha, \beta < \kappa$:

(i) Letting $\trianglelefteq_{\mathfrak{S}}$ be the reflexive closure of $\triangleleft_{\mathfrak{S}}$, $\alpha \leq \beta$ only if $\mathsf{ass}^{\mathfrak{i}}(\alpha, \mathfrak{S}) \trianglelefteq_{\mathfrak{S}} \mathsf{ass}^{\mathfrak{i}}(\beta, \mathfrak{S})$;

(ii) If, for some context $d \in D_{\mathfrak{S}}$, $\mathsf{ass}^{\mathfrak{i}}(\alpha, \mathfrak{S}) \triangleleft_{\mathfrak{S}} d$, then $\alpha < \beta$ only if $\mathsf{ass}^{\mathfrak{i}}(\alpha, \mathfrak{S}) \triangleleft_{\mathfrak{S}} \mathsf{ass}^{\mathfrak{i}}(\beta, \mathfrak{S})$;

[24] $\mathsf{max}(X)$ is the maximum of the set X under the contextually salient ordering (in this case, the standard well-ordering $\leq$ on the ordinals).

[25] $\mathsf{lub}(X)$ is the least upper bound of the set X under the contextually salient ordering (in this case, the standard well-ordering $\leq$ on the ordinals).

[26] Notice that, while theorem 1 can be strengthened so as to cover *every* index satisfying (C) *in its* $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}} \supseteq \mathbf{Intra}_{\mathcal{YTM}}$ *direction*, the theorem cannot be so strengthened *in its* $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}} \subseteq \mathbf{Intra}_{\mathcal{YTM}}$ *direction*. For consider, for example, an index $\mathfrak{i}$ satisfying (C) and such that, for every $\mathscr{L}$-structure $\mathfrak{S}$, for every $\alpha < \kappa$, it is not the case that both $\mathsf{pre}_{\mathfrak{S}}(\mathsf{ass}^{\mathfrak{i}}(\alpha, \mathfrak{S}))$ does not exist and $\mathsf{suc}_{\mathfrak{S}}(\mathsf{ass}^{\mathfrak{i}}(\alpha, \mathfrak{S}))$ exists (some such index exists, for, clearly, there is no $\mathscr{L}$-structure $\mathfrak{S}$ such that, for every $d \in D_{\mathfrak{S}}$, both $\mathsf{pre}_{\mathfrak{S}}(d)$ does not exist and $\mathsf{suc}_{\mathfrak{S}}(d)$ exists): $\neg\mathcal{Y}(\varphi \vee \neg\varphi) \wedge \mathcal{M}(\varphi \vee \neg\varphi) \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}}} \varphi \wedge \neg\varphi$ then holds, whereas $\neg\mathcal{Y}(\varphi \vee \neg\varphi) \wedge \mathcal{M}(\varphi \vee \neg\varphi) \vdash_{\mathbf{Intra}_{\mathcal{YTM}}} \varphi \wedge \neg\varphi$ does not hold.

(iii) If $\mathsf{suc}_{\mathfrak{S}}(\mathsf{ass}^{\mathfrak{i}}(\alpha, D_{\mathfrak{S}}))$ or $\mathsf{pre}_{\mathfrak{S}}(\mathsf{ass}^{\mathfrak{i}}(\beta, D_{\mathfrak{S}}))$ exists, then $\alpha+1=\beta$ only if $\mathsf{suc}_{\mathfrak{S}}(\mathsf{ass}^{\mathfrak{i}}(\alpha, \mathfrak{S})) = \mathsf{ass}^{\mathfrak{i}}(\beta, \mathfrak{S})$.[27]

(IM) in effect forces $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}}$, given an $\mathscr{L}$-structure, to look for necessary truth preservation taking for each following premise or conclusion a context not earlier than that of any preceding premise or conclusion ((i)); indeed, a later context whenever there is one such ((ii)); indeed, the immediately later context whenever there is one such ((iii)). This determines the informal use that is supposed to be made of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}}$ in real-life inference: namely, that of considering an argument assuming that each following premise or conclusion is uttered on a day not earlier than that on which any preceding premise or conclusion is uttered; indeed, on a later day whenever there is one such; indeed, on the immediately later day whenever there is one such.

Unsurprisingly, it can thus be established that, if $\mathfrak{i}$ satisfies (IM), $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}}$ captures the distinctive logic of $\mathcal{T}$ and $\mathcal{M}$ manifested in the Fregean Entailment:

Theorem 2. *If* $\mathfrak{i}$ *satisfies* (IM) *and* φ *does not contain* $\mathcal{Y}$, $\mathcal{T}$ *or* $\mathcal{M}$, *then* $\Gamma, \mathcal{M}\varphi \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}}} \mathcal{T}\varphi, \Delta$ *holds.*

Notice that the restriction on φ in theorem 2 is welcome, as it reflects an analogous restriction on the validity of the corresponding arguments in natural language: 'Tomorrow, it will be the case that today it's sunny' (understood as having as logical form $\mathcal{MT}P$) at c^{FE}_0 does not entail 'Today, it is the case that today it's sunny' (understood as having as logical form $\mathcal{TT}P$) at c^{FE}_1 (the former sentence is true in c^{FE}_0 while the latter false in c^{FE}_1 if the day of c^{FE}_0 is sunny while the day of c^{FE}_1 is cloudy). Notice also that the necessity of the restriction on φ in theorem 2 implies that, if $\mathfrak{i}$ satisfies (IM), $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}}$ is *not closed under uniform substitution* (for example, $\mathcal{M}P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}}} \mathcal{T}P$ holds, but the result of substituting in it $\mathcal{T}P$ for P—i.e. $\mathcal{MT}P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}}} \mathcal{TT}P$—does not hold).

Since an $\mathscr{L}$-structure can be any old strict linear ordering, and so possibly *non-discrete* (see fn 16), even if $\mathfrak{i}$ satisfies (IM) the corresponding argument involving $\mathcal{Y}$ rather than $\mathcal{M}$ (i.e. $\Gamma, \mathcal{T}\varphi \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}}} \mathcal{Y}\varphi, \Delta$, with φ not containing $\mathcal{Y}$, $\mathcal{T}$ or $\mathcal{M}$) is not valid. Again, such invalidity is welcome, as it reflects an analogous invalidity of the corresponding arguments in natural language: in at least some inferential situations of interest, 'Today, it's sunny' at c^{FE}_0 cannot be taken to entail 'Yesterday, it was sunny' at c^{FE}_1 (for example, if we modify the situation of the Fregean Entailment merely by stipulating [that the day of c^{FE}_0 is followed instead by a sequence of days with no earliest day (so that the day of c^{FE}_0 does not have a successor) but the day of c^{FE}_1 is still later than the day of c^{FE}_0], then, no matter how early a day later than the day of c^{FE}_0 we choose for being the day of c^{FE}_1, the day—if any—referred to by 'Yesterday' in c^{FE}_1 will also have to be later than the day of c^{FE}_0, and

[27] Notice that, given the properties of $<$ and $\lhd_{\mathfrak{S}}$, the result of replacing in (ii) the conditional consequent by its converse, but not the result of replacing in (iii) the conditional consequent by its converse, is entailed by (i). Notice also that (iii) could be strengthened by adding an analogous clause for limit ordinals. However, given that such strengthening does not play any role in the development of intercontextual logic in this paper, it has not been officially included in (IM).

so the former sentence is true in c_0^{FE} while the latter false in c_1^{FE} if the day of c_0^{FE} is sunny while the day before the day of c_1^{FE} is cloudy).[28] We still have however:

Theorem 3. *If* $\mathfrak{i}$ *satisfies* (IM) *and* φ *does not contain* $\mathcal{Y}$, $\mathcal{T}$ *or* $\mathcal{M}$, *then* $\Gamma, \mathcal{T}\varphi, \mathcal{Y}\neg\varphi \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}}} \Delta$ *holds.*

Conversely, even if $\mathfrak{i}$ satisfies (IM) the corresponding dual argument involving $\mathcal{M}$ rather than $\mathcal{Y}$ (i.e. $\Gamma \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}}} \Delta, \mathcal{M}\varphi, \mathcal{T}\neg\varphi$, with φ not containing $\mathcal{Y}$, $\mathcal{T}$ or $\mathcal{M}$) is not valid, for reasons analogous to those for which $\Gamma, \mathcal{T}\varphi \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}}} \mathcal{Y}\varphi, \Delta$ does not hold.[29]

Such invalidities make sense if we're contemplating the possibility that the structure of days may be non-discrete, but, in most inferential situations of interest, such possibility can safely be ignored. In fact, in most inferential situations of interest, it can safely be assumed that the structure of days is *discrete and unbounded in both directions*. In our framework, we can easily define a family of logics embodying these natural assumptions:

Definition 13. An *$\mathscr{L}$-$\mathbb{Z}$-structure* is an $\mathscr{L}$-structure isomorphic to $\langle \mathbb{Z}, < \rangle$.

Definition 14. $\Gamma \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta$ holds iff, for every $\mathscr{L}$-$\mathbb{Z}$-structure $\mathfrak{S}$ and $\mathscr{L}$-model $\mathfrak{M}$ on $\mathfrak{S}$, if, for every $\gamma \in \mathsf{dom}(\Gamma)$, $[\![\Gamma(\gamma)]\!]_{\mathsf{ass}^{\mathfrak{i}}(\gamma,\mathfrak{S}),\mathfrak{M}} = 1$, then, for some δ such that $\delta \in \mathsf{dom}(\Gamma, \Delta)$ and $\delta > \mathsf{max}(\mathsf{dom}(\Gamma))$ ($\delta \geq \mathsf{lub}(\mathsf{dom}(\Gamma))$ if $\mathsf{dom}(\Gamma)$ has no maximum element), $[\![\Gamma, \Delta(\delta)]\!]_{\mathsf{ass}^{\mathfrak{i}}(\delta,\mathfrak{S}),\mathfrak{M}} = 1$.

Given that the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s embodies assumptions that are enforced in most inferential situations of interest, we henceforth focus on them (although many results we'll discuss also hold for the larger family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}}$s). Correspondingly, we henceforth identify κ with ω.

In the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s, we recover the missing validities mentioned in the second last paragraph:

Theorem 4. *If* $\mathfrak{i}$ *satisfies* (IM) *and* φ *does not contain* $\mathcal{Y}$, $\mathcal{T}$ *or* $\mathcal{M}$, *then* $\Gamma, \mathcal{T}\varphi \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{Y}\varphi, \Delta$ *holds.*

Theorem 5. *If* $\mathfrak{i}$ *satisfies* (IM) *and* φ *does not contain* $\mathcal{Y}$, $\mathcal{T}$ *or* $\mathcal{M}$, *then* $\Gamma \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta, \mathcal{M}\varphi, \mathcal{T}\neg\varphi$ *holds.*

Theorem 6. *If* $\mathfrak{i}$ *satisfies* (IM) *and* φ *does not contain* $\mathcal{Y}$, $\mathcal{T}$ *or* $\mathcal{M}$, *then* $\Gamma \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta, \mathcal{T}\varphi, \mathcal{Y}\neg\varphi$ *holds.*

[28] Since $\mathfrak{i}$ satisfies (IM), actually many versions of the kind of possibility contemplated in the text cannot be turned into formal $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}}$-countermodels, for condition (iii) of (IM) is incompatible with the day of c_1^{FE} having a predecessor. Still, formal $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}}$-countermodels are available in which the day of c_1^{FE} does not have a predecessor (so that, in such models, the relevant instance of condition (iii) of (IM) is vacuously satisfied).

[29] To complete the picture, $\Gamma, \mathcal{M}\varphi, \mathcal{T}\neg\varphi \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}}} \Delta$ holds, but $\Gamma \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}}} \Delta, \mathcal{T}\varphi, \mathcal{Y}\neg\varphi$ does not hold, with φ not containing $\mathcal{Y}$, $\mathcal{T}$ or $\mathcal{M}$.

(IM) in effect forces $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$, given an $\mathscr{L}$–$\mathbb{Z}$-structure, to look for necessary truth preservation taking for each immediately following premise or conclusion the context immediately later than that of the immediately preceding premise or conclusion. This determines the informal use that is supposed to be made of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$ in real-life inference: namely, that of considering an argument assuming that each immediately following premise or conclusion is uttered on the day immediately later than that on which the immediately preceding premise or conclusion is uttered.

A cute feature that thus emerges of the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s restricted to $\mathfrak{i}$s satisfying (IM) is that *one can represent the pure passage of time in the logic*: letting $\top$ be any logical truth and $\bot$ any logical falsity, if $\mathfrak{i}$ satisfies (IM) an argument from φ to ψ in which, for example, ψ is at a context four days later than that at which φ is can be represented as $\varphi, \top, \top, \top \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \psi$ (or as $\varphi, \top, \top \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \bot, \psi$, or as $\varphi, \top \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \bot, \bot, \psi$ etc.). This observation also makes plain that, since the insertion or suppression of a logical truth or logical falsity implies a change in how the pure passage of time is represented to be, the traditional principles of *insertion or suppression of logical truths or logical falsities* fail in the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s restricted to $\mathfrak{i}$s satisfying (IM): for example, if $\mathfrak{i}$ satisfies (IM), $\mathcal{M}P, \top \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{Y}P$ holds, but neither $\mathcal{M}P, \top, \top \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{Y}P$ nor $\mathcal{M}P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{Y}P$ hold, and $\mathcal{M}P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \bot, \mathcal{Y}P$ holds, but neither $\mathcal{M}P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \bot, \bot, \mathcal{Y}P$ nor $\mathcal{M}P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{Y}P$ hold.

From a rather superficial point of view, even the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s restricted to $\mathfrak{i}$s satisfying (IM) might strike one as ridiculously weak, for they will typically lack many traditional *structural* properties:

Theorem 7. *If* $\mathfrak{i}$ *satisfies* (IM)*:*

(1) Reflexivity fails: $\varphi \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \varphi$ *does not hold;*

(2) Monotonicity fails: $\mathcal{M}P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{T}P$ *holds (as* per *theorem* 2*), but* $\mathcal{M}P, \psi \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{T}P$ *does not hold;*

(3) Transitivity fails: $\mathcal{M}P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{T}P$ *holds (as* per *theorem* 2*) and* $\mathcal{T}P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{Y}P$ *holds (as* per *theorem* 4*), but* $\mathcal{M}P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{Y}P$ *does not hold;*

(4) Contraction fails: $\mathcal{M}P, \mathcal{M}P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{Y}P$ *holds, but* $\mathcal{M}P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{Y}P$ *does not hold;*

(5) Commutativity fails: $\mathcal{T}P, \mathcal{Y}\neg P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta$ *holds (as* per *theorem* 3*), but* $\mathcal{Y}\neg P, \mathcal{T}P \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta$ *does not hold.*[30]

[30] I myself am a fervent believer in the *philosophical significance of substructural logics*: in earlier works, I've developed and defended an approach to *vagueness* which relies on the adoption of a *non-transitive* logic (see for example Zardini [2008a]) as well as an approach to *truth* which relies on the adoption of a *non-contractive* logic (see for example Zardini [2011]). Neither of those two approaches goes however so

Theorem 7 easily extends to other interesting classes of indices that do not satisfy (IM), although, given theorem 1, it does not of course extend to the class of indices satisfying (C). Theorem 7 is not in the least surprising. In the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s, many aspects of the *sequence structure* into which premises and conclusions are combined are no longer *mere artefacts* of the precise mathematical modelling (as happens for example with presentations of classical logic in sequence structure), but are on the contrary supposed to be *representationally significant*—in particular, to represent the contexts at which premises and conclusions are taken (see Shapiro [1998] for an influential discussion of the distinction, in logical modelling, between elements that mere artefacts and elements that are representationally significant). Given this significance of differences in sequence structure for the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s, it is just to be expected that properties like (1)–(5) amounting to the preservation of logical consequence under various manipulations of sequence structure—and thus to the obliteration of the logical import of such structure—are lacked by the family. Such *weakness* is thus merely the reflection of the very welcome fact that, contrary to stronger logics, in virtue of its attaching representational significance to many aspects of sequence structure the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s has enough *expressive power* as to be able to represent which sentences are at which contexts.

In fact, in a good sense, the sensitivity to many aspects of sequence structure exhibited by the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s is no sign of weakness at all. For it turns out to be perfectly compatible with each logic's ability fully to *recover* classical logic and intracontextual logic. *Classical logic (*$\mathbf{K}$*) on the non-context-dependent fragment of* $\mathscr{L}$ is already recoverable in an *intercontextual* fashion by exploiting the facts on which theorem 2 relies:

Theorem 8. *If* $\mathfrak{i}$ *satisfies* (IM)*, then* $\varphi_0, \varphi_1, \varphi_2 \ldots, \varphi_i \vdash_{\mathbf{K}} \psi_0, \psi_1, \psi_2 \ldots, \psi_j$ *holds iff* $\mathcal{M}(\varphi_0 \wedge \varphi_1 \wedge \varphi_2 \ldots \wedge \varphi_i) \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{T}(\psi_0 \vee \psi_1 \vee \psi_2 \ldots \vee \psi_j)$ *holds (with the atomic symbols of the language of* $\mathbf{K}$ *restricted to* $AS_{\mathscr{L}} \setminus \{\mathcal{Y}, \mathcal{T}, \mathcal{M}\}$*).*

The restriction on the atomic symbols of the language of $\mathbf{K}$ is owed to the same reasons that dictate an analogous restriction in theorem 2. But even such restriction can be lifted, with *both classical logic and intracontextual logic over full context-dependent* $\mathscr{L}$ being recoverable in an *intracontextual* fashion by exploiting the fact that the semantic foundations of the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}}$s are, as we've seen in section 3, completely standard:

Theorem 9. $\varphi_0, \varphi_1, \varphi_2 \ldots, \varphi_i \vdash_{\mathbf{Intra}_{\mathcal{YTM}}} \psi_0, \psi_1, \psi_2 \ldots, \psi_j$ *holds iff* $\langle . \rangle \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} (\varphi_0 \wedge \varphi_1 \wedge \varphi_2 \ldots \wedge \varphi_i) \supset (\psi_0 \vee \psi_1 \vee \psi_2 \ldots \vee \psi_j)$ *holds.*[31]

far as to abandon properties like *reflexivity*, *monotonicity* or *commutativity*, as the present approach to *context change* on the contrary does. Relatedly, and with a focus on failure of commutativity, while the non-transitive approach to vagueness is consistent with premises and conclusions being combined into *sets*, and while the non-contractive approach to truth is consistent with premises and conclusions being combined into *multisets*, the present approach to context change requires that premises and conclusions be combined into *sequences* (see Moruzzi and Zardini [2007], pp. 180–187; Zardini [2014a] for more relevant background on substructurality).

[31] $\langle . \rangle$ is the empty sequence (so that $\mathsf{dom}(\langle . \rangle) = \varnothing$).

Thus, the whole wealth of structure that makes up the beauty of classical logic and intracontextual logic is preserved in the family of $\mathbf{Inter}^{\mathsf{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s in the form of certain logical truths, which, being single sentences, are subtracted in the family of $\mathbf{Inter}^{\mathsf{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s from the disruptive play of context change (for in the family of $\mathbf{Inter}^{\mathsf{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s that play only takes place among different premises or conclusions).[32]

It is now time to notice that the focus on indices satisfying (IM) has merely been an expository convenience. For indices satisfying (IM) have the expository virtue of representing the relevant portion of the earlier-than ordering on days with the standard well-ordering on the ordinals (which is in turn mirrored by the usual left-to-right ordering on premises and conclusions), but, clearly, any index that represents the relevant portion of the earlier-than ordering on days with some well-ordering or other on ordinals (no matter how unnatural and gerrymandered that may appear to us) will generate a logic with essentially the same features, and thus a logic capable of capturing in its own way the Fregean Entailment and, more generally, the intercontextual logic of $\mathcal{Y}$, $\mathcal{T}$ and $\mathcal{M}$. More precisely, consider the *generalised immediate-monotonicity* constraint for an index $\mathfrak{i}$:

(GIM) For some designated permutation $\mathsf{per}^{\mathsf{i}}$ on the ordinals $< \omega$, for some index $\mathfrak{j}$ satisfying (IM), for every $\mathscr{L}$-$\mathbb{Z}$-structure $\mathfrak{S}$, $\mathsf{ass}^{\mathsf{i}}(\alpha, \mathfrak{S}) = \mathsf{ass}^{\mathsf{j}}(\mathsf{per}^{\mathsf{i}}(\alpha), \mathfrak{S})$.

It's easy to check that theorems 2–9 still hold if one relaxes (IM) to (GIM), replaces $\mathbf{Inter}^{\mathsf{i}}_{\mathcal{YTM}}$ with $\mathbf{Inter}^{\mathsf{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$ and, somewhat roughly speaking, permutes premises and conclusions using $\mathsf{per}^{\mathsf{i}}$ (appropriately filling in the resulting gaps with $\top$ or $\bot$, adding a

[32] The construction naturally suggests a variation whose distinctive feature would be to take *utterances* rather than sentences to be the logical-consequence bearers and which would roughly consist in requiring truth preservation from the premise utterances to the conclusion utterances taking each at the context in which they are made but allowing for reinterpretation of their non-logical vocabulary (see Zardini [2008b]; [2014c]; [2014e] for more relevant background on utterance truth). As *per* section 2, I should reiterate that the spirit of this paper is by no means opposed to this and other alternative ways of accounting for mid-argument context change and that the objective here is rather to show that, by exploiting the fine structure into which premises and conclusions can be combined, an account of mid-argument context change is still possible even in a standard framework in which sentences are the logical-consequence bearers. I should also note, however, that I do happen to think that the sentence-based account proposed here has certain advantages over the alternative utterance-based account just sketched. In addition to the three more general problems that I've already presented in section 2, let me now briefly introduce a more specific problem for the latter account. An utterance of 'If Dave is here, Dave is here' made successively pointing at two different places p_0 and p_1, with Dave being at p_0 but not at p_1, is false. But such utterance is true in the context in which it is made under any standard reinterpretation of its non-logical vocabulary (since any standard reinterpretation of its non-logical vocabulary assigns the same referent to the two occurrences of 'here'). Thus, under the utterance-based account just sketched, such utterance would be a *logical truth*, even though it is *false*, which does violence to our notion of logical truth. (In section 5, I'll introduce a conditional connective that makes available a reading of 'If Dave is here, Dave is here' such that an utterance of that sentence on that reading may not be a logical truth. Even so, the problem observed in this fn would still persist for an utterance of 'If Dave is here, Dave is here' on the $\supset$-reading.) Thanks to Peter Fritz for discussion of these issues.

negation when $\mathsf{per}^{\mathfrak{i}}$ permutes a premise into a conclusion or *vice versa* and adopting the natural extended understanding of the structural properties mentioned in theorem 7).[33]

In fact, there is arguably a good and precise sense in which the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s restricted to $\mathfrak{i}$s satisfying (GIM) really constitutes *one single logic*. This is straightforwardly so for indices satisfying (IM):

Theorem 10. *If* $\mathfrak{i}$ *and* $\mathfrak{j}$ *satisfy* (IM)*, then* $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}} = \mathbf{Inter}^{\mathfrak{j}}_{\mathcal{YTM}^{\mathbb{Z}}}$.

More generally for indices satisfying (GIM), it is first helpful to make explicit how they can be seen to impose a similar structure on arguments as indices satisfying (IM) do:

Definition 15. $\alpha \sqsubseteq^{\mathfrak{i}} \beta$ iff, for every $\mathscr{L}$-structure $\mathfrak{S}$, $\mathsf{ass}^{\mathfrak{i}}(\alpha, \mathfrak{S}) \trianglelefteq_{\mathfrak{S}} \mathsf{ass}^{\mathfrak{i}}(\beta, \mathfrak{S})$.

Informally, $\sqsubseteq^{\mathfrak{i}}$ orders positions in an argument according to the temporal ordering of their images under $\mathsf{ass}^{\mathfrak{i}}$ if this is invariant across $\mathscr{L}$-structures (with the consequences that, if $\mathfrak{i}$ satisfies (GIM), $\sqsubseteq^{\mathfrak{i}}$ is a linear ordering, and that, if $\mathfrak{i}$ satisfies (IM), $\sqsubseteq^{\mathfrak{i}}=\leq$). Thus, $\sqsubseteq^{\mathfrak{i}}$ is the ordering on the ordinals that more generally plays the role that in the specific case of indices satisfying (IM) is played by $\leq$ (and that, in that specific case, is in turn mirrored by the usual left-to-right ordering on premises and conclusions): that is, the role of ordering premises and conclusions according to the context at which they are taken. We can then establish an important connection between an index satisfying (GIM) and an index satisfying (IM) related to the former index in the way mentioned in (GIM):

Theorem 11. *Suppose that* $\mathfrak{i}$ *satisfies* (GIM) *and that, for every* $\mathscr{L}$*-*$\mathbb{Z}$*-structure* $\mathfrak{S}$, $\mathsf{ass}^{\mathfrak{i}}(\alpha, \mathfrak{S}) = \mathsf{ass}^{\mathfrak{j}}(\mathsf{per}^{\mathfrak{i}}(\alpha), \mathfrak{S})$*. Suppose further that* Δ_0 *and* Δ_1 *are of order type* ω *and that* $\Delta_0(\alpha) = \varphi$ *iff* $\Delta_1(\mathsf{per}^{\mathfrak{i}}(\alpha)) = \varphi$*. Then the following are equivalent:*

- $\langle . \rangle \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta_0$;
- $\langle . \rangle \vdash^{\mathfrak{j}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta_1$.

Putting together theorems 10 and 11, we can finally establish the desired connection between any two indices satisfying (GIM):

[33] In some cases, I find that the natural extended understanding of the structural properties mentioned in theorem 7 required by indices satisfying (GIM) but not (IM) is particularly illuminating. For example, the natural extended understanding of contraction required for theorem 7 to hold for indices satisfying (GIM) but not (IM) is such as to encompass contractions "*across the turnstile*", since it needs contraction to be counterexampled, for instance, by the fact that $\mathcal{MP}, \neg\mathcal{YP} \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \neg\mathcal{MP}$ holds, but $\mathcal{MP} \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \bot, \mathcal{YP}$ does not hold (if $\mathsf{per}^{\mathfrak{i}}(0) = 0$ and $\mathsf{per}^{\mathfrak{i}}(1) = 2$). Thus, the natural extended understanding of contraction required for theorem 7 to hold for indices satisfying (GIM) but not (IM) is such as to imply that the implication from $\varphi, \neg\bot \vdash \neg\varphi$ holding to $\varphi \vdash \bot, \bot$ holding is an instance of contraction, thereby revealing even the intuitionistically acceptable version of *reductio ad absurdum* (i.e. the implication from $\varphi \vdash \neg\varphi$ holding to $\langle . \rangle \vdash \neg\varphi$ holding) to be essentially just an instance of contraction (at least in those situations in which $\varphi \vdash \neg\varphi$ can be treated as equivalent with $\varphi, \neg\bot \vdash \neg\varphi$ and $\varphi \vdash \bot, \bot$ can be treated as equivalent with $\langle . \rangle \vdash \neg\varphi$). Zardini [2014b] has more discussion of the relation between contraction and *reductio ad absurdum*.

Corollary 1. *Suppose that* $\mathfrak{i}$ *and* $\mathfrak{j}$ *satisfy* (GIM). *Suppose further that* Δ_0 *and* Δ_1 *are of order type* ω *and that* $\Delta_0(\alpha) = \varphi$ *iff* $\Delta_1(\mathsf{per}^{\mathfrak{j}}(\mathsf{per}^{\mathfrak{i}}(\alpha))) = \varphi$. *Then the following are equivalent:*

- $\langle . \rangle \vdash^{\mathfrak{i}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta_0$;
- $\langle . \rangle \vdash^{\mathfrak{j}}_{\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta_1$.

Thus, restricting to $\mathfrak{i}$s and $\mathfrak{j}$s satisfying (GIM), $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$ is identical to $\mathbf{Inter}^{\mathfrak{j}}_{\mathcal{YTM}^{\mathbb{Z}}}$ (at worst) up to permutation of arguments' positions and restricting to zero-premise, ω-long arguments (the restrictions are not significant since any argument with premises can be represented as a zero-premise argument whose conclusions start with the negations of the premises, and any argument whose length is shorter than ω can be represented as an ω-long argument with $\perp$s added as vacuous conclusions).

In this good and precise sense, the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s restricted to $\mathfrak{i}$s satisfying (GIM) really constitutes one single intercontextual logic ($\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}$). Notice that $\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}$ is a *more abstract entity* than any subsets of the Cartesian product of the set of sequences of sentences of $\mathscr{L}$ with itself (subsets with which logics that combine premises and conclusions into sequences are usually identified). $\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}$ is sensitive to an abstract order on premises and conclusions, but is not sensitive to any of the particular ways in which such abstract order is represented by a particular index $\mathfrak{i}$ with the concrete $\sqsubseteq^{\mathfrak{i}}$.[34,35]

[34]Analogous situations are known to occur in many areas in which informal notions of *order* are represented using the formal notion of a *sequence*. Thus, to take a simple example, the relation of loving is sensitive to an abstract order on people (the one who loves must be distinguished from the one who is loved), but is not sensitive to any of the particular ways in which such abstract order is concretely represented by the sets of pairs $\{\langle x, y\rangle : x \text{ loves } y\}$, $\{\langle x, y\rangle : y \text{ loves } x\}$, $\{\langle x, y\rangle : \text{if } x \text{ and } y \text{ have the same height, } x \text{ loves } y\text{, otherwise } y \text{ loves } x\}$ etc. All these sets can be seen as equally good alternative ways of representing the relation of loving, just as, restricting to $\mathfrak{i}$s satisfying (GIM), all the $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s can be seen as equally good alternative ways of representing $\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}$. Thanks to Sebastiano Moruzzi and Eugenio Orlandelli for discussion of $\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}$.

[35]If we *generalise* the framework developed in this paper so as to cover *other fragments* of intercontextual logic beyond the one concerning 'yesterday', 'today' and 'tomorrow', the main idea should presumably be that *each index represents with a concrete ordering the way in which context changes mid-argument.* However, once a generalisation to other context-dependent expressions is made, there will typically be *many incompatible* ways in which context can change mid-argument: for example, considering 'you', 'I' and 'she', the public may become the agent, or, instead, may become a *demonstratum.* This contrasts with the particular case of the fragment concerning 'yesterday', 'today' and 'tomorrow', where the *only* way in which context can change mid-argument is determined by the passage of time. Since theorems 10 and 11 rely in effect both on the property of any index $\mathfrak{i}$ satisfying (GIM) of representing with $\sqsubseteq^{\mathfrak{i}}$ the passage of time and on the fact that such passage determines the same way in which context changes mid-argument, keeping fixed that property analogues of these theorems cannot be expected to hold once a generalisation to other context-dependent expressions is made (for then that fact is no longer guaranteed to obtain). An analogous comment applies if we generalise the framework developed in this paper so as to cover the case in which the structure of days is *not linear* (see fn 16). It is important to note that both these comments presuppose that premises and conclusions remain *linearly ordered* (so that, given the property mentioned in the second last sentence, each index satisfying (IM), and hence also each index

5 An Extension

In the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s, there is a sharp *distinction* between two sentences occurring in two different positions in an argument (in which case the two positions and hence the two sentences may correspond to two different days) and them occurring as components of a larger sentence in a single position in an argument (in which case the position and hence the two sentences must correspond to a single day). An immediate effect of this distinction is that *the deduction theorem fails*: for example, even if $\mathfrak{i}$ satisfies (IM) and φ does not contain $\mathcal{Y}$, $\mathcal{T}$ or $\mathcal{M}$, $\Gamma \vdash_{\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{M}\varphi \supset \mathcal{T}\varphi, \Delta$ does not hold, while, by theorem 2, $\Gamma, \mathcal{M}\varphi \vdash_{\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}} \mathcal{T}\varphi, \Delta$ holds.[36] Slightly more generally, while the family of $\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}$s allows for different premises and conclusions to be at different contexts, it still does not allow for occurrences of sentences that are all components of the same premise or conclusion to be at different contexts. That is in effect the feature of the framework developed so far that is exploited in theorem 9. In this section, I'll show however how the framework can be naturally so extended as to allow in a controlled fashion also for that possibility.[37]

satisfying (GIM), is in effect forced to track only a particular chain in each relevant partially ordered structure). Such presupposition is far from being unquestionable, and further philosophical reflection on and technical investigation of intercontextual logic may well lead to its motivated rejection. Thanks to an anonymous referee for comments that led to substantial changes in this fn.

[36] The deduction theorem fails not only in the sense that *implication* fails to *connect premises with conclusions*, but also in the sense that *conjunction* fails to *connect premises with premises* (even if $\mathfrak{i}$ satisfies (IM) and φ does not contain $\mathcal{Y}$, $\mathcal{T}$ or $\mathcal{M}$, $\Gamma, \mathcal{T}\varphi \wedge \mathcal{Y}\neg\varphi \vdash_{\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta$ does not hold, while, by theorem 3, $\Gamma, \mathcal{T}\varphi, \mathcal{Y}\neg\varphi \vdash_{\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta$ holds) and in the sense that *disjunction* fails to *connect conclusions with conclusions* (even if $\mathfrak{i}$ satisfies (IM) and φ does not contain $\mathcal{Y}$, $\mathcal{T}$ or $\mathcal{M}$, $\Gamma \vdash_{\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta, \mathcal{M}\varphi \vee \mathcal{T}\neg\varphi$ does not hold, while, by theorem 5, $\Gamma \vdash_{\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta, \mathcal{M}\varphi, \mathcal{T}\neg\varphi$ holds). It may be worth noting that, even so, the deduction theorem does not fail in the very weak sense that *negation* still *moves premises to conclusions and conclusions to premises* (if $\mathfrak{i}$ satisfies (IM), [$\Gamma, \varphi \vdash_{\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta$ holds only if $\Gamma \vdash_{\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}} \neg\varphi, \Delta$ holds, and $\Gamma \vdash_{\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}} \varphi, \Delta$ holds only if $\Gamma, \neg\varphi \vdash_{\mathbf{Inter}^{\mathfrak{i}}_{\mathcal{YTM}^{\mathbb{Z}}}} \Delta$ holds]).

[37] If we understand context so as to include, for example, discourse referents and the like, the broad tradition of *dynamic logics and semantics* provides many examples of logics that allow for different premises and conclusions to be at different contexts and that also allow for occurrences of sentences that are all components of the same premise or conclusion to be at different contexts (with the ensuing failure of the structural properties mentioned in theorem 7). While a detailed critical comparison of the relevant work done in that tradition with the present one lies beyond the scope of this paper, it is in order to comment on what, for our purposes, the main difference arguably is between the two. On the one hand, in the dynamic tradition it is *the occurrences of specific expressions* that trigger context change, doing so in virtue of and to the extent determined by certain semantic properties of such expressions; hence, given a particular language, *only specific kinds of context changes* are allowed (to take an influential example, in the dynamic predicate logic of Groenendijk and Stokhof [1991] it is *the occurrences of the existential quantifier* that trigger context change, doing so in virtue of and to the extent envisaged by the existential quantifier's semantic property of selecting assignments of discourse referents that can successfully be processed by the embedded expression; hence, given the language of dynamic predicate logic, *only change of discourse referents* is allowed). On the other hand, in intercontextual logic it is *the sheer numerical identities and differences of premises and conclusions* (and, as we'll see in this section, *the sheer numerical identities and differences of the relevant components of the same premise or*

The basic idea is to introduce a new conjunctive connective $\otimes$ that allows for the passage of time (and so for context change) from its left-hand side (lhs) to its right-hand side (rhs), while the rest of the connectives remains interpreted in the way assumed so far that idealises away from the fact that some of the occurrences they dominate[38] may be at different days (and so at different contexts). If we can put it this way, while those connectives see things *sub specie aeternitatis*, $\otimes$ sees things *sub specie durationis*.

As for syntax, we simply add the 2ary conjunctive connective $\otimes$ to $\mathscr{L}$ to form the new language $\mathscr{L}_{\otimes}$ (with $AS_{\mathscr{L}_{\otimes}}$ as its set of atomic symbols) and define a new 2ary disjunctive connective $\oplus$ and a new 2ary conditional connective $\rightarrow$ out of $\otimes$ and $\neg$ in a manner analogous to definition 3. Semantically, we keep our definitions of $\mathscr{L}$-$\mathbb{Z}$-structures and $\mathscr{L}$-$\mathbb{Z}$-models, and maintain our focus on these. In order to achieve the intended interpretation of $\otimes$, we need however to change our conception of contexts and circumstances from being single days to being days or pairs (of pairs of pairs...) of days:[39]

Definition 16. *The set $C_{\mathfrak{S}}$ of contexts and circumstances in an $\mathscr{L}$-$\mathbb{Z}$-structure* $\mathfrak{S}$ is defined by the recursion:

(I) If $d \in D_{\mathfrak{S}}$, then $d \in C_{\mathfrak{S}}$;

(II) If π_0 and $\pi_1 \in C_{\mathfrak{S}}$, then $\langle \pi_0, \pi_1 \rangle \in C_{\mathfrak{S}}$.

The truth definition adapts naturally to this extended framework, and it is now possible to give the intended interpretation to $\otimes$:

conclusion) that trigger context change, doing so independently of and to an extent unconstrained by the semantic properties of any expressions in the language; hence, even given a particular language, *any kind of context change* is in principle allowed (although in this paper I've focussed on the change of the time parametre). Concisely put, while in the dynamic tradition it is the interpretation of the language that governs context change, in intercontextual logic it is context change that governs the interpretation of the language. Thanks to an anonymous referee for discussion of the relation between the dynamic tradition and intercontextual logic.

[38]The precise syntax of the language is of some importance for the purposes of this section. Throughout, I assume a standard syntactic framework with the two primitive relations of *precedence* and *dominance*.

[39]The traditional notion of context is arguably associated with *two potentially different roles*: that of *being the kind of entity such that every utterance can eventually be interpreted with respect to one single such entity*, and that of *being the kind of entity that fixes the interpretation of certain expressions*. While these two roles are typically indiscriminable in formal semantics, they will come apart in the extension of intercontextual logic developed in this section: the first role will be played by *pairs (of pairs of pairs...) of days*, whereas the second role will be played by *days*. After some deliberation, I've chosen to use 'context' and its relatives ambiguously to refer to either kind of entities, leaving it to, hum, context to disambiguate. The rationale for this apparently perverse choice is that I think that doing so will actually facilitate seeing the very substantial connections between the extension of intercontextual logic developed in this section and the previous work done in this paper and, more generally, in formal semantics. Thus, for example, I mean the kind of entities playing the first role when I say that "we need [...] to change our conception of contexts", whereas I mean the kind of entities playing the second role when I say that "$\otimes$ [...] allows for the passage of time (and so for context change)". (Notice that, in some situations, some pairs (of pairs of pairs...) of days will not play the first role and will rather behave similarly to days.) An analogous comment applies for 'circumstance' and its relatives.

Definition 17. *Truth of an atom φ in a context π_0 at a circumstance π_1 in an $\mathscr{L}$-$\mathbb{Z}$-model* $\mathfrak{M}$ is defined by the recursion:

- If $\pi_1 = d$, then $[\![\varphi]\!]_{\pi_0,\pi_1,\mathfrak{M}} = \mathsf{int}_{\mathfrak{M}}(\varphi, d)$;
- If $\pi_1 = \langle \pi_2, \pi_3 \rangle$, then $[\![\varphi]\!]_{\pi_0,\pi_1,\mathfrak{M}} = \mathsf{min}([\![\varphi]\!]_{\pi_0,\pi_2,\mathfrak{M}}, [\![\varphi]\!]_{\pi_0,\pi_3,\mathfrak{M}})$.[40]

Definition 18. *Truth of a complex sentence in a context π_0 at a circumstance π_1 in an $\mathscr{L}$-$\mathbb{Z}$-model $\mathfrak{M}$ on an $\mathscr{L}$-$\mathbb{Z}$-structure* $\mathfrak{S}$ is defined by the recursion:

- $[\![\neg\varphi]\!]_{\pi_0,\pi_1,\mathfrak{M}} = 1 - [\![\varphi]\!]_{\pi_0,\pi_1,\mathfrak{M}}$;
- $[\![\varphi \wedge \psi]\!]_{\pi_0,\pi_1,\mathfrak{M}} = \mathsf{min}([\![\varphi]\!]_{\pi_0,\pi_1,\mathfrak{M}}, [\![\psi]\!]_{\pi_0,\pi_1,\mathfrak{M}})$;
- If $\pi_0 = d_0$, then:
  - If $\pi_1 = d$, then $[\![\varphi \otimes \psi]\!]_{\pi_0,\pi_1,\mathfrak{M}} = \mathsf{min}([\![\varphi]\!]_{\pi_0,\pi_1,\mathfrak{M}}, [\![\psi]\!]_{\pi_0,\pi_1,\mathfrak{M}})$;
  - If $\pi_1 = \langle \pi_2, \pi_3 \rangle$, then $[\![\varphi \otimes \psi]\!]_{\pi_0,\pi_1,\mathfrak{M}} = \mathsf{min}([\![\varphi]\!]_{\pi_0,\pi_2,\mathfrak{M}}, [\![\psi]\!]_{\pi_0,\pi_3,\mathfrak{M}})$;
- If $\pi_0 = \langle \pi_2, \pi_3 \rangle$, then:
  - If $\pi_1 = d$, then $[\![\varphi \otimes \psi]\!]_{\pi_0,\pi_1,\mathfrak{M}} = \mathsf{min}([\![\varphi]\!]_{\pi_2,\pi_1,\mathfrak{M}}, [\![\psi]\!]_{\pi_3,\pi_1,\mathfrak{M}})$;
  - If $\pi_1 = \langle \pi_4, \pi_5 \rangle$, then $[\![\varphi \otimes \psi]\!]_{\pi_0,\pi_1,\mathfrak{M}} = \mathsf{min}([\![\varphi]\!]_{\pi_2,\pi_4,\mathfrak{M}}, [\![\psi]\!]_{\pi_3,\pi_5,\mathfrak{M}})$;
- $[\![\mathcal{Y}\varphi]\!]_{\pi_0,\pi_1,\mathfrak{M}} = 1$ iff, for some $d \in D_{\mathfrak{S}}$, $\pi_0 = d$ and $[\![\varphi]\!]_{\pi_0,\mathsf{pre}_{\mathfrak{S}}(d),\mathfrak{M}} = 1$, 0 otherwise;
- $[\![\mathcal{T}\varphi]\!]_{\pi_0,\pi_1,\mathfrak{M}} = 1$ iff, for some $d \in D_{\mathfrak{S}}$, $\pi_0 = d$ and $[\![\varphi]\!]_{\pi_0,d,\mathfrak{M}} = 1$, 0 otherwise;
- $[\![\mathcal{M}\varphi]\!]_{\pi_0,\pi_1,\mathfrak{M}} = 1$ iff, for some $d \in D_{\mathfrak{S}}$, $\pi_0 = d$ and $[\![\varphi]\!]_{\pi_0,\mathsf{suc}_{\mathfrak{S}}(d),\mathfrak{M}} = 1$, 0 otherwise.

Definition 19. *Truth in a context π in an $\mathscr{L}$-$\mathbb{Z}$-model* $\mathfrak{M}$ is defined by the diagonalisation: $[\![\varphi]\!]_{\pi,\mathfrak{M}} = [\![\varphi]\!]_{\pi,\pi,\mathfrak{M}}$.

The first condition in the clauses for $\mathcal{Y}$, $\mathcal{T}$ and $\mathcal{M}$ in definition 18 is motivated by the fact that, intuitively, 'yesterday', 'today' and 'tomorrow' only manage to pick out a day if uttered in a context *restricted to a single day* (such as the contexts represented by days, see clause (I) in definition 16) rather than in a context *spread out over more than one day* (such as the contexts represented by pairs, see clause (II) in definition 16). Just as an expression like $\mathcal{T}$ that is sensitive to context (as well as a language like $\mathscr{L}$ that contains some such expression) is said to be "*context dependent*", it seems appropriate that an

[40] An alternative, *tolerant* rather than *strict* clause to the effect that $[\![\varphi]\!]_{\pi_0,\pi_1,\mathfrak{M}} = \mathsf{max}([\![\varphi]\!]_{\pi_0,\pi_2,\mathfrak{M}}, [\![\varphi]\!]_{\pi_0,\pi_3,\mathfrak{M}})$ would also have been possible. Given the other properties of the construction, the resulting system wouldn't have diverged greatly from the present one.

expression like $\otimes$ that is sensitive to context change (as well as a language like $\mathscr{L}_\otimes$ that contains some such expression) be said to be "*context-change dependent*".[41]

As with the family of $\mathbf{Inter}^{\mathsf{i}}_{\mathcal{YTM}}$s, with the semantics in place we now face the task of connecting it (and, in particular, its structures) with arguments in order to generate the logic. We focus for simplicity on a presentation of the logic that respects the idea behind immediate monotonicity. Since $\otimes$ is the only context-change-dependent connective, the guiding thought should be that each new occurrence of $\otimes$ in an argument marks its rhs as being at a context immediately following the context at which the lhs is, just as each new premise or conclusion marks it as being at a context immediately following the context at which the previous premise or conclusion is.

More formally, let the *atomisation of a sequence* Γ ($\mathsf{ato}(\Gamma)$) be the sequence that orders all the members of $AS_{\mathscr{L}_\otimes} \setminus \{(,)\}$ occurring in Γ according to the ordering among sentences represented by Γ and the syntactic precedence relations holding within sentences in Γ. Thus, while a sequence Γ of the kind considered so far only orders premises or conclusions, $\mathsf{ato}(\Gamma)$ "reads into" each premise and conclusion also ordering the members of $AS_{\mathscr{L}_\otimes} \setminus \{(,)\}$ occurring in them. Implementing the guiding thought in formal detail also requires that we model in our background mathematical theory the notion, already variously adumbrated in section 3 and in this section, of an expression *as it occurs* at a particular point in an atomisation. Since, on the one hand, the 'as'-clause is naturally interpreted as expressing a modification of the bearer expressed by the noun 'expression' and since, on the other hand, our standard background mathematical theory only envisages bearers and properties, but not modifications, our modelling of that notion will have to be somewhat unfaithful and proceed by introducing a new kind of entity: the *occurrence* of an expression ε at a point α in an atomisation (which, since there will never be ambiguity as to which atomisation is meant, we can simply denote with 'ε_α').[42] Let then the *decoration of a sequence* Γ

[41] On one natural understanding, a connective is a *monster* iff, roughly, it non-trivially selects contexts in which to evaluate the immediate components of the compound whose main connective it is. Under such understanding, a context-change-dependent connective like $\otimes$ is a monster. On one natural understanding, a connective is *intensional* iff, roughly, it non-trivially selects circumstances at which to evaluate the immediate components of the compound whose main connective it is. Under such understanding, a context-change-dependent connective like $\otimes$ is intensional. On one natural, weaker understanding, a connective is *context dependent* iff, roughly, it non-trivially selects contexts in or circumstances at which to evaluate the immediate components on the basis of the context in which the compound whose main connective it is is evaluated. Under such understanding, a context-change-dependent connective like $\otimes$ is context dependent. On another natural, stronger understanding, a connective is *context dependent* iff, roughly, it non-trivially selects circumstances at which to evaluate the immediate components on the basis of the context in which the compound whose main connective it is is evaluated. Under such understanding, a context-change-dependent connective like $\otimes$ is not context dependent.

[42] Focussing on occurrences of *sentences*, I stress that such introduction of entities that are finer-grained than sentences is in no tension with the philosophical picture sketched in section 2. First, in what follows occurrences will only play a *very limited role* that is fully compatible with the idea of section 2 that it is sentences (at contexts) that entail sentences (at contexts), since occurrences will only be appealed to in the *syntactic* part of the theory, in their (usual) role of bearers of the dominance relation; in particular, neither *logical* nor *semantic* properties will be attributed to occurrences (thus, in the latter respect, the theory remains firmly within the bounds of "expression-based semantics" in the sense of Salmon [2006]). Second, even restricted to this very limited role, occurrences are introduced as a *technical ersatz*

($\mathsf{dec}(\Gamma)$) be such that $(\mathsf{dec}(\Gamma))(\alpha) = \varepsilon_\alpha$ iff $\Gamma(\alpha) = \varepsilon$ (thus, while a sequence Γ of the kind considered so far orders expressions, $\mathsf{dec}(\Gamma)$ orders the corresponding occurrences). Also, let $\mathbb{S}^{\mathbb{Z}}$ be the set of $\mathscr{L}$-$\mathbb{Z}$-structures, and, finally, let the *squeeze of* α *into* Γ ($\mathsf{squ}_\Gamma(\alpha)$) be the (typically smaller) ordinal that in (the typically less discerning sequence) Γ represents the premise or conclusion in which $(\mathsf{dec}(\mathsf{ato}(\Gamma)))(\alpha)$ occurs.

Everything is now in place for defining assignments that change context not only *mid-argument* with each new premise or conclusion (as the assignments of the kind considered so far do), but also *mid-sentence* in going from the lhs of an occurrence of $\otimes$ to its rhs:

Definition 20. Let an indexed, sequence-relative function $\mathsf{ass}^{\mathsf{i}}_\Gamma : \omega \times \mathbb{S}^{\mathbb{Z}} \mapsto \bigcup_{\mathfrak{S} \in \mathbb{S}^{\mathbb{Z}}} (C_{\mathfrak{S}})$ be constrained as follows:

- $\mathsf{ass}^{\mathsf{i}}_\Gamma(\alpha, \mathfrak{S}) \in C_{\mathfrak{S}}$;
- If $(\mathsf{ato}(\Gamma))(\alpha) \in A_{\mathscr{L}}$, then $\mathsf{ass}^{\mathsf{i}}_\Gamma(\alpha, \mathfrak{S}) \in D_{\mathfrak{S}}$ and:
  - For every β such that:
    * $(\mathsf{ato}(\Gamma))(\beta) \in A_{\mathscr{L}}$;
    * $\mathsf{squ}_\Gamma(\beta) = \mathsf{squ}_\Gamma(\alpha)$;
    * For no γ, $[(\mathsf{ato}(\Gamma))(\gamma) = \otimes$ and $\alpha < \gamma < \beta]$,

    $\mathsf{ass}^{\mathsf{i}}_\Gamma(\beta, \mathfrak{S}) = \mathsf{ass}^{\mathsf{i}}_\Gamma(\alpha, \mathfrak{S})$;
  - For every β such that:
    * $(\mathsf{ato}(\Gamma))(\beta) \in A_{\mathscr{L}}$;
    * $\mathsf{squ}_\Gamma(\beta) = \mathsf{squ}_\Gamma(\alpha)$;
    * There is exactly one γ such that $[(\mathsf{ato}(\Gamma))(\gamma) = \otimes$ and $\alpha < \gamma < \beta]$,

    $\mathsf{ass}^{\mathsf{i}}_\Gamma(\beta, \mathfrak{S}) = \mathsf{suc}_{\mathfrak{S}}(\mathsf{ass}^{\mathsf{i}}_\Gamma(\alpha, \mathfrak{S}))$;
  - For every β such that:
    * $(\mathsf{ato}(\Gamma))(\beta) \in A_{\mathscr{L}}$;
    * $\mathsf{squ}_\Gamma(\beta) = \mathsf{squ}_\Gamma(\alpha) + 1$;
    * For no γ, $[(\mathsf{ato}(\Gamma))(\gamma) = \otimes$ and $\alpha < \gamma < \beta]$,

    $\mathsf{ass}^{\mathsf{i}}_\Gamma(\beta, \mathfrak{S}) = \mathsf{suc}_{\mathfrak{S}}(\mathsf{ass}^{\mathsf{i}}_\Gamma(\alpha, \mathfrak{S}))$;

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construct in the expressively poor background mathematical theory, with the only purpose of modelling in the theory the notion of [an expression as it occurs at a particular point in an atomisation], and that notion, on the construal proposed in the text, merely involves commitment to expressions, points and atomisations. Third, sentence occurrences are in any event *not to be identified with utterances* (or sentence-context pairs): taking for example the argument $\mathcal{M}P \vdash \mathcal{T}P$, there is only one occurrence of $\mathcal{M}P$ in that argument, even if there are indefinitely many utterances of $\mathcal{M}P$ (and indefinitely many sentence-context pairs figuring $\mathcal{M}P$ in their first coordinate) that are associated with that argument (relatedly, as I've indicated in fn 6, points do not represent any individual context nor any non-relationally specified type of context). (It is unfortunate that Kaplan [1989a], p. 522 deviated from the traditional usage of 'occurrence'—which I'm following—and called sentence-context pairs with the same word.)

- If $\mathsf{dec}((\mathsf{ato}(\Gamma)))(\alpha)$ is the occurrence of a 1ary connective immediately dominating ε_β, then $\mathsf{ass}^{\mathsf{i}}_{\Gamma}(\alpha, \mathfrak{S}) = \mathsf{ass}^{\mathsf{i}}_{\Gamma}(\beta, \mathfrak{S})$;

- If $\mathsf{dec}((\mathsf{ato}(\Gamma)))(\alpha)$ is $\wedge_\alpha$, immediately dominates ε_{0_β} and ε_{1_γ} and $\beta < \gamma$, then $\mathsf{ass}^{\mathsf{i}}_{\Gamma}(\alpha, \mathfrak{S}) = \mathsf{ass}^{\mathsf{i}}_{\Gamma}(\beta, \mathfrak{S})$;[43]

- If $\mathsf{dec}((\mathsf{ato}(\Gamma)))(\alpha)$ is $\otimes_\alpha$, immediately dominates ε_{0_β} and ε_{1_γ} and $\beta < \gamma$, then $\mathsf{ass}^{\mathsf{i}}_{\Gamma}(\alpha, \mathfrak{S}) = \langle \mathsf{ass}^{\mathsf{i}}_{\Gamma}(\beta, \mathfrak{S}), \mathsf{ass}^{\mathsf{i}}_{\Gamma}(\gamma, \mathfrak{S}) \rangle$

(notice that, in conformity with our focus, definition 20 already guarantees that sequence-relative assignments respect the idea behind immediate monotonicity).

With this new connection forged by definition 20 between $\mathscr{L}$-$\mathbb{Z}$-structures and arguments, we can define a new family of logics in a manner analogous to definitions 12 and 14. Let the *stretch of* α *from* Γ ($\mathsf{str}_\Gamma(\alpha)$) be the (typically greater) ordinal that in (the typically more discerning sequence) $\mathsf{dec}(\mathsf{ato}(\Gamma))$ represents the relevant occurrence of what is the main connective of $\Gamma(\alpha)$ (or the relevant occurrence of $\Gamma(\alpha)$ if $\Gamma(\alpha) \in A_{\mathscr{L}}$). Then:

Definition 21. $\Gamma \vdash_{\mathbf{Inter}^{\mathsf{i}}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}} \Delta$ iff, for every $\mathscr{L}$-$\mathbb{Z}$-structure $\mathfrak{S}$ and $\mathscr{L}$-$\mathbb{Z}$-model $\mathfrak{M}$ on $\mathfrak{S}$, if, for every $\gamma \in \mathsf{dom}(\Gamma)$, $[\![\Gamma(\gamma)]\!]_{\mathsf{ass}^{\mathsf{i}}_{\Gamma,\Delta}(\mathsf{str}_{\Gamma,\Delta}(\gamma),\mathfrak{S}),\mathfrak{M}} = 1$, then, for some δ such that $\delta \in \mathsf{dom}(\Gamma, \Delta)$ and $\delta > \mathsf{max}(\mathsf{dom}(\Gamma))$, $[\![\Gamma, \Delta(\delta)]\!]_{\mathsf{ass}^{\mathsf{i}}_{\Gamma,\Delta}(\mathsf{str}_{\Gamma,\Delta}(\delta),\mathfrak{S}),\mathfrak{M}} = 1$.

In fact, for reasons similar to those for which theorem 10 holds, the family of $\mathbf{Inter}^{\mathsf{i}}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}$ is *one single logic* ($\mathbf{Inter}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}$):

Theorem 12. *For every* i *and* j, $\mathbf{Inter}^{\mathsf{i}}_{\mathcal{YTM}\otimes^{\mathbb{Z}}} = \mathbf{Inter}^{\mathsf{j}}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}$.

$\mathbf{Inter}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}$ preserves the main features of $\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}$ studied in section 4. The analogues of theorems 2–4 and 7–9 hold (notice that the restrictions to indices satisfying (IM) become redundant and that $\mathbf{K}$ and $\mathbf{Intra}_{\mathcal{YTM}}$ are understood to be defined over $\mathscr{L}$ rather than $\mathscr{L}_\otimes$). The analogues of theorems 5 and 6 hold if the language is $\mathscr{L}$ rather than $\mathscr{L}_\otimes$. Again, such invalidities over full context-change-dependent $\mathscr{L}_\otimes$ are welcome, as they reflect analogous invalidities of the corresponding arguments in natural language: for example, if the language contains a connective 'first... and then...' allowing for mid-sentence context change, in at least some inferential situations of interest it cannot be taken that it is logically guaranteed that either 'Today, first it's sunny and then it's cloudy' holds at c_0^{FE} or 'Yesterday, it was not case that first it was sunny and then it was cloudy' holds at c_1^{FE} (for example, if we modify the situation of the Fregean Entailment merely by stipulating that both c_0^{FE} and c_1^{FE} are instead spread out over more than one day, neither 'today' in c_0^{FE} nor 'yesterday' in c_1^{FE} intuitively manage to pick out any day, and so the former sentence is false in c_0^{FE} and the latter false in c_1^{FE}).

[43]An alternative, *forwards-looking* rather than *backwards-looking* clause to the effect that $\mathsf{ass}^{\mathsf{i}}_{\Gamma}(\alpha, \mathfrak{S}) = \mathsf{ass}^{\mathsf{i}}_{\Gamma}(\gamma, \mathfrak{S})$ would also have been possible. Given the other properties of the construction, the resulting system wouldn't have diverged greatly from the present one.

The main innovative feature of $\mathbf{Inter}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}$ over $\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}$ is to represent the passage of time not only in the movement from one premise or conclusion to the next one, but also in the movement, triggered by $\otimes$, from one component to the next one *within the same premise or conclusion*. In fact, the new operators $\otimes$, $\oplus$ and $\rightarrow$ allow us to capture in the *object language* $\mathscr{L}_{\otimes}$ the *metalinguistic* notions of premise combination, conclusion combination and entailment respectively (contrast with the failures of $\wedge$, $\vee$ and $\supset$ at this task illustrated in fn 36 and in the text it is appended to):

Theorem 13. *$\Gamma_0, \varphi_0, \varphi_1, \varphi_2 \ldots, \varphi_i, \Gamma_1 \vdash_{\mathbf{Inter}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}} \Delta$ holds iff $\Gamma_0, \varphi_0 \otimes \varphi_1 \otimes \varphi_2 \ldots \otimes \varphi_i, \Gamma_1 \vdash_{\mathbf{Inter}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}} \Delta$ holds.*

Theorem 14. *$\Gamma \vdash_{\mathbf{Inter}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}} \Delta_0, \psi_0, \psi_1, \psi_2 \ldots, \psi_i, \Delta_1$ holds iff $\Gamma \vdash_{\mathbf{Inter}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}} \Delta_0, \psi_0 \oplus \psi_1 \oplus \psi_2 \ldots \oplus \psi_i, \Delta_1$ holds.*

Theorem 15. *$\Gamma, \varphi_0, \varphi_1, \varphi_2 \ldots, \varphi_i \vdash_{\mathbf{Inter}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}} \psi, \Delta$ holds iff $\Gamma \vdash_{\mathbf{Inter}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}} \varphi_0 \rightarrow \varphi_1 \rightarrow \varphi_2 \ldots \rightarrow \varphi_i \rightarrow \psi, \Delta$ holds.*

And from theorems 13–15 we straightforwardly get a strong version of the deduction theorem for $\mathbf{Inter}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}$:

Corollary 2. *$\Gamma, \varphi_0, \varphi_1, \varphi_2 \ldots, \varphi_i \vdash_{\mathbf{Inter}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}} \psi_0, \psi_1, \psi_2 \ldots, \psi_j, \Delta$ holds iff $\Gamma \vdash_{\mathbf{Inter}_{\mathcal{YTM}\otimes^{\mathbb{Z}}}} \varphi_0 \otimes \varphi_1 \otimes \varphi_2 \ldots \otimes \varphi_i \rightarrow \psi_0 \oplus \psi_1 \oplus \psi_2 \ldots \oplus \psi_j, \Delta$ holds.*[44]

[44] A well-known conjunctive connective broadly similar to $\otimes$ is the '*and next*'-connective $\boxtimes$ introduced by von Wright [1963], pp. 28–34 and further studied for example in von Wright [1965]. Von Wright's favoured understanding of $\boxtimes$ (which is officially defined simply in terms of an axiomatic system) treats it as an intensional 2ary connective that is *neither context-change dependent* (as for example $\otimes$ is) *nor context dependent* (as for example $\mathcal{T}$ is), thus resembling in these respects for example the Priorean intensional 1ary 'it will be the case that'-connective (see for example Prior [1955]). More specifically, adapted to the framework of this paper, $\boxtimes$ enjoys the kind of basic semantics developed in section 3 and can be taken to be governed by the following clause for an $\mathscr{L}$-$\mathbb{Z}$-model $\mathfrak{M}$ on an $\mathscr{L}$-$\mathbb{Z}$-structure:

- $[\![\varphi \boxtimes \psi]\!]_{d_0,d_1,\mathfrak{M}} = \mathsf{min}([\![\varphi]\!]_{d_0,d_1,\mathfrak{M}}, [\![\psi]\!]_{d_0,\mathsf{suc}_{\mathfrak{S}}(d_1),\mathfrak{M}})$.

$\boxtimes$ thus differs fundamentally from $\otimes$ in that it does not really allow for the passage of time from its lhs to its rhs, but it only partially mimics it by *shifting the circumstance at which its rhs is evaluated to the circumstance immediately following the circumstance at which its lhs is evaluated*, with the consequence that such shift is only *temporary* in the sense that it only concerns the evaluation of the rhs of the relevant occurrence of $\boxtimes$ rather than the evaluation of the whole argumentative material following the occurrence (so that in particular the circumstance at which what immediately follows the rhs is evaluated is in effect *reset* to be the circumstance at which the lhs is evaluated). For example, while $(\varphi \otimes \psi) \otimes \neg\psi$ is consistent (since, when testing for consistency, $\neg\psi$ is evaluated in the context and at the circumstance immediately following the context in and the circumstance at which ψ is evaluated), $(\varphi \boxtimes \psi) \boxtimes \neg\psi$ is not (since $\neg\psi$ is evaluated (in the same context and) at the same circumstance as ψ). An interesting consequence of this fundamental difference is that, while, as is indicated among other things by theorem 13 and the properties of sequences, $\otimes$ is "*basically associative*" in the sense that, *as a premise or conclusion*, $\varphi_0 \otimes \varphi_1 \otimes \varphi_2$ is intersubstitutable with $(\varphi_0 \otimes \varphi_1) \otimes \varphi_2$ *salva validitate* (since, when taken as a premise or conclusion, $\varphi_0 \otimes \varphi_1 \otimes \varphi_2$ evaluates φ_1 in the context and at the circumstance immediately following the context in and the circumstance at which φ_0 is evaluated and evaluates φ_2 in the context and at the circumstance immediately following the context in and the circumstance at which φ_1 is evaluated, just as $(\varphi_0 \otimes \varphi_1) \otimes \varphi_2$ does), as von Wright [1965], p. 297 himself observes $\boxtimes$ is not (while $\varphi_0 \boxtimes \varphi_1 \boxtimes \varphi_2$ evaluates φ_1 at the

6 Conclusion

In a rhetorically overtoned slogan, intracontextual logics are the logics of monologue, isolation, being and identity; intercontextual logics are the logics of dialogue, relation, becoming and difference. More soberly, intercontextual logics allow for context change *among* premises or conclusions (as done by $\mathbf{Inter}_{\mathcal{YTM}^{\mathbb{Z}}}$) and also *within* premises or conclusions (as done by $\mathbf{Inter}_{\mathcal{YTM}_{\otimes}^{\mathbb{Z}}}$). Given the pervasiveness of context change, this feature makes intercontextual logics a much more *realistic* model of the human use of language when compared with the decidedly *idealistic* model offered by intracontextual logics. This paper has undertaken some first steps in the exploration of such logics, showing how, discerning more *structure* in logical consequence than is usually done, such exploration can be pursued in full compliance with the standard assumption of *sentences* as logical-consequence bearers.

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circumstance immediately following the circumstance at which φ_0 is evaluated and evaluates φ_2 at the circumstance immediately following the circumstance at which φ_1 is evaluated, $(\varphi_0 \boxtimes \varphi_1) \boxtimes \varphi_2$ does evaluate φ_1 at the circumstance immediately following the circumstance at which φ_0 is evaluated but evaluates φ_2 at the same circumstance as φ_1). (For completeness, I note that, while basically associative in the sense explained above, $\otimes$ is *not unrestrictedly associative* in the sense that, *as a component of a premise or conclusion*, $\varphi_0 \otimes \varphi_1 \otimes \varphi_2$ is not intersubstitutable with $(\varphi_0 \otimes \varphi_1) \otimes \varphi_2$ *salva validitate*: for example, while $(P \otimes Q) \wedge (R \otimes \neg R \otimes S)$ is consistent, $(P \otimes Q) \wedge ((R \otimes \neg R) \otimes S)$ is not.) A related difference emerges if we restrict to $\otimes$'s and $\boxtimes$'s semantics *in the absence of context change* (and so to situations in which the only relevant contexts are days): letting both the context and the circumstance be d_0, while $\varphi \otimes \psi$ becomes indiscriminable from $\varphi \wedge \psi$ in that its truth in d_0 at d_0 requires ψ to be true in d_0 *at* d_0, $\varphi \boxtimes \psi$ retains its temporal connotation in that its truth in d_0 at d_0 requires ψ to be true in d_0 *at* $\mathsf{suc}_{\mathfrak{S}}(d_0)$. Summing up in somewhat pictorial but hopefully helpful terms, while, in the presence of context change, both $\otimes$ and $\boxtimes$ function in such a way as to have their rhs evaluated at the circumstance immediately following the circumstance at which their lhs is evaluated, $\otimes$ achieves this effect by immerging itself into the passage of time and having its two sides "look inward" at the days on which they are respectively uttered, whereas $\boxtimes$ achieves the same effect by subtracting itself from the passage of time and, although having its lhs "look inward" at the day on which both it and the rhs are uttered, also having its rhs "look forward" at the day immediately following the day on which both it and the lhs are uttered. Thanks to an anonymous referee for recommending a comparison between $\otimes$ and $\boxtimes$.

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