Confirming the Less Likely, Discovering the Unknown
Dogmatisms: Surd and Doubly Surd, Natural, Flat and Doubly Flat*

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1 Prelude

This paper is a sustained attempt at finding a probabilistic framework harmonious with a certain (in my opinion, quite appealing) view about perceptual justification that has arguably momentous anti-sceptical consequences. The view is in fact more properly analysed as a family of views, constituted by a core minimal position and a structure of very interesting, partly connected and jointly compatible strengthenings thereof, either along the axis of claims about the acquisition and presence of justification or along the axis of claims about the relationships between justification (including its defeat) and probability. The upshot of the dialectic will be that all the members of the family enjoy a natural, well-behaved and illuminating probabilistic representation within the framework of a well-known non-classical theory of the structure and dynamics of probabilities.

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The rest of this paper is organised as follows. After giving some background, section 2 introduces the core minimal position—dogmatism. Section 3 presents Moore's "proof of the existence of the external world" and recommends to the dogmatist an appealingly bold anti-sceptical take on it. Dogmatism, and in particular some of its strengthenings, have recently been subject to two influential objections assuming the framework of Bayesian confirmation theory which are rehearsed in section 4. Section 5 starts to take up the challenge issued by the first Bayesian objection, offering a particularly illuminating style of counterexample to a modal principle of quasi-monotonicity of degrees of probability on degrees of justification. Section 6 turns that counterexample into a counterexample to a temporal principle of quasi-monotonicity of degrees of probability on degrees of justification. Section 7 makes explicit how those results afford a warranted line of reply to the first Bayesian objection. However, section 8 observes that, in addition to the second Bayesian objection still being left unanswered, that line of reply is not easily available for a certain strengthening of dogmatism. Section 9 suspends temporarily consideration of the dynamics of probabilities to focus instead on consideration of their structure, arguing that a certain other strengthening of dogmatism is inconsistent with the classical theory of probabilities. Section 10 proposes the Dempster-Shafer theory as a more hospitable framework for that kind of dogmatism. Section 11 proceeds to argue in favour of the adoption of Dempster's rule of combination as a theory of learning which fits well the cases of learning from experience relevant for this paper. Within this framework, section 12 discusses some fine-tuning modelling choices, eventually finding a theory of the structure and dynamics of probabilities fully harmonious with dogmatism and all its strengthenings. Section 13 briefly recapitulates the main themes of the paper.

2 Dogmatism about Perceptual Justification

The version of *dogmatism* we'll focus on in this paper is dogmatism about *propositional perceptual* justification. Before saying what the view is, a couple of words of explanation are in order concerning what it is about. Firstly, propositional justification can be understood as a *relation* between a subject and a proposition, thus contrasting with *doxastic* justification, which is the usual *property* of beliefs typically considered in epistemological discussions. Roughly, one has a propositional justification for believing that P iff one has something such that, were one to form a belief that P by basing it on that, one would have formed a doxastically justified belief that P (see Firth [1978], p. 218 for the introduction of the propositional/doxastic distinction).[1] Secondly, and equally roughly, one has a perceptual justification for believing that P iff one has something such that, were one to form a perceptual belief that P by basing it on that, one would have formed a doxastically justified perceptual belief that P.[2,3]

[1] I'll henceforth use 'justification' and its relatives for 'propositional justification' and its relatives.

[2] In turn, we can rest content with an intuitive understanding of what it is for a belief to be perceptual.

[3] I'll henceforth use 'perceptual' and its likes for 'visual' and its likes. While dogmatism about *visual* perceptual justification is quite appealing, dogmatism about the justification afforded by *other* sensory

Dogmatism about perceptual justification holds then, for any proposition P belonging to a certain class $\mathcal{P}$, that one's having a (perceptual) experience as though P[4] suffices[5] to provide one with an at least *prima facie* justification for believing that P. A couple of words of explanation concerning this time the view itself are in order. Firstly, the justification is only *prima facie* rather than *all-things-considered*: it can be *defeated* in many ways.[6] More precisely, dogmatic (perceptual) justification exhibits an intriguing combination of *resistance* to *overriding* defeat and *permeability* to *undermining* defeat (see Pollock [1974], pp. 39–46 for the introduction of the overriding/undermining distinction). Dogmatic justification cannot be easily overridden. For example, a prior justification for believing that there is no cube (because, say, one was so told by the secretary) does not typically suffice to defeat one's dogmatic justification for believing that there is a cube (the kind of prior justification that would do the trick needs to be something much stronger like, say, the fact that one has appreciated the fine details of some sort of Eleatic argument and cannot find any fault in them). Dogmatic justification can however be easily undermined. For example, a prior justification for believing that the room contains devices producing random perceptual illusions (because, say, one was so told by the secretary) suffices to undermine one's dogmatic justification for believing that there is a cube (the kind of prior justification that would not do the trick needs to be something much weaker like, say, one's justification, given by the relevant quantum-mechanical considerations, for believing that there is a .000000001 chance that one has become an envatted brain disconnected from the external world). The

modalities—like, for example, smell—seems to be less appealing: vision seems to present states-of-affairs concerning the external world with an immediacy lacked by the presentations offered by other sensory modalities. Relatedly, I emphasise that, throughout, against a certain usage of the phrase 'perceptual presentation' and of its likes, I take perceptual presentations *not to be factive*: on my usage of the phrase, one can have a perceptual presentation of the state-of-affairs of its being the case that P even if it is not the case that P (with the plausible consequence, once a probabilistic framework has been introduced in section 4, that a perceptual presentation of the state-of-affairs of its being the case that P does not provide certainty that P). Thanks to Miguel Ángel Fernández for pressing me on this latter point.

[4]This and similar phrases are supposed to be intended in a colloquial, uncontroversial sense, carrying no commitment to any view on exactly what content—if any—experiences have. For any view holding (plausibly, in my opinion) that experiences do not have as contents propositions that are typically expressed by 'as though'-clauses or are contents of perceptual beliefs, there is an important challenge of explaining how the latter propositions are "*extracted*" from experience. This is, as far as I know, an open question in empirical psychology that may well have significant consequences for the dogmatism/anti-dogmatism debate. However, in this paper I won't touch further on this important issue and I'll assume in particular that such "extraction"—if it is indeed needed—happens in a dogmatist-friendly way (i.e. that it does not draw on one's beliefs about the external world).

[5]Throughout, I understand this sufficiency only in the very intensional and epistemic—but nevertheless still extremely controversial—terms of *epistemic dependence*: that is, the justification provided by the experience does not epistemically depend on any other justification one may have—it only epistemically depends on the experience itself. I'll henceforth use 'dependence' and its relatives for 'epistemic dependence' and its relatives.

[6]I'll henceforth use 'justification' and its relatives for '*prima facie* justification' and its relatives. However, to keep things concise, in a couple of places in which I discuss certain cases of justification I'll ignore the possibility that the justification in question is defeated (at least by the explicit features of the case), so that my applying the expression 'justification' in these cases will imply that the justification in question is not defeated (at least not by the explicit features of the case).

senses, and vision in particular, are a court that cannot be easily overruled, but that can be easily defamed. To give a systematic account of what defeats dogmatic justification is an important task facing the dogmatist—a task that can thankfully be left for another occasion.

Secondly, it is a matter of great delicacy to specify what exactly $\mathcal{P}$ is, since it would seem that, on all reasonable understandings of what it is to have an experience as though P (see fn 4), one can have an experience as though P without plausibly having any dogmatic justification for believing that P, so that $\mathcal{P}$ cannot be identified with the class of all propositions (or, on the most natural way of understanding what it is for a belief to be perceptual, with the class of propositions that can be perceptually believed). For example, while it would seem that, at the stadium, one can have an experience as though Maradona had just scored a goal, it just isn't plausible that that experience suffices to provide one with a dogmatic justification for believing that Maradona has just scored a goal—one presumably needs to have (and usually does have) some independent[7] justification for believing that a football match is in progress and that Maradona is playing in it in order for that experience to justify the proposition that Maradona has just scored a goal rather than the weaker proposition that a shortish guy with curly hairs has just kicked a ball beyond a white line. Quite generally, we should distinguish between two properties of concepts of material objects:

(P_1) A concept's being applicable *on the basis of experience*;

(P_2) A concept's being applicable *with dogmatic justification*.

Perhaps surprisingly, many concepts (like the concept of a goal, of Maradona, of a shop etc.) exemplify property (P_1) without exemplifying property (P_2), whereas obviously every concept exemplifying property (P_2) exemplifies property (P_1) as well (and, unsurprisingly, some concepts—like the concept of a CIA spy—exemplify neither). It is this wider extension of property (P_1) with respect to property (P_2) that requires the restriction to $\mathcal{P}$. To give a systematic account of which propositions belong to $\mathcal{P}$ is another one of the important tasks facing the dogmatist—and another one of the tasks that can thankfully be left for another occasion (for illustrative purposes, I assume throughout that ⟨There is a cube⟩[8] does belong to $\mathcal{P}$).

The above will be our official understanding of what dogmatism about perceptual justification[9] amounts to. In the rest of this paper, we'll consider and develop further this *core minimal position* as well as some very interesting, partly connected and jointly compatible *strengthenings* thereof, which will run either along the axis of claims about the *acquisition and presence of justification* or along the axis of claims about the *relationships between justification (including its defeat) and probability*. Dogmatism is to be contrasted

[7]The reader will be able to extrapolate from the second remark in fn 22 why the 'independent'-gloss is needed in such contexts.

[8]Throughout, $\langle\varphi\rangle$ denotes the proposition expressed by φ.

[9]I'll henceforth use 'dogmatism' and its relatives for 'dogmatism about perceptual justification' and its relatives.

with *anti-dogmatist* views according to which, for example, one's experience as of a cube, although it may well have *some positive epistemic force*, does not by itself suffice to provide one with a *justification* for believing that there is a cube. On such views, there needs to be *additional epistemic support* in order for one to have such justification—to consider just but one specific anti-dogmatist view, it may for example be required that one have a justification for believing that there is an external world whose manifest features are usually faithfully reflected in one's experiences.

I take it that dogmatism enjoys a certain intuitive plausibility over anti-dogmatism. Beyond this intuitive plausibility, it would constitute a positive argument in favour of the view to give a systematic account of why an experience as though P should have such a strong epistemic bias as the dogmatist contends in favour of $\langle P \rangle$ (rather than, say, in favour of $\langle$Although it looks as though P, it is not the case that $P\rangle$). However, rather than engaging in offering some such positive argument in favour of dogmatism, in this paper I'll be concerned with defending the view, and in particular the various strengthenings of the core minimal position, from two recent influential objections assuming the framework of Bayesian confirmation theory; in the process, I'll motivate and develop a natural, well-behaved and illuminating probabilistic representation of the view within the framework of a well-known non-Bayesian theory of the structure and dynamics of probabilities. Before introducing the Bayesian objections, however, some more background is required.

3 Dogmatism, Scepticism and Moorean Arguments

Dogmatism has an obvious *anti-sceptical punch*: it allows one to have a justification for believing that there is a cube in a way that does not depend on one's having additional epistemic support for ruling out the sceptical hypothesis that one is victim of a global deceit. This anti-sceptical punch has however appeared to many epistemologists to be objectionably strong, as revealed by the following problem. Moore [1939] put forth something like the following argument (which has subsequently been interpreted) as a proof of the existence of the external world:

(M_1) Here is a hand;

(M_2) If here is a hand, there exists an external world;

(M_3) Therefore, there exists an external world.

The argument (call it 'MOORE') is clearly known to be valid (it is an instance of *modus ponens*) and premise (M_2) is justified, let's assume, by reflection on what it is to be a hand and what it takes for there to exist an external world. Premise (M_1) would also seem to have to be justified (by a familiar kind of experience) if scepticism is to be false. Yet, even though the premises are justified and the argument is known to be valid, there would seem to be a substantial sense in which MOORE *fails*—in particular, a substantial sense in which MOORE is *viciously circular*. The advertised problem for the dogmatist is that, given her view, it is not clear how she can find any such fault in MOORE.

Here is, for example, one natural explanation of what is *viciously circular* about MOORE (see Wright [2007], pp. 36–39):

(VC_1) Assume the specific anti-dogmatist view briefly mentioned in section 2: an experience as of a hand provides one with a justification for believing (M_1) only in conjunction with an independent justification for believing that there exists an external world whose manifest features are usually faithfully reflected in one's experiences;

(VC_2) From this, one can infer[10] that an experience as of a hand provides one with a justification for believing (M_1) only in conjunction with an independent justification for believing (M_3);[11]

(VC_3) It clearly follows that one cannot use MOORE to acquire a *first* justification for believing (M_3);

(VC_4) Moreover, one might think that it also follows that one cannot use MOORE to acquire a *new* justification for believing (M_3).

(VC_4) (if not already (VC_3)) seems to explain satisfactorily what the felt vicious circularity of MOORE consists in.[12]

[10] At least by closure of justification, see principle (C) below in the text.

[11] The inference to (VC_2) is more problematic on apparently minor variations of the specific anti-dogmatist view briefly mentioned in section 2. For example, according to one such variation, an experience as of a hand provides one with a justification for believing (M_1) only in conjunction with an independent justification for believing that one's experiences are generally reliable. But it's hard to see in what sense ⟨One's experiences are generally reliable⟩ could *entail* ⟨There exists an external world⟩ (one's experiences might be generally reliable in the absence of an external world, if their deliverances did not generally consist in reports about material objects), and so it's hard to see in what way the inference to (VC_2) could be vindicated on this apparently minor variation of the specific anti-dogmatist view briefly mentioned in section 2. I believe that this train of thought hints at a crucial difficulty for anti-dogmatist explanations of what is viciously circular about MOORE, but elaboration of this point lies beyond the scope of this paper.

[12] The *gap* between (VC_3) and (VC_4) is well worth emphasising against a certain tendency in the literature to conflate them: it is unclear why an argument justification for believing whose premises or knowledge of whose validity necessarily requires an independent justification for believing its conclusion could nevertheless not possibly be used so as to acquire a justification for believing its conclusion *additional to the one required to be independently had*. However, *insofar* as the vicious circularity of MOORE is supposed to be cashed out in the epistemic terms of acquisition of justification, it would seem odd to think that the problem is merely with acquisition of a first justification and that everything is alright with acquisition of a new justification. Unfortunately, (VC_3) is all that clearly follows from (VC_2), and hence the present explanation, risking to break down at the step from (VC_3) to (VC_4), is actually in danger of being severely incomplete. I believe that this train of thought hints at another crucial difficulty for anti-dogmatist explanations of what is viciously circular about MOORE, but elaboration of this point also lies beyond the scope of this paper.

On this explanatory scheme, and assuming—against the doubts aired in fn 12—that it can help itself up to (VC_4), the *transmission* principle for justification:

(TN) If one has a justification for believing that P_0, one has a justification for believing that P_1, one has a justification for believing that P_2... and one knows that $\langle P_0\rangle$, $\langle P_1\rangle$, $\langle P_2\rangle$... entail $\langle Q\rangle$, in virtue of this one has a new justification for believing that Q

fails: one has a justification for believing (M_1) and a justification for believing (M_2), and one knows that they entail (M_3), but, given (VC_4), it is not the case that in virtue of this one has a new justification for believing (M_3). Crucially, (TN) fails without the *closure* principle for justification:

(C) If one has a justification for believing that P_0, one has a justification for believing that P_1, one has a justification for believing that P_2... and one knows that $\langle P_0\rangle$, $\langle P_1\rangle$, $\langle P_2\rangle$... entail $\langle Q\rangle$, one has a justification for believing that Q

failing: indeed, on the explanatory scheme under consideration, it is precisely because one can only have a justification for believing (M_1) by having an (independent) justification for believing (M_3) that (TN) fails (moreover, as pointed out in fn 11, the step from (VC_1) to (VC_2) is only valid given (C))!

This explanatory scheme is however *not* available to the dogmatist, who rejects (VC_1). What alternative explanation—if any—is then available to her? From a certain appealing dogmatist perspective, (TN) actually does not fail in the case of MOORE and its likes, and so, in a sense, MOORE and its likes are perfectly good anti-sceptical arguments (let's dub this view—a strengthening of dogmatism along the axis of claims about the acquisition and presence of justification—'*dogmatism*$^{\surd}$').[13] In particular, the dogmatist$^{\surd}$ thinks that one can use MOORE to acquire a new justification for believing the negation of a sceptical hypothesis which only depends on a dogmatic justification for believing (M_1), on a reflective justification for believing (M_2) and on knowledge of *modus ponens*.

The dogmatist$^{\surd}$ is then free to hold that the failure of MOORE is much more restricted and philosophically less interesting than anti-dogmatists usually make it out to be. One promising dogmatist$^{\surd}$-friendly explanatory scheme would run as follows.[14] Arguments are too coarse-grained entities for bearing in themselves the properties of

[13] I emphasise that dogmatism does not entail dogmatism$^{\surd}$: a dogmatist can maintain that an experience as of a cube suffices to provide a justification for believing that there is a cube while also maintaining—on grounds different from those of the anti-dogmatist's (VC_1)–(VC_4)-explanation—that (TN) does fail in the case of MOORE and its likes (see Silins [2007]).

[14] The scheme is generally inspired by some considerations of Pryor [2004], pp. 362–370 though diverging from them in important details. Pryor [2004] focusses entirely on the *radicalised* version of scepticism mentioned in fn 18 and mostly on why *subjects* who believe *without justification* the relevant defeating proposition may not use MOORE to acquire a new *doxastic* justification for its conclusion. I mostly focus on a *less radical* version of scepticism and take as central the *epistemic state* that provides a *justification* for

failing or succeeding. What fails or succeeds is not simply an argument, but an argument *together with the specific justifications offered for its premises* (there may be no good justification for one of the argument's premises), and whether that fails or succeeds is also *relative to a given subject's epistemic state* (the subject may have a defeater for the justification offered for one of the argument's premises). With this in mind, it should have been clear from the start that *absolutely any* argument may not be used to acquire a new justification for its conclusion by a subject who has a defeater for the justification offered for one of the argument's premises,[15] and the argument together with that justification might well be regarded as *viciously circular relative to that subject's epistemic state* if the defeater for the justification ultimately depends on a defeater that is actually so strong as to:

(TJP) Defeat any ordinary justification that may be offered for the premise;

(TJA) Defeat any ordinary justification that may be offered for the relevantly analogous premise of any relevantly analogous argument;

(TJO) Defeat any ordinary justification that may otherwise provide a route to the argument's conclusion.

Under plausible assumptions, we can then explain why MOORE together with the familiar perceptual justification that is ordinarily offered for (M_1) is viciously circular *relative to the epistemic state of a sceptic about the senses*:

(VC'_1) We may plausibly assume that a sceptic about the senses has an all-things-considered justification for believing that perception does not provide a justification for believing propositions about the external world;

(VC'_2) By (C), such sceptic has:

- (TJP^*) A justification for believing that perception does not provide a justification for believing (M_1);
- (TJA^*) A justification for believing that perception does not provide a justification for believing any proposition relevantly analogous to (M_1);
- (TJO^*) A justification for believing that perception does not otherwise provide a route to (M_3).

such scepticism; I explain why in such a state one may not use MOORE to acquire a new *propositional* justification for its conclusion and in what sense this is due to a *vicious circularity* exhibited by MOORE relative to that state, deriving from this explanation a *contextual* explanation of what the felt vicious circularity of MOORE consists in.

[15]The considerations to follow in the text extend in a natural way to cases in which a subject believes without justification the relevant defeating proposition: the subject, if reflective enough, will still believe to be in the kind of situation described in the text.

(VC_3') From this, one can infer[16] that such sceptic has:

- (TJP^{**}) A defeater for any ordinary (i.e. perceptual) justification that may be offered for (M_1);
- (TJA^{**}) A defeater for any ordinary (i.e. perceptual) justification that may be offered for the premise relevantly analogous to (M_1) of any argument relevantly analogous to MOORE;
- (TJO^{**}) A defeater for any ordinary (i.e. perceptual) justification that may otherwise provide a route to (M_3).

(VC_3') explains why MOORE together with the familiar perceptual justification that is ordinarily offered for (M_1) is viciously circular relative to the epistemic state of a sceptic about the senses. From the perspective of such scepticism, MOORE together with the familiar perceptual justification that is ordinarily offered for (M_1) is the vain attempt at establishing something relying on a basis that has already been quite generally discredited as unsuitable for that purpose. Consequently, given that such scepticism is very naturally *salient* in *typical* contexts in which MOORE is considered,[17] (VC_3') would seem to explain satisfactorily what the felt vicious circularity of MOORE consists in.[18]

[16] The step from a starred claim to its double-starred relative need not in any way appeal to the general *restricted-factivity* principle that, if one has a justification for believing that one does not have a justification for believing that P, one does not have a justification for believing that P (which in turn would be a characteristic theorem of a **D4** logic for justification), nor to the slightly more specific principle that, if one has a justification for believing that *a source* does not provide a justification for believing certain propositions, one does not have a justification for believing those propositions *on the basis of that source*. That step is rather intuitively warranted by the fine details of the sceptic's all-things-considered justification: a justification *of such a reflective, deep-reaching and subverting kind* for believing that perception does not provide a justification for believing propositions about the external world intuitively defeats any ordinary (i.e. perceptual) justification that may be offered for (M_1), any ordinary (i.e. perceptual) justification that may be offered for the premise relevantly analogous to (M_1) of any argument relevantly analogous to MOORE and any ordinary (i.e. perceptual) justification that may otherwise provide a route to (M_3).

[17] Though it is not salient in *all* contexts in which MOORE is considered. For example, if before opening one's eyes for the first time, one is uncertain whether one lives in an external world or in a solipsistic world of completely chaotic sensations (where orderly experiences such as that as of a hand would be impossible), and, upon opening one's eyes, one has an experience as of a hand and considers MOORE, such scepticism would not be salient (and, unsurprisingly, in such context MOORE together with the familiar perceptual justification that is ordinarily offered for (M_1) would be an unexceptionable way of settling the question at hand).

[18] In fact, many discussions of the sceptic about the senses seem to assume something along the lines of the idea that such sceptic has an all-things-considered justification not only for believing that perception does not provide a justification for believing propositions about the external world, but also for *withholding judgement about the existence of an external world*. Relative to the epistemic state of such a radicalised sceptic, MOORE would be even more dramatically viciously circular, as *any (ordinary or not ordinary) all-things-considered justification for believing its conclusion would amount to a straightforward overriding defeater for what would now be the key sceptical justification*. From the perspective of such a radicalised scepticism, MOORE together with the familiar perceptual justification that is ordinarily offered for (M_1) is the vain attempt at refuting something that has already subverted the basis on which

4 Bayesian Dogmatic Learning?

With so much background in place, we can proceed to introduce two influential objections assuming the framework of Bayesian confirmation theory which have recently been raised against dogmatism, and in particular against some of its strengthenings. Following Bayesian confirmation theory, let's assume, for the time being, that the *classical* theory of *probabilities* correctly describes the degrees of support that a given subject's epistemic state at a given time lends to the elements belonging to the relevant σ-algebra $\mathcal{S}$ of propositions of interest.[19] And, again following Bayesian confirmation theory, let's also assume, again for the time being, that *learning* upon getting evidence E goes by *conditionalisation*:

CONDITIONALISATION $\mathfrak{P}_E(H) = \mathfrak{P}(H/E)$,

where the classical probability function $\mathfrak{P}_E$ measures the degrees of support lent to the propositions in $\mathcal{S}$ by any epistemic state got by merely adding E as evidence to any epistemic state whose degrees of support lent to the propositions in $\mathcal{S}$ are measured by the classical probability function $\mathfrak{P}$),[20] and where $\mathfrak{P}(X/Y)$ is defined in the usual way:

CONDITIONAL PROBABILITY $\mathfrak{P}(X/Y) = \dfrac{\mathfrak{P}(X \wedge Y)}{\mathfrak{P}(Y)}$.

Suppose then that three propositions E, H and H^* and a classical probability function $\mathfrak{P}$ satisfy the conditions:

(EHH^*_1) H^* entails E and is incompatible with H;

(EHH^*_2) $0 < \mathfrak{P}(E) < 1$ and $\mathfrak{P}(H^*) > 0$.

Here are a couple of general facts following from (EHH^*_1) and (EHH^*_2):

LOWERING $\mathfrak{P}(\neg H^*/E) < \mathfrak{P}(\neg H^*)$;

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the attempt relies (and, unsurprisingly, in contexts in which such a radicalised scepticism is salient MOORE together with the familiar perceptual justification that is ordinarily offered for (M_1) does come across as even less helpful).

[19]Thus, throughout, I use 'probability' and its likes for '*epistemic* probability' and its likes (notice that the "support" in question may simply amount to providing a reason for thinking that the relevant proposition is to a certain extent likely). Focus on epistemic probability is of course controversial for the many Bayesians who reject the intelligibility or at least the usefulness of the notion. Still, I think that that is by far the most natural and revealing setting in which to frame our whole discussion. In any event, the essence of the dialectic would remain unaltered if we framed our discussion in terms of *subjective* probability instead. Thanks to Dylan Dodd for discussions of these issues.

[20]This presupposes that all that is relevant for determining the degrees of support lent to the propositions in $\mathcal{S}$ by the former states are the degrees of support lent to the propositions in $\mathcal{S}$ by the latter states (plus, of course, E), irrespective of whatever *non-quantitative* differences there may be among these latter states. That plausible but non-trivial presupposition will henceforth be made.

CAPPING $\mathfrak{P}(H/E) \leq \mathfrak{P}(\neg H^{*}/E) < \mathfrak{P}(\neg H^{*})$.

Now, let:

- $E = EXPCUBE = \langle$One has an experience as of a cube$\rangle$;
- $H = CUBE = \langle$There is a cube$\rangle$;
- $H^{*} = DECEIT = \langle$A global deceit makes it look to one as though there were a cube when in fact there is no cube$\rangle$.

EXPCUBE, *CUBE* and *DECEIT* and the classical probability functions that measure the degrees of support lent to the propositions in $\mathcal{S}$ by most of our epistemic states satisfy (EHH^{*}_{1}) and (EHH^{*}_{2}), and hence we have the corresponding instances of LOWERING and CAPPING for them.

Alas, this seems to create a couple of glitches at least for certain strengthenings of dogmatism. Let $\mathfrak{P}$ be the classical probability function that measures the degrees of support lent to the propositions in $\mathcal{S}$ by one's epistemic state before having an experience as of a cube. Firstly, recall from section 3 that a dogmatist$^{\surd}$ maintains not just that an experience as of a cube suffices to provide one with a justification for believing *CUBE*, but also that one can use that justification to acquire a new justification for believing $\neg$*DECEIT* (exploiting (TN) and the fact that one knows that *CUBE* entails $\neg$*DECEIT*).[21] However, given **CONDITIONALISATION**, $\mathfrak{P}_{EXPCUBE}(\neg DECEIT) = \mathfrak{P}(\neg DECEIT/EXPCUBE)$ and, given LOWERING, $\mathfrak{P}(\neg DECEIT/EXPCUBE) < \mathfrak{P}(\neg DECEIT)$, so that $\mathfrak{P}_{EXPCUBE}(\neg DECEIT) < \mathfrak{P}(\neg DECEIT)$. In other words, upon having an experience as of a cube, one's probability for $\neg$*DECEIT* *goes down*! If the experience as of a cube lowers one's probability for $\neg$*DECEIT*, it would seem that it cannot afford one a new justification for believing it (see Hawthorne [2004], pp. 73–77; Cohen [2005], pp. 424–425; White [2006], pp. 531–537; Weatherson [2007]; Silins [2007], pp. 123–128 for early discussions of this or similar issues)?

Secondly, from a certain appealing dogmatist perspective, an experience as of a cube and its likes may suffice to provide one with a justification for believing *CUBE* and its likes even though one has no independent justification for believing $\neg$*DECEIT* and its likes (let's dub this view—another strengthening of dogmatism along the axis

[21] It might be worried that dogmatism$^{\surd}$ should not be applied to $\neg$*DECEIT*, as it might be worried that *DECEIT* is not a "real sceptical hypothesis" on the grounds that it could easily be falsified (if the relevant experience is not as of a cube). However, it turns out that the relevant experience is as of a cube, and so *DECEIT* turns out to be just as unfalsifiable as other sceptical hypotheses. Thanks to an anonymous referee for raising this worry.

of claims about the acquisition and presence of justification—'*dogmatism*$^\natural$').[22,23] However, given **CONDITIONALISATION**, $\mathfrak{P}_{EXPCUBE}(CUBE) = \mathfrak{P}(CUBE/EXPCUBE)$ and, given CAPPING, $\mathfrak{P}(CUBE/EXPCUBE) < \mathfrak{P}(\neg DECEIT)$. In other words, upon having an experience as of a cube, one's probability for *CUBE cannot be higher* (and indeed *must be strictly lower*) than one's prior probability for $\neg$*DECEIT*! If one's probability for *CUBE* upon having an experience as of a cube is high enough as to provide one with a justification for believing *CUBE*, it would seem that one's prior probability for $\neg$*DECEIT* was already high enough as to provide one with a justification for believing $\neg$*DECEIT* that was independent of one's experience as of a cube (see Schiffer [2004], pp. 174–176; White [2006], pp. 533–534; Silins [2007], pp. 129–134; Wright [2007], p. 42 for early discussions of this or similar issues).

There are various moves a friend of dogmatism and of its strengthenings could make when faced with these objections. Without aiming at exhaustivity, let me briefly list what seem to me to be the most promising avenues of reply open to her. With regard to the objection from LOWERING, the dogmatist could:

(L_1) Deny that a drop in probability is always incompatible with acquisition of new justification;

(L_2) Deny that one can transmit the justification for *CUBE* to $\neg$*DECEIT* (thus in effect giving up dogmatism$^{\surd}$);

(L_3) Reject some of the principles of the classical theory of probabilities and/or **CONDITIONALISATION**.

With regard to the objection from CAPPING, the dogmatist could:

(C_1) Deny that the higher prior probability of $\neg$*DECEIT* implies the existence of an independent justification for believing it;

[22] I emphasise that dogmatism does not entail dogmatism$^\natural$: a dogmatist can maintain that an experience as of a cube *suffices* to provide a justification for believing that there is a cube while also maintaining that the existence of such justification *entails* the existence of an independent justification for believing $\neg$*DECEIT* (see Silins [2007], pp. 129–134). Quite generally, the fact that its being the case that *P entails* its being the case that *Q* does not imply that its being the case that *P depends on* its being the case that *Q* in any sense (see fn 5): for example, its being the case that snow is white entails its being the case that 'Snow is white' is true, but it certainly does not depend on it in any sense. As for the 'independent'-gloss in the formulation of dogmatism$^\natural$, notice that that is needed because dogmatism$^\natural$ should be compatible with the dogmatist$^{\surd}$ idea that, by (TN), the existence of a justification for believing *CUBE* (plus knowledge of the validity of the argument from *CUBE* to $\neg$*DECEIT*) entails the existence of a justification for believing $\neg$*DECEIT that depends on the former justification* (so that dogmatism$^\natural$ can also be compatible with (C)).

[23] A prominent kind of situation relevant for dogmatism$^\natural$—as well as for some of the other strengthenings of dogmatism below in the text—is the one in which a subject is *at the ideal beginning of empirical inquiry*, at the very last moment before opening her eyes for the first time but already fully possessed of her epistemic faculties (an admittedly far-fetched situation, but clearly one of extreme epistemological significance).

(C_2) Deny that an experience as of a cube may suffice to provide one with a justification for believing *CUBE* even though one has no independent justification for believing $\neg$*DECEIT*, while maintaining that such an experience would suffice to provide one with a justification for believing *CUBE* (thus in effect giving up dogmatism$^{\natural}$);

(C_3) Reject some of the principles of the classical theory of probabilities and/or **CONDITIONALISATION**.

5 Justification and Probability

I want to start the development of my favoured way of making probabilistic sense of dogmatism and its strengthenings by first focussing on the objection from Lowering. In particular, I want to argue that, against the background of a certain independently appealing assumption about the relationships between justification and probability, move (L_1) is perfectly warranted for the dogmatist$^{\surd}$. The assumption I have in mind implies the existence of striking counterexamples to the *modal* principle of *quasi-monotonicity of degrees of probability on degrees of justification*:

(MDPJ) For every subject s, proposition P and worlds w_0 and w_1, if in w_0 s has more justification for believing that P with $\mathfrak{P}^{s,w_0}(\langle P\rangle) = \mathfrak{r}_0$[24] than s has in w_1 with $\mathfrak{P}^{s,w_1}(\langle P\rangle) = \mathfrak{r}_1$, then $\mathfrak{r}_0 \geq \mathfrak{r}_1$.

Here is what is in my view a particularly illuminating style of counterexample to (MDPJ) (see Smith [2010]).

Firstly, suppose that in w_0 Tom is attending the drawing of a fair lottery, being privy to the information concerning its fairness and number of tickets, and has an experience as though ticket #i had been drawn (let *IWON* be ⟨Ticket #i won⟩). After that experience, what is Tom's probability that ticket #j ($i \neq j$) lost (let *JLOST* be ⟨Ticket #j lost⟩)? Well, at least in typical cases, it would seem that Tom cannot completely rule out that [the winning ticket is actually ticket #j but something funny is going on in that experience][25] (let *ERROR* be ⟨The winning ticket is ticket #j but something funny is going on in Tom's experience⟩), and it would seem that, letting t be Tom, that ought to reflect in $\mathfrak{P}^{t,w_0}(\neg ERROR)$ being (very high but) < 1. Since *JLOST* entails $\neg$*ERROR*, $\mathfrak{P}^{t,w_0}(JLOST) < 1$ as well.

Secondly, let $\mathfrak{l}$ be a plausible value for $\mathfrak{P}^{t,w_0}(JLOST)$. Suppose then that in w_1 the fair lottery has n tickets, with n such that $1/n < 1 - \mathfrak{l}$, and that Tom is no longer attending the drawing of the lottery, still being privy though to the information concerning its fairness and number of tickets. Then, given the natural assumption:

[24]Throughout, $\mathfrak{P}^{s,w,t}$ is the classical probability function that correctly describes the degrees of support that s's epistemic state in w at t lends to the propositions in $\mathcal{S}$. In those contexts in which worlds and times are not an issue, I omit the second and third superscript respectively. The notation extends to the non-classical probability functions considered below in the text.

[25]Throughout, I use square brackets to disambiguate constituent structure in English.

(L) If, upon getting evidence E, it would be certain for a subject s in a world w at a time t that x is the ticket of a fair lottery with l tickets and, upon getting evidence E, s would have no other relevant information, then $\mathfrak{P}_{E}^{s,w,t}(\langle x \text{ wins}\rangle) = 1/l$,

$\mathfrak{P}^{t,w_1}(JLOST) = 1 - 1/n > \mathfrak{l} = \mathfrak{P}^{t,w_0}(JLOST)$.

However, it seems quite clear that Tom in w_0 has *more justification* for believing $JLOST$ than he has in w_1, and so (MDPJ) fails. Moreover, it also seems quite clear that Tom in w_0 has a justification for believing $JLOST$ that he *does not have* in w_1, a justification that does not depend on any justification for believing that the lottery is fair and has a certain number of tickets.[26]

6 Confirming the Less Likely

Let's now proceed to see how the style of counterexample offered in section 5 against (MDPJ) can be used to buttress move (L_1). (MDPJ) has a natural *temporal* weakening:

(TDPJ) For every subject s, proposition P, world w and times t_0 and t_1, if in w at t_0 s has less justification for believing that P with $\mathfrak{P}^{s,w,t_0}(\langle P\rangle) = \mathfrak{r}_0$ than s has in w at t_1 with $\mathfrak{P}^{s,w,t_1}(\langle P\rangle) = \mathfrak{r}_1$, then $\mathfrak{r}_0 \leq \mathfrak{r}_1$.

And, with some ingenuity, the previous counterexample to (MDPJ) can be turned into a counterexample to (TDPJ).

Firstly, suppose that in a world w at a time t_0 before the drawing Tom's probability for the lottery's having n tickets is $\mathfrak{n}$ (let N be $\langle$The lottery has n tickets$\rangle$). Suppose also, merely for simplicity, that in w at t_0 Tom's probability for the lottery's having m tickets ($m < n$) is $1 - \mathfrak{n}$ (let M be $\langle$The lottery has m tickets$\rangle$: thus, for Tom in w at t_0 $N \vee M$ is certain). Then:

$$\begin{aligned}
\mathfrak{P}^{t,w,t_0}(JLOST) &= \mathfrak{P}^{t,w,t_0}((JLOST \wedge N) \vee (JLOST \wedge \neg N)) \\
&= \mathfrak{P}^{t,w,t_0}((JLOST \wedge N) \vee (JLOST \wedge M)) \\
&= \mathfrak{P}^{t,w,t_0}(JLOST \wedge N) + \mathfrak{P}^{t,w,t_0}(JLOST \wedge M) \\
&= \mathfrak{P}^{t,w,t_0}(JLOST/N)\mathfrak{P}^{t,w,t_0}(N) + \mathfrak{P}^{t,w,t_0}(JLOST/M)\mathfrak{P}^{t,w,t_0}(M) \\
&= \mathfrak{P}_{N}^{t,w,t_0}(JLOST)\mathfrak{P}^{t,w,t_0}(N) + \\
&\quad + \mathfrak{P}_{M}^{t,w,t_0}(JLOST)\mathfrak{P}^{t,w,t_0}(M) \qquad \text{by } \textbf{CONDITIONALISATION} \\
&= (1 - 1/n)\mathfrak{n} + (1 - 1/m)(1 - \mathfrak{n}) \qquad \text{by (L)}
\end{aligned}$$

Secondly, suppose that in w at a later time t_1 Tom goes on to attend the drawing of the lottery in the same way as he attends it in w_0, with the addition that the same experience

[26] Thanks to an anonymous referee for comments that led to a refinement of this example.

now also reveals to Tom that the number of tickets is m rather than n. Suppose also, merely for simplicity, that the evidence *EXPIWONM* provided by the experience is all the evidence got by Tom between t_0 and t_1, and that for Tom in w at t_1 it is certain that the experience constitutes a genuine apprehension of the state-of-affairs described by M although it is not certain that it constitutes a genuine apprehension of the state-of-affairs described by *IWON* (let *APPRIWON* be ⟨Tom's experience constitutes a genuine apprehension of the state-of-affairs described by *IWON*⟩). Let $\mathfrak{l}$ be a plausible value for $\mathfrak{P}^{t,w,t_0}_{EXPIWONM}(APPRIWON)$. Then:

$$\begin{aligned}
\mathfrak{P}^{t,w,t_1}(JLOST) &= \mathfrak{P}^{t,w,t_0}_{EXPIWONM}(JLOST)\\
&= \mathfrak{P}^{t,w,t_0}_{EXPIWONM}((JLOST \wedge APPRIWON) \vee (JLOST \wedge \neg APPRIWON))\\
&= \mathfrak{P}^{t,w,t_0}_{EXPIWONM}(JLOST \wedge APPRIWON) + \mathfrak{P}^{t,w,t_0}_{EXPIWONM}(JLOST \wedge \neg APPRIWON)\\
&= \mathfrak{P}^{t,w,t_0}_{EXPIWONM}(JLOST/APPRIWON)\mathfrak{P}^{t,w,t_0}_{EXPIWONM}(APPRIWON) +\\
&\quad + \mathfrak{P}^{t,w,t_0}_{EXPIWONM}(JLOST/\neg APPRIWON)\mathfrak{P}^{t,w,t_0}_{EXPIWONM}(\neg APPRIWON),
\end{aligned}$$

and so, given that $\mathfrak{P}^{t,w,t_0}_{EXPIWONM}(JLOST/APPRIWON) = 1$:

$$\begin{aligned}
&= \mathfrak{P}^{t,w,t_0}_{EXPIWONM}(APPRIWON) +\\
&\quad + \mathfrak{P}^{t,w,t_0}_{EXPIWONM}(JLOST/\neg APPRIWON)\mathfrak{P}^{t,w,t_0}_{EXPIWONM}(\neg APPRIWON)\\
&= \mathfrak{l} + (1 - 1/m)(1 - \mathfrak{l}) \qquad \text{by (L)}
\end{aligned}$$

Clearly, given a reasonable value for $\mathfrak{l}$, m, n and $\mathfrak{n}$ can be assigned reasonable values such that:

$$\begin{aligned}
\mathfrak{P}^{t,w,t_0}(JLOST) &= (1 - 1/n)\mathfrak{n} + (1 - 1/m)(1 - \mathfrak{n})\\
&> \mathfrak{l} + (1 - 1/m)(1 - \mathfrak{l})\\
&= \mathfrak{P}^{t,w,t_1}(JLOST)
\end{aligned}$$

For example, let $\mathfrak{l} = .9$, $m = 2$, $n = 100$ and $\mathfrak{n} = .95$. Then:

$$\begin{aligned}
\mathfrak{P}^{t,w,t_0}(JLOST) &= (1 - 1/n)\mathfrak{n} + (1 - 1/m)(1 - \mathfrak{n})\\
&= (.99 \times .95) + (.5 \times .05)\\
&= .9405 + .025\\
&= .9655\\
&> .95\\
&= .9 + .05\\
&= .9 + (.5 \times .1)\\
&= \mathfrak{l} + (1 - 1/m)(1 - \mathfrak{l})\\
&= \mathfrak{P}^{t,w,t_1}(JLOST)
\end{aligned}$$

However, it again seems quite clear that Tom at t_0 has *less justification* for believing *JLOST* than he has at t_1, and so (TDPJ) fails. Moreover, it also seems quite clear that Tom at t_1 has a justification for believing *JLOST* that he *does not have* at t_0, a justification that does not depend on any justification for believing that the lottery is fair and has a certain number of tickets.[27]

7 Dogmatically Confirming the Less Likely

It now remains to make explicit the connection between the failure of (TDPJ) observed in section 6 with move (L_1). Clearly, if Tom at t_1 has a justification for believing *JLOST* that he does not have at t_0, Tom at t_1 has acquired a new justification for believing *JLOST*, although his probability for *JLOST* has gone down. Given this, it is very unclear what remains of the objection from Lowering. Acquiring a new justification for *H* on the basis of *E* has just been shown (by Tom's acquiring a new justification for *JLOST* on the basis of *EXPIWONM*) to be in general compatible with *E* lowering the probability of *H*—why shouldn't this also happen in the specific case of acquiring a new justification for ¬*DECEIT* on the basis of *EXPCUBE*?

In fact, it is not just that there are counterexamples to (TDPJ); the style of counterexample offered in section 6 precisely exemplifies an *abstract structure* that it is natural for the dogmatist$^{\surd}$ to think to be also exemplified in the case of the argument from *CUBE* to ¬*DECEIT*:

(AS_1) *DECEIT* is a *very specific* hypothesis (it has to be, if it is to entail *EXPCUBE*). So, before having the experience as of a cube, it is plausibly taken to be *quite unlikely on statistical grounds* (as spelt out by some suitably relaxed principle of indifference or frequency), given one's good evidence concerning the minority of experiences exactly like that described by *EXPCUBE* (and so, conversely, ¬*DECEIT* is plausibly taken to be quite likely)—just like, before having the experience as of ticket #*i* being drawn among *m* tickets, ¬*JLOST* is plausibly taken to be quite unlikely on statistical grounds (as spelt out by (L)), given one's good evidence concerning the number of tickets (and so, conversely, *JLOST* is plausibly taken to be quite likely).

(AS_2) Upon having the experience as of a cube, those specific statistical grounds are *completely defeated* by *EXPCUBE* becoming certain—just like, upon having the

[27] Kung [2010] proposes and studies broadly similar but much weaker examples concerning acquisition of *reason for being confident to a certain extent* rather than of *justification for believing*. Vogel [2013] proposes and studies broadly similar but slightly weaker examples concerning *acquisition of* but not *improvement in* justification. Weatherson [2013] critically discusses what is essentially the same example but concerning *learning* rather than *acquisition of justification* (alongside other examples concerning *acquisition of knowledge*). A discussion of the approaches developed in those papers to the relevant examples will have to wait for another occasion.

experience as of ticket #*i* being drawn among *m* tickets, those other specific statistical grounds are completely defeated by *EXPIWONM* (and hence *M*) becoming certain.

(AS_3) While defeating those grounds, the experience as of a cube does however also provide a jolly good justification *of a different kind* for believing ¬*DECEIT*, for it provides a straightforward perceptual justification for believing *CUBE*, which obviously entails ¬*DECEIT*—just like, while defeating those other grounds, the experience as of ticket #*i* being drawn among *m* tickets does however also provide a jolly good justification of a different kind for believing *JLOST*, for it provides a straightforward perceptual justification for believing *IWON*, which obviously entails *JLOST*.[28]

[28] An interesting disanalogy between the two cases is that, while the argument from *IWON* to *JLOST* sounds perfectly *fine* (if the justification offered for *IWON* is the experience as of ticket #*i* being drawn among *m* tickets), the argument from *CUBE* to ¬*DECEIT* sounds pretty *funny* (if the justification offered for *CUBE* is the experience as of a cube). I've already offered in section 3 one explanation of the perceived funniness. However, that explanation is arguably *partial* because it applies *equally well* to the argument from *CUBE* to ⟨There exists an external world⟩ (if the justification offered for *CUBE* is the experience as of a cube), which too does sound a bit funny, but noticeably less so than the analogous argument concluding to ¬*DECEIT* instead. What I'm arguing in the text is that it is not a good explanation of the additional funniness to say that (TN) fails in the case of the latter argument because the relevant ¬*DECEIT*-instance of Lowering is true while the relevant ⟨There exists an external world⟩-instance of Lowering is not. (That this is not a good explanation, and, more strongly, that one who accepts (TN) for the argument from *CUBE* to ⟨There exists an external world⟩ could equally accept it for the argument from *CUBE* to ¬*DECEIT* is already indicated by the fact that the argument from *CUBE* to ⟨There exists an external world and it is not the case that [a global deceit makes it look to one as though there were a cube when in fact there is no external world]⟩ sounds just as funny as the argument from *CUBE* to ¬*DECEIT*, but the conclusion of the latter argument is equivalent with ⟨There exists an external world⟩. Hence, the relevant ⟨There exists an external world and it is not the case that [a global deceit makes it look to one as though there were a cube when in fact there is no external world]⟩-instance of Lowering is not true, and, more strongly, (TN) should not discriminate between that argument and the argument from *CUBE* to ⟨There exists an external world⟩ (since they are identical up to equivalence), from which it follows that the fact that also the argument from *CUBE* to ¬*DECEIT* sounds pretty funny should not be taken as a reason to think that (TN) fails in its case. Even more strongly, the fact that the argument from *CUBE* to ¬*DECEIT* sounds pretty funny should arguably not be taken as pointing to *any* kind of epistemic flaw in the argument. For absolutely every proposition *P*, the argument from ⟨*P*⟩ to ⟨*P* or it is not the case that a global deceit makes it look to one as though *P*⟩ sounds just as funny as the argument from *CUBE* to ¬*DECEIT*, but I think that, on reflection, we should all agree that many such arguments have no epistemic flaw.) A better explanation, I think, will start by observing the quite general fact that acceptance (in the sense of *explicit acceptance*) of a proposition ⟨*P* or *Q*⟩ is standardly taken to imply (in the sense of *involving*) that one's grounds for accepting it (defeasibly) licence an *inference* from ⟨It is not the case that *P*⟩ (from ⟨It is not the case that *Q*⟩) to ⟨*Q*⟩ (to ⟨*P*⟩)—that is, that one's grounds for accepting it are strong enough as to *survive* (typical) refutations of either disjunct. Given the relevant De Morgan equivalence, a similar phenomenon occurs also in the case of negated conjunctions: acceptance of a proposition ⟨It is not the case that [*P* and *Q*]⟩ is standardly taken to imply that one's grounds for accepting it (defeasibly) licence an inference from ⟨*P*⟩ (from ⟨*Q*⟩) to ⟨It is not the case that *Q*⟩ (to ⟨It is not the case that *P*⟩)—that is, that one's grounds for accepting it are strong enough as to survive (typical) proofs of either conjunct. Now, it is arguable that offending propositions like ¬*DECEIT* need to "have something like" the form of a negated conjunction, for their negation needs to be such as, on the one hand, to be in *tension* (typically, be straightforwardly *incompatible*) with a proposition like *CUBE* and,

(AS_4) Assuming the probabilistic necessary condition on justification:

($JP^{\geq .5}$) If s has a justification for believing that P, $\mathfrak{P}^s(\langle P \rangle) \geq .5$,

the experience as of a cube *also* makes $\neg$*DECEIT* still more likely than not, but it does not make it as likely as the defeated statistical grounds made it—just like the experience as of ticket #*i* being drawn among *m* tickets also makes *JLOST* still more likely than not, but it does not make it as likely as the defeated statistical grounds made it.

(AS_5) Since the justification that the experience as of a cube provides for believing $\neg$*DECEIT* is that represented by the perceptual presentation of a state-of-affairs (described by *CUBE*) which obviously entails $\neg$*DECEIT*, that justification is nevertheless not only different from but also *better* than any justification represented by statistical grounds—just like, since the justification that the experience as of ticket #*i* being drawn among *m* tickets provides for believing *JLOST* is that represented by the perceptual presentation of a state-of-affairs (described by *IWON*) which obviously entails *JLOST*, that justification is nevertheless not only different from but also better than any justification represented by statistical grounds.

It is in view of this striking correspondences between the independently motivated style of counterexample to (TDPJ) offered in section 6 and a natural way for the dogmatist$^{\surd}$ to think about the argument from *CUBE* to $\neg$*DECEIT* that I believe we can conclude that move (L_1) is perfectly warranted for the dogmatist$^{\surd}$.

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on the other hand, to be *coherent* with (typically, straightforwardly *entail*) a proposition like *EXPCUBE*. Given these circumstances, the dogmatist can offer a satisfactory explanation of the additional funniness attached to the argument from *CUBE* to $\neg$*DECEIT* (if the justification offered for *CUBE* is the experience as of a cube). For, as has just been observed, acceptance of $\neg$*DECEIT* is standardly taken to imply that one's grounds for accepting it (defeasibly) licence an inference from ⟨A global deceit makes it look to one as though there were a cube⟩ (from $\neg$*CUBE*) to *CUBE* (to ⟨It is not the case that a global deceit makes it look to one as though there were a cube⟩)—that is, that one's grounds for accepting it are strong enough as to survive (typical) proofs of either conjunct. However, according to the dogmatist, if the justification offered for *CUBE* is the experience as of a cube, one's grounds for accepting $\neg$*DECEIT* are *nothing like that*: in particular, they do not (even defeasibly) licence an inference from $\neg$*CUBE* to ⟨It is not the case that a global deceit makes it look to one as though there were a cube⟩—that is, one's grounds for accepting $\neg$*DECEIT* are not strong enough as to survive (typical) proofs of $\neg$*CUBE*. (Typical anti-dogmatists would disagree with these claims, at least in the specific case in which the offending proposition is $\neg$*DECEIT*.) In fact, one's grounds for accepting $\neg$*DECEIT* would be even more dramatically discredited by a (typical) proof of $\neg$*CUBE*, for the truth of ⟨A global deceit makes it look to one as though there were a cube⟩ would actually explain why $\neg$*CUBE* is true in spite of one's experience as of a cube. Thus, according to the dogmatist, the argument from *CUBE* to $\neg$*DECEIT* (if the justification offered for *CUBE* is the experience as of a cube) *quite dramatically contradicts a standard implication* associated with acceptance of $\neg$*DECEIT*, and this can plausibly be taken as a satisfactory explanation of the additional funniness attached to that argument. (Notice that, as against many other alternatives—some of which have been mentioned in this paper—and assuming that acceptance of a proposition ⟨*P* and *Q*⟩ is standardly taken to imply that one accepts both ⟨*P*⟩ and ⟨*Q*⟩, such explanation has the virtue of smoothly extending to those cases in which the offending proposition is *equivalent* with *CUBE*, like *CUBE* $\wedge$ $\neg$*DECEIT*.) Thanks to Filippo Ferrari, Eugenio Orlandelli, Sven Rosenkranz and Martin Smith for urging me to consider these issues.

8 New and First Justification

I've been arguing that the relevant instance of (TN) is compatible with LOWERING. There is however a slightly stronger transmission principle for justification that has implications about *first* rather than simply *new* justifications:

(TF) If one has a justification for believing that P_0, one has a justification for believing that P_1, one has a justification for believing that P_2... and one knows that $\langle P_0 \rangle$, $\langle P_1 \rangle$, $\langle P_2 \rangle$... entail $\langle Q \rangle$, in virtue of this one can have a first justification for believing that Q.

Now, from a certain appealing dogmatist perspective, (TF) too actually does not fail in the case of MOORE and its likes, and so, in an even stronger sense, MOORE and its likes are perfectly good anti-sceptical arguments (let's dub this view—another strengthening of dogmatism along the axis of claims about the acquisition and presence of justification—'*dogmatism*$^{\surd\surd}$').[29] In particular, the dogmatist$^{\surd\surd}$ thinks that one can use MOORE to acquire a first justification for believing the negation of a sceptical hypothesis that only depends on a dogmatic justification for believing (M_1), on a reflective justification for believing (M_2) and on knowledge of *modus ponens*.

Is the relevant instance of (TF) compatible with LOWERING? It is not given both ($JP^{\geq .5}$) and its converse:

($PJ^{\geq .5}$) If $\mathfrak{P}^s(\langle P \rangle) \geq .5$, s has a justification for believing that P,

and, more generally, for a threshold $\mathfrak{t}$, given both analogues of ($JP^{\geq .5}$) and ($PJ^{\geq .5}$) with $\mathfrak{t}$ substituted for .5 (the higher $\mathfrak{t}$, the stronger ($JP^{\geq \mathfrak{t}}$) and the weaker ($PJ^{\geq \mathfrak{t}}$); any such pair amounts in effect to $\mathfrak{t}$'s being the *probabilistic threshold for justification*).[30] For, by ($JP^{\geq \mathfrak{t}}$),

[29] I emphasise that dogmatism$^{\surd}$ (and hence dogmatism) does not entail dogmatism$^{\surd\surd}$ (while the converse entailment obviously holds, since any first justification is obviously a new justification): a dogmatist$^{\surd}$ can maintain that the existence of a justification for believing the premises of MOORE and its likes and knowledge of such arguments' validity entails the existence of an independent justification for believing the arguments' conclusions, and that, nevertheless, MOORE and its likes can be used so as to acquire a justification for believing their conclusions *additional to the one entailed to be independently had*. This also makes clear, however, that, although neither dogmatism$^{\surd}$ nor dogmatism$^{\natural}$ entails dogmatism$^{\surd\surd}$, dogmatism$^{\surd\natural}$ does (throughout, concatenation of different superscripts denotes conjunction of the corresponding characteristic claims).

[30] Notice that acceptance of a probabilistic threshold for justification is perfectly compatible with rejection of (MDPJ) and (TDPJ), and, more generally, with move (L_1) in reply to the objection from LOWERING. For that move only requires that *justification* not be *reduced* to (or, in any event, *identified* with) probability, lest *degrees* of justification be reduced to (or, in any event, identified with) *degrees* of probability. A probabilistic threshold for justification need not however have any such implication: rather than the reflection of an underlying *identity*, it may be the reflection of an underlying deep *connection* between two distinct (and, as witnessed by the counterexamples to (MDPJ) and (TDPJ), sometimes dramatically diverging) properties. Notice also that acceptance of a probabilistic threshold for justification will be an essential component of one of the strengthenings of dogmatism along the axis of claims about the relationships between justification (including its defeat) and probability which will properly be introduced in section 11.

one's justification for believing $\neg$*DECEIT* requires one's probability for $\neg$*DECEIT* upon getting *EXPCUBE* to be $\geq$ t, and, by Lowering and **CONDITIONALISATION**, that implies that one's prior probability for $\neg$*DECEIT* is also $\geq$ t, so that, by ($PJ^{\geq t}$), one already has a justification for believing $\neg$*DECEIT*.

However, while ($JP^{\geq t}$) is extremely plausible for a wide range for t, ($PJ^{\geq t}$), while appealing, is not completely uncontroversial for just about every t. One prominent reason for this is that one's probability for $\langle P \rangle$ may be $\geq$ t in virtue of *merely statistical grounds* and it is in general not completely uncontroversial whether a merely statistical ground favouring $\langle P \rangle$ ever suffices to provide a justification for believing that *P* (rather than simply a justification for believing that *it is likely that P*, see e.g. Nelkin [2000]). If it does not, then the way is open to the dogmatist$^{\surd\surd}$ in effect to appropriate move (C_1) for her own predicament and claim—with an argument exactly analogous to the one I've given in sections 6 and 7—that even the relevant instance of (TF) is true in spite of the truth of Lowering.[31] And even if it does, the argument I gave in those sections (and especially point (AS_1)) clearly indicates a dogmatist$^{\surd}$-friendly way for the relevant instance of (TF) to fail. It fails because it is part of the set-up that, even before having an experience as of a cube, $\neg$*DECEIT* is plausibly taken to be quite likely on statistical grounds (as spelt out by some suitably relaxed principle of indifference or frequency), given one's good evidence concerning the minority of experiences like that described by *EXPCUBE*, and the relevant ($PJ^{\geq t}$)-principle will now suffice to turn those grounds into a justification for believing $\neg$*DECEIT*.

Doesn't all this depend on an unnecessary feature of the particular example (i.e. that, even before having an experience as of a cube, $\neg$*DECEIT* is plausibly taken to be quite likely on statistical grounds)? No. If the objector tried to take a conclusion whose probability before the relevant experience is $\sim .5$, she would have to take its negation also to have probability before the experience $\sim .5$. But that is incompatible with one's having a justification for believing the premise of the relevant instance of (TF) for at least two (related) reasons (which would obviously apply with even greater force if the objector ingenuously tried to take a conclusion whose probability before the relevant experience is $\ll .5$):

(R_1) Since the negation of the conclusion is inconsistent with the premise of the relevant instance of (TF) (and entails the occurrence of the relevant experience), one would presumably have a defeater for one's justification for believing the premise of the relevant instance of (TF);

(R_2) By Capping and **CONDITIONALISATION**, the probability of the premise after having the relevant experience would at best be $\sim .5$, which, given many extremely plausible ($JP^{\geq t}$)-principles, would be incompatible with one's having a justification for believing the premise of the relevant instance of (TF).

[31] That being noted, given the appeal of a probabilistic sufficient condition for justification I'll henceforth set aside move (C_1). And, given the appeal of dogmatism$^{\surd}$ and dogmatism$^{\natural}$, I've already set aside moves (L_2) and (C_2). I stress however that, although they do fall outside of the course I'm steering in this paper, I consider all these moves interesting and worthy of future investigation.

9 Justification, Defeat and Probability

For all of its interest, the final part of the dialectic of section 8, predicated on the assumption of an appealing pair of a ($PJ^{\geq t}$)-principle and its corresponding ($JP^{\geq t}$)-principle, rescues dogmatism$^{\surd}$ but at the expense of forsaking dogmatism$^{\surd\surd}$. Dogmatism$^{\surd\surd}$ also entails dogmatism$^{\natural}$: if one can use the argument from *CUBE* to $\neg$*DECEIT* to acquire a first justification for believing $\neg$*DECEIT*, then one can have a justification for believing *CUBE* even though one has no independent justification for believing $\neg$*DECEIT*. And, although dogmatism$^{\natural}$ does not conversely entail dogmatism$^{\surd\surd}$ (see fn 29), given that $\mathfrak{P}_{EXPCUBE}(CUBE) \leq \mathfrak{P}_{EXPCUBE}(\neg DECEIT)$ the dialectic of section 8 clearly applies to dogmatism$^{\natural}$ just as well as to dogmatism$^{\surd\surd}$ (in fact, it then becomes a way of making sharp the objection from Capping, which was left unanswered in section 8). In view of these connections, I propose to step back for a while from the dialectic involving dogmatism$^{\surd\surd}$, which merely concerns the *dynamics* of probabilities (i.e. how probabilities evolve upon getting new evidence), and enter instead what I think is a more fundamental dialectic involving dogmatism$^{\natural}$, which concerns nothing less than the *structure itself* of probabilities (i.e. how probabilities are distributed on a σ-algebra of propositions at a given time).

Thus, one might think that my two-pronged (R_1)–(R_2)-reply in section 8, while answering the objection from Lowering for the dogmatist$^{\surd}$, exploits facts that are shown to be problematic for the dogmatist$^{\natural}$ by the objection from Capping. However, I think that the way I've exploited those facts—especially in the reply's prong (R_1)—also hints at the fact that, for at least one kind of dogmatist$^{\natural}$, the classical theory of probabilities distorts their real structure. For reflect that dogmatism in general is a view that draws an *absolutely sharp distinction*, for a proposition P, between *lacking* $\langle P \rangle$ *as a defeater* and *having a justification for believing that it is not the case that* P.[32] For example, according to dogmatism, one's having an all-things-considered justification for believing *CUBE* on the basis of an experience as of a cube depends on one's lacking *DECEIT* as a defeater, but does not depend on one's having a justification for believing $\neg$*DECEIT*. Dogmatism$^{\natural}$ strengthens the point slightly and maintains that that distinction is such

[32]Given that just about any justification for believing any proposition can be defeated in some way or other, the distinction in the text (or at least the possibly weaker distinction between lacking $\langle P \rangle$ as a defeater and having an all-things-considered justification for believing that it is not the case that P) should be drawn in at least some cases *by most epistemologies* already on *purely structural* grounds, on pain of accepting that all-things-considered justification for believing any proposition depends on all-things-considered justification for believing some other propositions, which would force the relation of (epistemic) dependence to be non-well-founded in some way or other. Moreover, the distinction in the text should be drawn in at least some cases by most epistemologies also on *more direct* grounds: for example, it is very intuitive that, while one's all-things-considered justification for believing that $2 + 2 = 4$ does depend on one's lacking ⟨One has taken a pill causing one to make dramatic arithmetical miscalculations⟩ as a defeater, it does not depend on one's having a justification for believing that one has not taken a pill causing one to make dramatic arithmetical miscalculations. If this is correct, then, under plausible assumptions, the whole dialectic of this paper will actually apply not just to dogmatism about perceptual justification, but to most epistemologies. Having made these suggestive remarks, a proper investigation of this issue must however be left for another occasion.

that, in the relevant cases, one's lacking $\langle P\rangle$ as a defeater does not even entail that one has a justification for believing that it is not the case that P. For example, according to dogmatism$^{\natural}$, one may have a justification for believing *CUBE* on the basis of one's experience as of a cube even though one has no independent justification for believing $\neg$*DECEIT*, but one merely lacks *DECEIT* as a defeater.

Now, in that example, assuming a plausible relative of (PJ$^{\geq .5}$) for defeaters:

(PD$^{\not\ll .5}$) If $\langle P\rangle$ is a defeater for a certain justification for believing that Q and $\mathfrak{P}^s(\langle P\rangle) \not\ll .5$, s has $\langle P\rangle$ as a defeater for that justification for believing that Q,

a defeater would be had by a subject s if $\mathfrak{P}^s(DECEIT) \not\ll .5$. Hence, s's lack of a defeater implies that $\mathfrak{P}^s(DECEIT) \ll .5$. But then, since the *classical* theory of probabilities has it that:

DIFFERENTIALITY $\mathfrak{P}(\neg X) = 1 - \mathfrak{P}(X)$,

it follows that $\mathfrak{P}^s(\neg DECEIT) \gg .5$. And, unless one is prepared to reject (PJ$^{\gg .5}$), that in turn implies that s has a justification for believing $\neg$*DECEIT*. Moreover, since the lack of a defeater must be determined by s's epistemic state independently of the experience as of a cube, (PD$^{\not\ll .5}$) and (PJ$^{\gg .5}$) are highly plausibly interpreted as implying, in the framework of the classical theory, that s's justification for believing $\neg$*DECEIT* is independent of the experience as of a cube. And this contradicts dogmatism$^{\natural}$. The argument of course generalises to other appealing pairs of a (PD$^{>t}$)-principle and its corresponding (PJ$^{\geq 1-t}$)-principle. By (PD$^{>t}$), a defeater would be had by s if $\mathfrak{P}^s(DECEIT) > t$. Hence, s's lack of a defeater implies that $\mathfrak{P}^s(DECEIT) \not> t$. But then, by **DIFFERENTIALITY**, it follows that $\mathfrak{P}^s(\neg DECEIT) \geq 1 - t$. And, unless one is prepared to reject (PJ$^{\geq 1-t}$), that in turn implies that s has a justification for believing $\neg$*DECEIT*. Moreover, since the lack of a defeater must be determined by s's epistemic state independently of the experience as of a cube, (PD$^{>t}$) and (PJ$^{\geq 1-t}$) are highly plausibly interpreted as implying, in the framework of the classical theory, that s's justification for believing $\neg$*DECEIT* is independent of the experience as of a cube. And this contradicts dogmatism$^{\natural}$.

Now, from a certain appealing dogmatist perspective, at least one pair of a (PD$^{>t}$)-principle and its corresponding (PJ$^{\geq 1-t}$)-principle are true (let's dub this view—a strengthening of dogmatism along the axis of claims about the relationships between justification (including its defeat) and probability—'*dogmatism*$^{\flat}$'). Thus, while the objections from Lowering and Capping—relying as they do on **CONDITIONALISATION**—merely bring out a problem for how the dogmatist$^{\surd}$ and the dogmatist$^{\natural}$ conceive of the *dynamics* of probabilities, the argument just given—relying as it does only on **DIFFERENTIALITY** (plus (PD$^{>t}$) and (PJ$^{\geq 1-t}$))—shows that a dogmatist$^{\natural\flat}$ cannot but deny that the *structure itself* of probabilities is correctly described by the classical theory of probabilities. In particular, the argument just given shows that, according to the dogmatist$^{\natural\flat}$, **DIFFERENTIALITY** should fail, and so, under the minimal assumption that $\mathfrak{P}(X \vee \neg X) = 1$:

ADDITIVITY If X and Y are incompatible, $\mathfrak{P}(X \vee Y) = \mathfrak{P}(X) + \mathfrak{P}(Y)$

should fail too. For instance, in the example just considered, since s lacks a defeater $\mathfrak{P}^s(DECEIT) \ll .5$ (by (PD$^{\not\ll .5}$)), but, since s also does not have a justification for believing $\neg DECEIT$, $\mathfrak{P}^s(\neg DECEIT) \not\gg .5$ (by (PJ$^{\gg .5}$)), and so **DIFFERENTIALITY** (and hence **ADDITIVITY**) fails.

10 A Theory of Non-Additive Probabilities

A theory of non-additive probabilities hospitable to dogmatism$^{\natural\flat}$ is represented by the *Dempster-Shafer theory of probabilities* (see e.g. Shafer [1976]). Very interestingly, it will turn out that developing this theory in a particular way with respect to learning *also* yields a framework hospitable to dogmatism$^{\surd\surd}$.

For our purposes, it will be useful to take as basic the notion of a *mass*. Given a finite set U, a mass on U is a function $\mathfrak{M} : \wp(U) \mapsto \mathbb{R}$ such that:

(1) $\mathfrak{M}(\emptyset) = 0$;

(2) $\sum\limits_{X \subseteq U} (\mathfrak{M}(X)) = 1$.

In our context, a mass can be taken to represent the degree to which in a given subject's epistemic state at a given time there are *reasons* in favour of a hypothesis considered *in its specificity* (i.e. not because of its being a weaker consequence of a hypothesis in whose favour there are certain reasons). Probabilities can then be defined in terms of masses. Given a finite set U and a mass $\mathfrak{M}$ on U, a DS *probability function* on U is a function $\mathfrak{D}^{\mathfrak{M}} : \wp(U) \mapsto \mathbb{R}$ such that:

(DS) $\mathfrak{D}^{\mathfrak{M}}(X) = \sum\limits_{Y \subseteq X} (\mathfrak{M}(Y))$.

If $\mathfrak{D}$ is a DS probability function on U, the following fundamental properties are easily derivable from (DS):[33]

[33] I emphasise that, typically, in the literature on the Dempster-Shafer theory of probabilities properties (i)–(iv) (or equivalents thereof) are actually taken as *definitional* of a DS probability function. As I've said, however, for our purposes it will be more useful to take instead as basic the notion of a mass, and define a DS probability function in terms of it by means of (DS). (The difference is immaterial if—as I'm assuming—the members of the elements of $\mathcal{S}$ are *finitely many*, for then every DS probability function directly defined in terms of properties (i)–(iv) can also be defined in terms of the notion of a mass (setting $\mathfrak{M}(X) = \sum\limits_{Y: Y \subseteq X} (-1^{|X|-|Y|}\mathfrak{D}(Y))$). The difference is however of some consequence if the members of the elements of $\mathcal{S}$ are *infinitely many*, for then, although it is still the case that every mass defines by means of (DS) a DS probability function with properties (i)–(iv), it is no longer the case that every DS probability function directly defined in terms of properties (i)–(iv) can also be defined in terms of the notion of a mass. For this reason, I'm working under the simplifying but innocuous assumption that the members of the elements of $\mathcal{S}$ are finitely many.)

(i) $\mathfrak{D}(\emptyset) = 0$;

(ii) $\mathfrak{D}(U) = 1$;

(iii) If $X \subseteq Y$, then $\mathfrak{D}(X) \leq \mathfrak{D}(Y)$;

(iv) For every $X_0, X_1, X_2 \ldots, X_n \subseteq U$,
$$\mathfrak{D}\Big(\bigcup_{0\leq m\leq n}(X_m)\Big) \geq \sum_{\emptyset\neq I\subseteq\{0,1,2\ldots,n\}}\Big((-1)^{|I|+1}\mathfrak{D}\Big(\bigcap_{i\in I}(X_i)\Big)\Big).$$

Note in particular that property (iv) yields as a special case:

SUPER-ADDITIVITY If X and Y are incompatible, $\mathfrak{D}(X \vee Y) \geq \mathfrak{D}(X) + \mathfrak{D}(Y)$,

and as a consequence of **SUPER-ADDITIVITY** and property (ii):

SUB-DIFFERENTIALITY $\mathfrak{D}(\neg X) \leq 1 - \mathfrak{D}(X)$.

Thus, while in the classical theory of probabilities there is an *equality* between the probability of a disjunction of incompatible propositions and the sum of the probabilities of its disjuncts (**ADDITIVITY**), in the Dempster-Shafer theory there is only an *inequality*, with the probability of the disjunction only bounding from above the sum of the probabilities of its disjuncts (**SUPER-ADDITIVITY**): such sum cannot be higher than the probability of the disjunction but it can be (and typically is) lower. As a consequence, while in the classical theory there is an *equality* between the probability of a negation and the difference between 1 and the probability of its *negatum* (**DIFFERENTIALITY**), in the Dempster-Shafer theory there is only an *inequality*, with the probability of the negation only bounding from below the difference between 1 and the probability of its *negatum* (**SUB-DIFFERENTIALITY**): such difference cannot be lower than the probability of the negation but it can be (and typically is) higher.

It is thanks to the features represented by **SUPER-ADDITIVITY** and **SUB-DIFFERENTIALITY** that the Dempster-Shafer theory of probabilities is much more hospitable to dogmatism$^{\natural\flat}$ than the classical theory is. For instance, in the example considered in section 9, $\mathfrak{D}^s(\mathit{DECEIT})$ can be $\ll$ .5 (so that s can lack a defeater without violating (PD$^{\not\ll .5}$)) while $\mathfrak{D}^s(\neg \mathit{DECEIT})$ can also be $\not\gg$ .5 (so that s can lack a justification for believing $\neg \mathit{DECEIT}$ without violating (PJ$^{\gg .5}$)). Indeed, it is a great virtue of the Dempster-Shafer theory that, thanks to **SUPER-ADDITIVITY** and **SUB-DIFFERENTIALITY**, it allows us to represent the probabilistic features of a typical situation—as conceived of by the dogmatist$^{\natural\flat}$—before s has an experience as of a cube: a situation where there is *absolutely no danger of defeat* from *DECEIT* (so that $\mathfrak{D}^s(\mathit{DECEIT}) = 0$) while there is still *absolutely no reason in favour* of $\neg \mathit{DECEIT}$ (so that $\mathfrak{D}^s(\neg \mathit{DECEIT}) = 0$ too).

11 Dempsterian Dogmatic Learning

It is now time to connect the dialectic about the dogmatist$^{\natural\flat}$'s conception of the structure of probabilities developed in sections 9 and 10 with the dialectic about the dogmatist$^{\surd\surd}$'s conception of the dynamics of probabilities developed in section 8. To do so, we must ask how *learning* should proceed in a dogmatist$^{\natural\flat}$ framework adopting the Dempster-Shafer theory of probabilities. I should stress right at the outset of this section that I'm not presupposing that this question has a uniform general answer, and that in any event I'm not trying to determine what the *general* answer (uniform or non-uniform) to the question is; in this paper, I'm only interested in the much more modest project of providing a theory of learning which fits well into a dogmatist$^{\natural\flat}$ framework adopting the Dempster-Shafer theory *in the cases of learning from experience relevant for our context* (with an eye at developing a theory that, at least in such cases, is not only dogmatist$^{\natural\flat}$-, but also dogmatist$^{\surd\surd}$-friendly).

One might think that **CONDITIONALISATION** carries over its high plausibility from the classical theory of probabilities to the Dempster-Shafer theory, but this is not so, since, on this approach, by **CONDITIONAL PROBABILITY** the posterior DS probability function would only be defined if the probability of the evidence according to the prior DS probability function is > 0. This kind of limitation is of course familiar from the classical theory. However, while the limitation is arguably not so crippling in the classical case (since, on typical applications of the theory, *very few if any* propositions that potentially are pieces of evidence should ever be assigned probability 0), it is doubtlessly devastating in the case of the Dempster-Shafer theory (since, as was illustrated in section 10, on typical applications of the theory *very many* propositions that potentially are pieces of evidence should often—i.e. in all contexts in which there are no reasons in favour of them—be assigned probability 0).

Another approach would be to use a super-additive analogue of the classical *primitive* conditional-probability functions as introduced for example by Rényi [1955]. In our context, such primitive conditional-probability functions are however objectionable in that, at least in the case in which the evidence has prior probability 0, they completely disregard the information contained in the prior DS probability function about propositions which are compatible with but not entailed by the evidence.

A better approach takes its lead from the fact that every DS probability function $\mathfrak{D}$ can be thought of as the *lower probability* $\mathscr{P}^{\downarrow}$ of the set of classical probability functions $\mathscr{P} = \{\mathfrak{P} : \mathfrak{P}(X) \geq \mathfrak{D}(X)\}$ (that is, $\mathscr{P}^{\downarrow}(X) = \mathrm{glb}(\{\mathfrak{P}(X) : \mathfrak{P} \in \mathscr{P}\})$). One can then take $\mathfrak{D}_E$ to be the lower probability of the set $\mathscr{P}_E = \{\mathfrak{P}_E : \mathfrak{P} \in \mathscr{P}$ and $\mathfrak{P}(E) \neq 0\}$, with learning in that set going by **CONDITIONALISATION**. Although such approach does allow the probability of the evidence according to the prior DS probability function to be = 0 and does not disregard the information contained in the prior DS probability function about propositions which are compatible with but not entailed by the evidence, there are at least three problems with its use in our context.

Firstly, the approach yields counterintuitive results *in many cases in which one gets* *good although not conclusive evidence that some of the classical probability functions in the prior set are misguided*. A glaring example of this is offered precisely by our context. If we try to preserve the idea, dear to the dogmatist$^{\natural\flat}$'s conception of the structure of probabilities developed in sections 9 and 10, that $\mathscr{P}^{\downarrow}(\neg DECEIT) = 0$, then, whether we achieve this by including in $\mathscr{P}$ a classical probability function assigning probability 0 to $\neg DECEIT$ or by only including in $\mathscr{P}$ classical probability functions assigning arbitrarily low positive probability to $\neg DECEIT$, we are stuck with the consequence that $\mathscr{P}^{\downarrow}_{EXPCUBE}(CUBE) \leq \mathscr{P}^{\downarrow}_{EXPCUBE}(\neg DECEIT) = 0$, which contradicts dogmatism (at least assuming the extremely plausible ($\mathrm{JP}^{>0}$)). The argument generalises to a low enough positive value $\mathfrak{r}$, since we are then stuck with the consequence that $\mathscr{P}^{\downarrow}_{EXPCUBE}(CUBE) \leq \mathscr{P}^{\downarrow}_{EXPCUBE}(\neg DECEIT) \leq \mathfrak{r}$, which contradicts dogmatism (at least assuming the plausible ($\mathrm{JP}^{>\mathfrak{r}}$)).

Secondly, even if we decide to abandon the idea that $\mathscr{P}^{\downarrow}(\neg DECEIT) = 0$ or even the idea that $\mathscr{P}^{\downarrow}(\neg DECEIT)$ is reasonably low, *instances of the problematics arising from* Lowering *and* Capping would still be with us. For reflect that condition (EHH^{*}_{2}) makes clear that any classical probability function $\mathfrak{P}$ which both is in $\mathscr{P}_{EXPCUBE}$ and does not satisfy Lowering is such that either $\mathfrak{P}(EXPCUBE) = 1$ (in which case it follows that $\mathfrak{P}_{EXPCUBE}(\neg DECEIT) = \mathfrak{P}(\neg DECEIT)$) or $\mathfrak{P}(DECEIT) = 0$ (in which case the more specific consequence follows that $\mathfrak{P}_{EXPCUBE}(\neg DECEIT) = \mathfrak{P}(\neg DECEIT) = 1$), and so such that $\mathfrak{P}_{EXPCUBE}(\neg DECEIT) \leq \mathfrak{P}(\neg DECEIT)$. Thus, every classical probability function in $\mathscr{P}_{EXPCUBE}$ is such that $\mathfrak{P}_{EXPCUBE}(\neg DECEIT) \leq \mathfrak{P}(\neg DECEIT)$, and so $\mathscr{P}^{\downarrow}_{EXPCUBE}(\neg DECEIT) \leq \mathscr{P}^{\downarrow}(\neg DECEIT)$. Consider now an appealing pair of a ($\mathrm{PJ}^{\geq \mathfrak{t}}$)-principle and its corresponding ($\mathrm{JP}^{\geq \mathfrak{t}}$)-principle. By ($\mathrm{PJ}^{\geq \mathfrak{t}}$), a subject s would have a justification for believing $\neg DECEIT$ if $\mathscr{P}^{\downarrow s}(\neg DECEIT) \geq \mathfrak{t}$—indeed would highly plausibly have such a justification independently of the experience as of a cube, given that $\mathscr{P}^{\downarrow s}$ correctly describes the degrees of support that s's epistemic state before that experience lends to the propositions in $\mathcal{S}$. Hence, s's lack of an independent justification for believing $\neg DECEIT$ implies that $\mathscr{P}^{\downarrow s}(\neg DECEIT) < \mathfrak{t}$. But then, since we've just established that $\mathscr{P}^{\downarrow s}_{EXPCUBE}(\neg DECEIT) \leq \mathscr{P}^{\downarrow s}(\neg DECEIT)$, it follows that $\mathscr{P}^{\downarrow s}_{EXPCUBE}(CUBE) \leq \mathscr{P}^{\downarrow s}_{EXPCUBE}(\neg DECEIT) < \mathfrak{t}$. And, unless one is prepared to reject ($\mathrm{JP}^{\geq \mathfrak{t}}$), that in turn implies that, upon having the experience as of a cube, s still has no justification for believing *CUBE*. Now, from a certain appealing dogmatist perspective, at least one triple of a ($\mathrm{PJ}^{\geq \mathfrak{t}}$)-principle and its corresponding ($\mathrm{PD}^{>1-\mathfrak{t}}$)-principle *and* ($\mathrm{JP}^{\geq \mathfrak{t}}$)-principle are true (let's dub this view—another strengthening of dogmatism along the axis of claims about the relationships between justification (including its defeat) and probability—'*dogmatism*$^{\flat\flat}$'). We thus have it that the lower-probability approach contradicts dogmatism$^{\natural\flat\flat}$.[34]

[34]Clearly, it is just one of the essential components of dogmatism$^{\flat\flat}$—the acceptance of a probabilistic threshold for justification—that is really at work in the argument in the text. Probabilistic thresholds for justification have already made their appearance in section 8 and, as the dialectic of that section makes clear, in the framework of Bayesian confirmation theory such thresholds are incompatible both with dogmatism$^{\surd\surd}$ and with dogmatism$^{\natural}$. Also, the other essential component of dogmatism$^{\flat\flat}$—the acceptance of a probabilistic sufficient condition for defeat set at the difference between 1 and the

Thirdly, the approach carries over from the previous flat-footed Bayesian framework we've worked with up to section 8 the modelling of learning from experience as consisting in a proposition about one's *experience* becoming *certain* and *indirectly* making *likely* a proposition about one's *surroundings*. That is at best unnatural for a dogmatist. The modelling strongly suggests (if not implies) that the proposition about one's surroundings becoming likely (as every other relevant change in one's epistemic state) depends on the proposition about one's experience becoming certain, and thus, henceforth assuming that facts about justification correlate in a reasonably tight way with facts about probability, that the proposition about one's surroundings becoming justified depends on the proposition about one's experience becoming (maximally?) justified. That would seem to contradict the dogmatist idea that the justification provided by the experience does not depend on any other justification one may have.

This objection may invite the conciliatory (and relatively uninteresting) rejoinder to the effect that, while there is in effect a contradiction between the modelling and its target philosophical view, that is simply to be regarded as an aspect in which the modelling—as many other modellings of many other views—is *partially distorting*. Such rejoinder does nothing but deepen the need for a better modelling of learning from experience. The objection may however also invite the less conciliatory (and more interesting) rejoinder to the effect that there is actually no contradiction between the modelling and its target philosophical view: the justification provided by the experience does not depend on any other justification one may have in the sense that it does not need the *independent concourse* of any such justification, but the justification provided by the experience does operate only in a *mediated* fashion, by directly justifying the proposition about one's experience which in turn directly justifies the proposition about one's surroundings (so that the experience only indirectly justifies the proposition about one's surroundings).

Although the "*mediated dogmatist*" view sketched in this rejoinder certainly deserves further investigation, it does seem bound to clash with dogmatism$^\natural$ and even with dogmatism. For how must one's epistemic state be before the relevant experience so that, upon having that experience, a proposition like *EXPCUBE* being justified can determine that a proposition like *CUBE* is justified? It's hard to see how any such normal state could fail to provide a justification for believing in a *link between experience and reality* (along the lines of ⟨Typically, if *EXPCUBE* is true, *CUBE* is true⟩), which contradicts dogmatism$^\natural$.[35] Indeed, it's also hard to see how it could not be at least partly *thanks* to the independent concourse of the justification for believing in this

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probabilistic threshold for justification—determines either that one's reasons in favour of ⟨P⟩ can suffice to defeat one's justification for believing that Q even if they do not suffice to provide one with a justification for believing that P (in the much more plausible case in which the probabilistic threshold for justification is $> .5$) or that one's reasons in favour of ⟨P⟩ can suffice to provide one with a justification for believing that P even if they may not suffice to defeat one's justification for believing that Q (in the much less plausible case in which the probabilistic threshold for justification is $< .5$).

[35] Although propositions along the lines of ⟨Typically, if *EXPCUBE* is true, *CUBE* is true⟩ are strictly speaking logically independent from ¬*DECEIT*, throughout I count them as one of the "likes" of ¬*DECEIT* referred to in the definition of 'dogmatism$^\natural$'.

link that, upon having the experience, a proposition like *EXPCUBE* being justified can determine that a proposition like *CUBE* is justified, which contradicts dogmatism.[36]

All these (arguably interrelated) problems with the lower-probability approach can be overcome by shifting to an alternative, more dogmatist$^{\natural\flat\flat}$-friendly approach. Conceptually, the key move consists in thinking of the contribution of experience as being *epistemologically of exactly the same kind* as the prior epistemic state the experience contributes to (in the sense of being representable as a full-blooded DS probability function), against the flat-footed Bayesian framework we've worked with up to section 8 and the lower-probability approach discussed in this section, both of which think of the contribution of experience as being *epistemologically of a different—and much less structured—kind* than the prior epistemic state the experience contributes to (in the sense of being representable as a simple assignment of probability 1 to a certain proposition). Thus, we now take an experience to contribute its own mass $\mathfrak{M}_1$ to a given subject's epistemic state at a given time, mass which is then combined by the operation $\otimes$ with the prior mass $\mathfrak{M}_0$ following *Dempster's rule of combination* (see Dempster [1967]):

COMBINATION $$\mathfrak{M}_0 \otimes \mathfrak{M}_1(X) = \begin{cases} 0 & \text{if } X = \varnothing \\ \mathfrak{n} \sum\limits_{Y_0, Y_1 : Y_0 \cap Y_1 = X} (\mathfrak{M}_0(Y_0)\mathfrak{M}_1(Y_1)) & \text{otherwise,} \end{cases}$$

where $\mathfrak{n}$ is the normalising factor $1/\sum\limits_{Y_0, Y_1 : Y_0 \cap Y_1 \neq \varnothing} (\mathfrak{M}_0(Y_0)\mathfrak{M}_1(Y_1))$.

Coming back to the *caveat* entered at the beginning of this section, I emphasise that I'm only proposing **COMBINATION** as a theory of learning which fits well into a dogmatist$^{\natural\flat\flat}$ framework adopting the Dempster-Shafer theory of probabilities *in the cases of learning from experience relevant for our context*.[37] More specifically, such cases are

[36] Another problem with mediated dogmatism (and, more generally, with the modelling under discussion in the text) is worth a brief mention. Let's assume that an experience as of a cube has something along the lines of *CUBE* as its content (see the *caveat* in fn 4). Now, support is typically supposed to go by *contents*, but *CUBE* does not seem to make *EXPCUBE* particularly likely (let alone certain). Thus, it would seem that that the mere experience does not have the effect of making *EXPCUBE* certain (or even likely), contrary to what mediated dogmatism (and, more generally, the modelling under discussion in the text) requires. Note that this problem goes in a direction somewhat converse to the one in which the problem developed in the text goes: while that problem concerns how to get from *EXPCUBE* (being justified) to *CUBE* (being justified), this problem concerns how to get from *CUBE* (being presented by the experience) to *EXPCUBE* (being justified).

[37] It might be worth observing that, contrary to the problems suffered by other familiar alternatives to **CONDITIONALISATION**, **COMBINATION** straightforwardly enjoys epistemologically nice algebraic properties such as *commutativity* ($\mathfrak{M}_0 \otimes \mathfrak{M}_1 = \mathfrak{M}_1 \otimes \mathfrak{M}_0$) and *associativity* ($(\mathfrak{M}_0 \otimes \mathfrak{M}_1) \otimes \mathfrak{M}_2 = \mathfrak{M}_0 \otimes (\mathfrak{M}_1 \otimes \mathfrak{M}_2)$). It does not enjoy *idempotency* (it is not always the case that $\mathfrak{M}_0 \otimes \mathfrak{M}_0 = \mathfrak{M}_0$), but this possibly surprising feature should actually be welcomed once it is realised that $\mathfrak{M}_0 \otimes \mathfrak{M}_1$ is quite generally supposed to represent the combination of the masses that correctly describe the degrees of support that *two wholly distinct epistemic states* lend to the propositions in $\mathcal{S}$. Now, in the extreme case in which two wholly distinct epistemic states *completely agree*, the same mass (i.e. the same function) will correctly describe the degrees of support that each of them lends to the propositions in $\mathcal{S}$, but, very intuitively, the epistemic state resulting from pooling together any two such completely agreeing epistemic states will typically differ from them, making *even more likely* what they deem to be most likely. The failures of idempotency of $\otimes$ determined by **COMBINATION** correctly deliver this kind of result.

characterised by the fact that, on just about anyone's view (including the dogmatist's), the subject's epistemic state before the experience on the one hand and the experience itself on the other hand can be thought of, a bit roughly, as two epistemic states each of which supports propositions that are *compatible* with the propositions supported by the other epistemic state. It is this characteristic that makes **COMBINATION** a sensible rule to use, for **COMBINATION** is well-known to yield aberrant results in those cases in which there is no such compatibility.[38] Indeed, precisely because of this characteristic, the specific application of **COMBINATION** to be made in section 12 will be relatively uncontroversial, in the sense that its result will agree with what is delivered by many other proposed methods of combination, since many other such methods diverge from **COMBINATION** only in cases which do not exhibit that characteristic.

12 Dogmatically Discovering the Unknown

To appreciate the workings of **COMBINATION** in our context, we should now ask how the relevant masses should exactly be specified. Let's start with a very simplified but hopefully non-distorting powerset- (and hence σ-) algebra $\mathcal{D}$ over the set $U^{\mathcal{D}} = \{1, 2, 3, 4\}$, where the propositions relevant for our purposes are:

- $EXPCUBE = \{1, 2\}$;
- $CUBE = \{2, 3\}$;
- $DECEIT = \{1\}$.

Let's also follow the idea, dear to the dogmatist$^{\natural}$, that the epistemic state at a time t_0 immediately before an experience as of a cube may be a state of *complete ignorance* (with respect to the propositions in $\mathcal{D}$), and so let's assume that the mass $\mathfrak{M}^{t_0}$ that correctly describes the degrees of support that such state lends to the propositions in $\mathcal{D}$ is such that $\mathfrak{M}^{t_0}(U^{\mathcal{D}}) = 1$. Letting $\mathfrak{M}^{e}$ be the mass contributed by an experience as of a cube and taking a time t_1 immediately after that experience, by **COMBINATION** the mass

[38] Such aberrance can already be seen abstractly by noting that the normalisation operated by $\mathfrak{n}$ in **COMBINATION** is such as to discard all portions of the two masses to be combined which support incompatible propositions. The aberrance can be made vivid with a well-known example (originally due to Zadeh [1984], p. 82). Suppose that a patient with neurological symptoms has been examined by two doctors d_0 and d_1. The mass $\mathfrak{M}^{d_0}$ that correctly describes the degrees of support that d_0's epistemic state after the examination lends to the relevant alternative diagnoses is such that $\mathfrak{M}^{d_0}(MENINGITIS) = .99$ and $\mathfrak{M}^{d_0}(TUMOR) = .01$, and the mass $\mathfrak{M}^{d_1}$ that correctly describes the degrees of support that d_1's epistemic state after the examination lends to the relevant alternative diagnoses is such that $\mathfrak{M}^{d_1}(CONCUSSION) = .99$ and $\mathfrak{M}^{d_1}(TUMOR) = .01$. Disappointingly, **COMBINATION** yields that $\mathfrak{M}^{d_0} \otimes \mathfrak{M}^{d_1}(MENINGITIS) = \mathfrak{M}^{d_0} \otimes \mathfrak{M}^{d_1}(CONCUSSION) = 0$, and, even more disappointingly, that $\mathfrak{M}^{d_0} \otimes \mathfrak{M}^{d_1}(TUMOR) = 1$! The problem of how to combine two *conflicting* masses has been the subject of an extensive literature (see e.g. Shafer [1976]; Dubois and Prade [1986]; Yager [1987]; Inagaki [1991]; Zhang [1994] for several early proposals).

$\mathfrak{M}^{t_1}$ that correctly describes the degrees of support that one's epistemic state at t_1 lends to the propositions in $\mathcal{D}$ is such that $\mathfrak{M}^{t_1} = \mathfrak{M}^{t_0} \otimes \mathfrak{M}^e$ (assuming, of course, that the experience as of a cube is the only epistemically significant event occurring between t_0 and t_1). Our question reduces then to how $\mathfrak{M}^e$ should exactly be specified.

A first attempt at defining $\mathfrak{M}^e$ would follow the previous flat-footed Bayesian framework we've worked with up to section 8 and the lower-probability approach discussed in section 11, and simply set $\mathfrak{M}^e(\mathit{EXPCUBE}) = 1$. That would however imply that $\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e(\mathit{EXPCUBE}) = 1$, and hence that $\mathfrak{D}^{\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e}(\mathit{CUBE}) = 0$ (since $\mathit{EXPCUBE} \nsubseteq \mathit{CUBE}$)—a most unwelcome consequence.

This first attempt fails because, in a nutshell, its mass is distributed *too unspecifically*, and in particular no positive mass is assigned to any proposition entailing *CUBE*. A second much better attempt at defining $\mathfrak{M}^e$ would therefore be to have a mass that more finely *discriminates into EXPCUBE*, and hence set $\mathfrak{M}^e(\mathit{EXPCUBE} \wedge \mathit{CUBE}) = \mathfrak{c}$ and $\mathfrak{M}^e(\mathit{EXPCUBE} \wedge \mathit{DECEIT}) = 1 - \mathfrak{c}$ (with $0 < \mathfrak{c} < 1$, thus representing—among other things—the fact that an experience as of a cube does point in favour of *CUBE*, although not conclusively so). That would imply that $\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e(\mathit{EXPCUBE} \wedge \mathit{CUBE}) = \mathfrak{c}$ and $\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e(\mathit{EXPCUBE} \wedge \mathit{DECEIT}) = 1 - \mathfrak{c}$, and so that $\mathfrak{D}^{\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e}(\mathit{EXPCUBE}) = 1$, $\mathfrak{D}^{\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e}(\mathit{CUBE}) = \mathfrak{D}^{\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e}(\neg \mathit{DECEIT}) = \mathfrak{c}$ and $\mathfrak{D}^{\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e}(\mathit{DECEIT}) = \mathfrak{D}^{\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e}(\neg \mathit{CUBE}) = 1 - \mathfrak{c}$. Assuming that $\mathfrak{c} \geq \mathfrak{t}$ (with $\mathfrak{t}$ being the value figuring in the relevant triple of principles characteristic of dogmatism$^{\natural\flat\flat}$), such consequences at last conform with dogmatism$^{\natural\flat\flat}$: one starts without having *DECEIT* as a defeater but with no justification for believing $\neg$*DECEIT* ($\natural$), and upon having an experience as of a cube one acquires a justification for believing *CUBE*, with all this being reflected by the probabilities ($\flat\flat$). Indeed, very interestingly, such consequences conform with dogmatism$^{\surd\surd}$ as well, in the sense that the behaviour of $\neg$*DECEIT* is probabilistically exactly the same as that of *CUBE*. Thus, given that in this framework $\mathfrak{D}^{\mathfrak{M}^{t_0}}(\cdot)$ and $\mathfrak{D}^{\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e}(\cdot)$ play the same role in learning as $\mathfrak{P}(\cdot)$ and $\mathfrak{P}(\cdot/\mathit{EXPCUBE})$ respectively do in the flat-footed Bayesian framework we've worked with up to section 8, we have that the results of substituting the former for the latter in the relevant instances of Lowering and Capping fail.

This explanation should make abundantly clear that and how the first and second problem identified for the lower-probability approach in section 11 are solved. It is not equally clear, however, that the third problem introduced in that section has also been effectively addressed, especially in the form that problem had taken as an objection against mediated dogmatism. True, strictly speaking we are now modelling one's epistemic state at t_0 with the least specific mass $\mathfrak{M}^{t_0}$, and hence as being a state of complete ignorance (with respect to the propositions in $\mathcal{D}$) which in particular does not provide any justification for believing in a link between experience and reality. Nevertheless, some such link seems simply to have been *built instead into the experience as of a cube*: it is that experience itself that now in effect carries the information that, given *EXPCUBE*, *CUBE* is more likely than *DECEIT* (with a ratio of $\mathfrak{c}/1 - \mathfrak{c}$). And that, while compatible with dogmatism$^{\natural\flat\flat}$, attributes to the experience as of a cube rather sophisticated and biased information about conditional likelihoods which, on most views, it just is not plausible to assume to be carried by any ordinary experience.[39]

In addition to this new form taken by the objection against mediated dogmatism raised in section 11 (as well as the problem briefly mentioned in fn 36), there are at least two other problems with this second attempt. Firstly, the attempt's definition of $\mathfrak{M}^e$ implies that an experience as of a cube, making *DECEIT* to a certain extent likely, makes to a certain extent likely its own *falsidicality* (and hence, since, making *CUBE* likely, it also makes likely its own veridicality, the experience is in a certain sense in conflict with itself). That contradicts the idea, natural albeit not forced for the dogmatist$^{\natural\flat\flat}$, that an experience as of a cube *univocally* albeit *not conclusively* points in favour of *CUBE*. Interestingly, this intuitive distinction between univocity and conclusiveness is obliterated in the classical theory of probabilities. For, in that theory, if an epistemic state points to some extent in favour of X non-conclusively (that is, if $0 < \mathfrak{P}(X) < 1$), then, by **DIFFERENTIALITY**, it also points to some extent in favour of $\neg X$ (that is, $\mathfrak{P}(\neg X) > 0$)—to the extent to which an epistemic state does not point in favour of a hypothesis, it has to point in favour of other hypotheses incompatible with it. This arguably distorting feature of the classical theory is eliminated in the Dempster-Shafer theory, so that, on the latter theory, it becomes possible for there to be an epistemic state which points in favour of a hypothesis univocally albeit not non-conclusively, with the opposing idea that univocity implies conclusiveness being exposed as a vestige of additive thinking.[40]

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[39] The implausibility would go away if such information were *built instead into one's epistemic state at* t_0, against our assumptions about $\mathfrak{M}^{t_0}$. This would mean in fact to revert to $\mathfrak{M}^e$ as defined in the previous attempt and to define $\mathfrak{M}^{t_0}$ to be such that $\mathfrak{M}^{t_0}(\neg DECEIT) = \mathfrak{c}$ and $\mathfrak{M}^{t_0}(DECEIT) = 1 - \mathfrak{c}$ (or something along these lines). That too would imply that $\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e(EXPCUBE \wedge CUBE) = \mathfrak{c}$ and $\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e(EXPCUBE \wedge DECEIT) = 1 - \mathfrak{c}$. But, on this alternative definition, the sophisticated and biased information about conditional likelihoods would more plausibly be carried by one's epistemic state at t_0 rather than by the experience as of a cube. Although it may be appealing to some anti-dogmatists, for dogmatists this alternative definition would however be subject to something like the original form of the objection against mediated dogmatism raised in section 11.

[40] This obliteration of an intuitive distinction in the classical theory of probabilities is sometimes obfuscated by a quite inadequate interpretation of the relationships between the classical theory and the Dempster-Shafer theory. Thus, it is sometimes suggested that a probability .5 in the classical theory should just be equated with a probability 0 in the Dempster-Shafer theory, and that, consequently, what is left for a probability < .5 to mean in the classical theory is *merely* that there are *negative* reasons *against* the relevant hypothesis. Were such interpretation tenable, it would belie the assumption, implicit in the argument given in the text, that any *positive (however low) probability* in the classical theory means (possibly among other things) that there is some *positive (however weak) reason* in favour of the hypothesis. But that interpretation is not tenable (at least with respect to standard uses of probability assignments): just as, if a subject s has 6 out of the 10 tickets of a fair lottery, the fact that s's probability of winning the lottery is .6 means (possibly among other things) that there are positive reasons of a certain strength in favour of s's winning the lottery, so, if s has 4 out of the 10 tickets of a fair lottery, the fact that s's probability of winning the lottery is .4 also means (possibly among other things) that there are positive reasons of a certain strength in favour of s's winning the lottery. It does not merely mean that there are negative reasons of a certain strength against s's winning the lottery. Of course, given **DIFFERENTIALITY**, the fact that s's probability of winning the lottery is .4 entails that s's probability of not winning the lottery is .6, which does mean (possibly among other things) that there are negative reasons of a certain strength against s's winning the lottery. Because of this, a probability < .5 in the classical theory does entail (and, in at least one reasonable sense of 'mean', does mean among other things) that there are negative

Secondly, the attempt's definition of $\mathfrak{M}^e$ also implies that the probabilities of $\neg CUBE$ and *DECEIT* too *go up* upon having an experience as of a cube. That may be problematic for dogmatism$^{\natural\flat\flat}$ (and dogmatism$^{\surd\surd}$) for reasons similar to those for which Lowering appeared to be in tension with dogmatism$^{\surd}$. For reflect that those reasons really split into two distinct thoughts:

(U) One cannot acquire a new justification for believing that P on the basis of a certain experience if, upon having that experience, one's probability for $\langle \neg P \rangle$ goes up;

(D) One cannot acquire a new justification for believing that P on the basis of a certain experience if, upon having that experience, one's probability for $\langle P \rangle$ goes down.

While (U) and (D) are equivalent in the classical theory of probabilities (since, by **DIFFERENTIALITY**, $\mathfrak{P}(X/Y) < \mathfrak{P}(X)$ iff $\mathfrak{P}(\neg X/Y) > \mathfrak{P}(\neg X)$), they are no longer so in the Dempster-Shafer theory, as witnessed by the previous model (in which the probability of *DECEIT* goes up—thus triggering (U)—even if the probability of $\neg DECEIT$ goes up too and hence does not go down—thus failing to trigger (D)). Now, (U) as well as (D) *in their full generality* have been shown not to hold by the style of counterexample offered in section 6. However, as the discussion in section 7 makes clear, that specific style of counterexample crucially relies on the assumption that one's epistemic state before the experience already supports to a high degree certain propositions, an assumption which emphatically does not hold in the case of the state of complete ignorance (with respect to the propositions in $\mathcal{D}$) represented by the least specific mass $\mathfrak{M}^{t_0}$ and typically envisaged by dogmatism$^{\natural\flat\flat}$ (and dogmatism$^{\surd\surd}$). Nevertheless, a similar style of counterexample to (U) (but not to (D)) could be contemplated by supposing that, starting from a similar state of complete ignorance, one gets evidence that both makes it likely to a high degree $\mathfrak{d}$ that P and makes it likely to a low degree $\leq 1 - \mathfrak{d}$ that it is not the case that P. It is not clear that this new style of counterexample to (U) would be successful, in particular because it is not clear for which values of $\mathfrak{t}$ (if any) (PD$^{>\mathfrak{t}}$) fails.[41] And even if there were in general such values, it is unclear that they would fall

reasons against the relevant hypothesis, but that entailment is due to **DIFFERENTIALITY** rather than to a differential interpretation of probabilities > .5 on the one hand and probabilities . < 5 on the other hand ("positive" in the former case and "negative" in the latter case). It should thus not in the least obfuscate the fact that, in the classical theory just as well as in the Dempster-Shafer theory, any positive (however low) probability means (possibly among other things) that there is some (however weak) positive reason in favour of the hypothesis.

[41]In fact, given that, as I've tried to argue in sections 5–7, slacks between justification (including its defeat) and probability are only to be expected, one might think that the very important *qualitative* difference marked by the *quantitative* difference between a *probability* 0 and a *positive probability*—that is, the qualitative difference between *having no reason* in favour of a certain proposition and *having reasons* in its favour (see fn 40)—is such as to play a major role in the dynamics of defeat, and in particular such that it (at least typically) determines defeat even if, for at least some positive but low degrees $\mathfrak{d}_0$ and $\mathfrak{d}_1$ (with $\mathfrak{d}_0 < \mathfrak{d}_1$), probability raising from $\mathfrak{d}_0$ to $\mathfrak{d}_1$ does not (at least not typically) determine defeat. (Such view would find a natural although probably even less plausible counterpart in the view that the very important *qualitative* difference marked by the *quantitative* difference between a *probability* 1 and a *non-maximum probability*—that is, the qualitative difference between *having conclusive reasons* in favour of

within the range of the plausible values of $1 - \mathfrak{c}$ employed in the attempt's definition of $\mathfrak{M}^{t_0}$. In any event, even if (U) turned out to fail also in some cases of states of complete ignorance, it would seem strange if dogmatism$^{\natural\flat\flat}$ (or dogmatism$^{\surd\surd}$) itself entailed that there are violations of it. Thus, although the real extent to which (U) holds remains unclear, it would seem both incautious and gratuitous to saddle the dogmatist$^{\natural\flat\flat}$ (or the dogmatist$^{\surd\surd}$) with violations of it in the case of the state of complete ignorance (with respect to the propositions in $\mathcal{D}$) represented by the least specific mass $\mathfrak{M}^{t_0}$ and typically envisaged by dogmatism$^{\natural\flat\flat}$ (and dogmatism$^{\surd\surd}$).[42]

All these remaining (arguably interrelated) problems with the previous attempt at defining $\mathfrak{M}^e$ can be overcome by shifting to an alternative, more dogmatist$^{\surd\surd\natural\flat\flat}$-friendly model. The key move consists in relinquishing the unnatural modelling of learning from experience as requiring that a proposition about one's experience become certain. **COMBINATION** makes it particularly easy to relinquish that modelling choice, as it allows to model the mass contributed by an experience as of a cube as *directly pointing in favour of CUBE itself* rather than in favour of $EXPCUBE \wedge CUBE$—thus solving once and for all the third problem introduced in section 11 (as well as the problem briefly mentioned in fn 36)—and, moreover, as doing so both *univocally albeit not conclusively*—thus solving the first new problem discussed in this section—and *without violating* (U) *and* (D)—thus solving the second new problem discussed in this section. For example, keeping fixed our assumptions about $\mathcal{D}$ and $\mathfrak{M}^{t_0}$, we can set $\mathfrak{M}^e(CUBE) = \mathfrak{c}$ and $\mathfrak{M}^e(U^{\mathcal{D}}) = 1 - \mathfrak{c}$. That would imply that $\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e(CUBE) = \mathfrak{c}$ and $\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e(U^{\mathcal{D}}) = 1 - \mathfrak{c}$, and so that $\mathfrak{D}^{\mathfrak{M}^{t_0}\otimes\mathfrak{M}^e}(CUBE) = \mathfrak{D}^{\mathfrak{M}^{t_0}\otimes\mathfrak{M}^e}(\neg DECEIT) = \mathfrak{c}$ and $\mathfrak{D}^{\mathfrak{M}^{t_0}\otimes\mathfrak{M}^e}(DECEIT) = \mathfrak{D}^{\mathfrak{M}^{t_0}\otimes\mathfrak{M}^e}(\neg CUBE) = 0$. Such consequences at last fully resonate with dogmatism$^{\surd\surd\natural\flat\flat}$:[43] one starts without having *DECEIT* or $\neg$*CUBE* as a defeater but with no justification for believing $\neg$*DECEIT* or *CUBE* ($\natural$), and, upon having an experience as of a cube, one acquires a first justification for believing *CUBE* and $\neg$*DECEIT* ($\surd\surd$), but no defeater such as *DECEIT* and $\neg$*CUBE*, with all this being reflected by the probabilities ($\flat\flat$).

In the model just offered, for every $X \subseteq EXPCUBE$, $\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e(X) = 0$, and so $\mathfrak{D}^{\mathfrak{M}^{t_0}\otimes\mathfrak{M}^e}(EXPCUBE) = 0$. This *prima facie* surprising consequence is actually intended in the light of the point made in fn 36. That there is a cube is not a very good reason

a certain proposition and *not having conclusive reasons* in its favour—is such as to play a major role in the dynamics of justification, and in particular such that it (at least typically) determines loss of justification even if, for at least some non-maximum but high degrees $\mathfrak{d}_0$ and $\mathfrak{d}_1$ (with $\mathfrak{d}_0 < \mathfrak{d}_1$), probability lowering from $\mathfrak{d}_1$ to $\mathfrak{d}_0$ does not (at least not typically) determine loss of justification.)

[42] Thanks to Alejandro Mosqueda and Brian Weatherson for discussions of (U).

[43] In the model just offered in the text, the relevant instances of Lowering and Capping, with the relevant DS probability functions substituted for the classical probability functions, fail. In terms of the taxonomy offered in section 4 of possible replies to the objections from Lowering and Capping, I'm thus recommending to the friend of dogmatism and of its strengthenings full-heartedly to embrace all the appealing strengthenings we've seen and to endorse moves (L_3) and (C_3) respectively. (At least for the case of the state of complete ignorance (with respect to the propositions in $\mathcal{D}$); once one has gathered enough experiences as to make *EXPCUBE* (and hence *DECEIT*) rather unlikely, move (L_1) becomes available and the objection from Capping less pressing, although moves (L_3) and (C_3) also continue to be available.)

for thinking that anyone has an experience as of a cube, and so *CUBE* does not seem to make *EXPCUBE* particularly likely—indeed, for ordinary epistemic states, it is not implausible to think that it does not make it likely at all. But then, assuming that an experience as of a cube has something along the lines of *CUBE* as its content and that support goes by contents, it follows that *EXPCUBE* is not made likely at all by an experience as of a cube *in and of itself*.

I hasten to add that this rather draconian view is fully compatible with the idea that, for normal human subjects, the fact that one has an experience as of a cube is typically accessible by *introspection*, so that, for those subjects, one's *overall* epistemic state after such an experience supports not only *CUBE* (by courtesy of the experience itself) but also *EXPCUBE* (by courtesy of the accompanying introspection). Still, since we're investigating the *particular* epistemic import *of experience itself* rather than the *total* epistemic import *of the sundry factors that for normal human subjects correlate with experience*, screening off such factors from our model is only appropriate. I should also add that, although the framework I'm proposing (contrary to the flat-footed Bayesian framework we've worked with up to section 8 and to the alternatives explored in section 11) is *hospitable* to such draconian view, it does *not force* it, *even if we require satisfaction of the desiderata* that an experience as of a cube directly point in favour of *CUBE* itself, that it do so both univocally albeit not conclusively and without violating (U) and (D). For example, keeping fixed our assumptions about $\mathcal{D}$ and $\mathfrak{M}^{t_0}$, a non-draconian approach can set $\mathfrak{M}^e(CUBE) = \mathfrak{c} - \mathfrak{e}$, $\mathfrak{M}^e(EXPCUBE) = \mathfrak{e}$ and $\mathfrak{M}^e(U^{\mathcal{D}}) = 1 - \mathfrak{c}$. That would imply that $\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e(CUBE) = \mathfrak{c} - \mathfrak{e}$, $\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e(EXPCUBE) = \mathfrak{e}$ and $\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e(U^{\mathcal{D}}) = 1 - \mathfrak{c}$, and so that $\mathfrak{D}^{\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e}(CUBE) = \mathfrak{D}^{\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e}(\neg DECEIT) = \mathfrak{c} - \mathfrak{e}$, $\mathfrak{D}^{\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e}(EXPCUBE) = \mathfrak{e}$ and $\mathfrak{D}^{\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e}(DECEIT) = \mathfrak{D}^{\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e}(\neg CUBE) = 0$, thus satisfying all of the above desiderata while modelling the idea that *EXPCUBE* is made *likely to degree* $\mathfrak{e}$ by an experience as of a cube.[44]

Of course, even on the draconian view I'm espousing, the previous kind of model would still be a more appropriate one for modelling the total epistemic import of the sundry factors (such as introspection) that for normal human subjects correlate with experience rather than the particular epistemic import of experience itself. For this specific purpose, it would however seem even more appropriate to think of the contribution of introspection as being *epistemologically of exactly the same kind* as the contribution of experience (in the sense of being representable as a full-blooded DS probability function), against the non-draconian approach sketched in the previous paragraph which thinks of the contribution of introspection as being epistemologically of a different—and much less structured—kind than the contribution of experience (in the sense of being representable as a simple assignment of a certain probability to a

[44] Notice however that, keeping fixed our assumptions about $\mathcal{D}$ and $\mathfrak{M}^{t_0}$, the framework I'm proposing together with the desideratum that an experience as of a cube directly point in favour of *CUBE itself* is inconsistent with the stronger (and even less plausible) idea that *EXPCUBE* is made *certain* by an experience as of a cube. More generally, keeping fixed our assumptions about $\mathcal{D}$ and $\mathfrak{M}^{t_0}$, the framework I'm proposing together with the desideratum that an experience as of a cube directly point in favour of *CUBE itself* to degree $\mathfrak{c}$ puts an upper bound of $1 - \mathfrak{c}$ to the degree to which *EXPCUBE* is made likely by an experience as of a cube.

certain proposition *qua* part of the totality of assignments in which the contribution of experience consists). Thus, we now take introspection to contribute its own mass $\mathfrak{M}^i$, which we can set to be such that $\mathfrak{M}^i(EXPCUBE) = \mathfrak{e}$ and $\mathfrak{M}^i(U^{\mathcal{D}}) = 1 - \mathfrak{e}$. That would imply that $(\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e) \otimes \mathfrak{M}^i(CUBE) = \mathfrak{c}(1 - \mathfrak{e})$, $(\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e) \otimes \mathfrak{M}^i(EXPCUBE) = \mathfrak{e}(1 - \mathfrak{c})$, $(\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e) \otimes \mathfrak{M}^i(EXPCUBE \wedge CUBE) = \mathfrak{ce}$, and so that $\mathfrak{D}^{(\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e) \otimes \mathfrak{M}^i}(CUBE) = \mathfrak{D}^{(\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e) \otimes \mathfrak{M}^i}(\neg DECEIT) = \mathfrak{c}$, $\mathfrak{D}^{(\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e) \otimes \mathfrak{M}^i}(EXPCUBE) = \mathfrak{e}$ and $\mathfrak{D}^{(\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e) \otimes \mathfrak{M}^i}(DECEIT) = \mathfrak{D}^{(\mathfrak{M}^{t_0} \otimes \mathfrak{M}^e) \otimes \mathfrak{M}^i}(\neg CUBE) = 0$, thus satisfying all of the desiderata of the previous paragraph while modelling the idea that *EXPCUBE* is made *likely to degree* $\mathfrak{e}$ by the introspection that for normal human subjects correlates with an experience as of a cube (with the crucial difference that, contrary to what fn 44 noted about the non-draconian approach sketched in the previous paragraph, $\mathfrak{e}$ is no longer bounded from above by $1 - \mathfrak{c}$ and can indeed be 1).[45]

We can now fully appreciate how this final model can be so interpreted as to offer a radical solution to the various forms that the third problem introduced in section 11 has taken. As I've already noted in section 11, on this final model the contribution of an experience is *epistemologically of exactly the same kind* as the prior epistemic state the experience contributes to: experience too offers a global (typically uncertain) view of how things are, *including how they are with respect to propositions about the objective world like CUBE*. Accordingly, an experience and the prior epistemic state the experience contributes to *need to be combined together "as peers", including their take on propositions about the objective world like CUBE*. Thus, there is *no asymmetry in epistemic authority* between the experience and the prior epistemic state the experience contributes to. In particular, it is not the case that the prior epistemic state the experience contributes to calls the shots by *quarantining* the information about the *objective* world carried by the experience and *accepting* only the reifying and neutralising reflection about the experience encoded in propositions about the *subjective* world like *EXPCUBE* (with the consequence that, in order to bring these to bear on propositions about the objective world like *CUBE*, appeal needs to be made to some kind of link between experience and reality). Rather, the experience is *epistemologically just as authoritative as* the prior epistemic state the experience contributes to: both states play exactly the same role in one's epistemic life as *independent moments directly feeding information about the objective world*; neither state can be identified as *the real locus of one's epistemic life* and thus as a *privileged vantage point* from which one has to try to make sense of the other state's bearing on propositions about the objective world (for example, by appealing to some kind of link between experience and reality).[46] Moreover, the information carried by an

[45]Thanks to Grant Reaber, Martin Smith and Crispin Wright for discussions of these issues.

[46]The relationship between the two states is thus very much like the relationship between the epistemic state one is in after gathering some evidence and the epistemic state one is in after gathering some other evidence: in both cases, an epistemic subject is no less oneself in one state than in the other state, and so what is called for is *reciprocal mediation* between the states rather than *unilateral appropriation* of an allegedly less privileged state by an allegedly more privileged state. (That being said, as I've already noted in section 11, the case of an experience and the prior epistemic state the experience contributes to has the characteristic that, a bit roughly, the experience typically supports propositions that are *compatible* with the propositions supported by the prior epistemic state the experience contributes to, and so, in

experience is *purely objective*: it only speaks to propositions about the objective world like *CUBE* and is silent on propositions about the subjective world like *EXPCUBE*. Thus, not only need the prior epistemic state the experience contributes to not provide any justification for believing in a link between experience and reality; no such justification needs to be provided by the experience either.[47,48] Perceptual justification accrues even

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that case, the mediation assumes the form of a *cumulation* rather than of a *revision*.)

[47]Obviously, by (C), justification for believing *CUBE* entails justification for believing the *material implication* from *EXPCUBE* to *CUBE*. But justification for a simple material implication does not constitute any epistemic link between the implication's antecedents and its consequents (for one, we can observe that, for the same reasons, justification for believing *CUBE* entails justification for believing the material implication from any proposition to *CUBE*). Notice also that it would be extremely problematic to think that justification for believing in a link between experience and reality can be acquired by the simple inductive procedure come to be known as "*bootstrapping*" (see Vogel [2000] for an early discussion). Thus, on the interpretation I'm proposing, such justification, when indeed present, is way further down the line with respect to basic cases of perceptual (and introspective) justification.

[48]Interestingly, part of the interpretation just sketched and part of the probabilistic features I've been recommending are also in principle available on a more conservative approach that agrees with the classical theory of probabilities as far as their structure is concerned but thinks that learning from experience need not be modelled as consisting in a proposition becoming certain but, more generally, as consisting in a proposition E becoming *likely to a certain extent* $\mathfrak{e}$, and interacting with the prior epistemic state whose degrees of support lent to the propositions in $\mathcal{S}$ are measured by the classical probability function $\mathfrak{P}^t$ to produce a posterior epistemic state whose degrees whose degrees of support lent to the propositions in $\mathcal{S}$ are measured by the classical probability function $\mathfrak{P}^u$. Such approach would naturally substitute **CONDITIONALISATION** with:

J-CONDITIONALISATION $\mathfrak{P}^u(H) = \mathfrak{P}^t(H|E)\mathfrak{e} + \mathfrak{P}^t(H|\neg E)(1 - \mathfrak{e})$

(see Jeffrey [1983], pp. 165–183, who actually proposes a slightly more general version of **J-CONDITIONALISATION** whose details are not necessary for our purposes). (Notice that $\mathfrak{e}$, contrary to the relevant value given by the mass contributed by the experience, cannot generally be interpreted as the degree to which the experience *in itself* supports E, since, necessarily, $\mathfrak{P}^u(E) = \mathfrak{e}$, and $\mathfrak{P}^u(E)$ might well be influenced by features of the prior epistemic state; if one wishes to recover that value on a **J-CONDITIONALISATION** approach, one would have to *reparametrise* **J-CONDITIONALISATION** along the lines proposed by Field [1978], which would also have the nice consequence of making it *commutative*.) For example, we can set $\mathfrak{P}^{t_0}(DECEIT|\neg CUBE) = \mathfrak{d}$ and let $\mathfrak{c}$ be the degree to which *CUBE* becomes likely upon having the experience as of a cube. That would imply that $\mathfrak{P}^{t_1}(CUBE) = \mathfrak{c}$, $\mathfrak{P}^{t_1}(\neg DECEIT) = \mathfrak{c} + (1 - \mathfrak{d})(1 - \mathfrak{c})$, $\mathfrak{P}^{t_1}(DECEIT) = \mathfrak{d}(1 - \mathfrak{c})$ and $\mathfrak{P}^{t_1}(\neg CUBE) = 1 - \mathfrak{c}$, which might be thought to deliver the probabilistic features I've been recommending. Not so quick. In our context, the main problem with a **J-CONDITIONALISATION** approach is that, in agreeing with the classical theory of probabilities about their *structure*, it too is inconsistent with dogmatism$^{\natural\flat}$. In our context, a **J-CONDITIONALISATION** approach also suffers from problems concerning the *dynamics* of probabilities. To begin with, it is not even clear that, on this approach, the relevant instances of Lowering and Capping, with $\mathfrak{P}^{t_1}$ obtained as above substituted for $\mathfrak{P}^{t_0}(\cdot|EXPCUBE)$, fail. Assuming, very plausibly, that $\mathfrak{c}$ is $\sim .9$, the relevant instances of Lowering and Capping still hold if $\mathfrak{P}^{t_0}(\neg DECEIT)$ is extremely high and not insignificantly higher than $\mathfrak{P}^{t_0}(\neg DECEIT|\neg CUBE)$ (for example if $\mathfrak{P}^{t_0}(\neg DECEIT) = .999$ and $\mathfrak{P}^{t_0}(\neg DECEIT|\neg CUBE) = .98$); but, on this approach, one would expect $\mathfrak{P}^{t_0}(\neg DECEIT)$ to be precisely something like that, since an experience as of a cube is only one of the myriad possible kinds of experience of shapes whereas $\neg$*CUBE* eliminates one way in which $\neg$*DECEIT* (but not *DECEIT*) could hold. I've explained in sections 7 and 8 how the fact that the relevant instances of Lowering and Capping hold on these grounds is compatible with dogmatism$^{\surd}$, but the point remains that, at least if the further assumption (characteristic of dogmatism$^{\flat\flat}$) of a probabilistic threshold for justification is made, that fact is not compatible with dogmatism$^{\surd\surd}$ or dogmatism$^{\natural}$. Moreover, even if one opts for (as I've just argued, in the total absence of justification for believing in a link between experience and reality.[49]

controversial) modelling choices on which the relevant instances of LOWERING and CAPPING fail, a related problem still emerges when we consider what is now the further episode of learning consisting in one's introspecting that one has an experience as of a cube. Assuming, very plausibly, that the degree to which *EXPCUBE* becomes likely upon introspecting that one has an experience as of a cube is ~ 1, on this approach it is still the case that $\mathfrak{P}^{t_2}(DECEIT) > \mathfrak{P}^{t_1}(DECEIT)$: on this approach, ordinary introspection still oddly enough raises the probability that one is victim of a global deceit.

[49]In recommending to the dogmatist to reply to the objections from LOWERING and CAPPING by endorsing moves (L_3) and (C_3) respectively (see fn 43), the outlook of this paper chimes with the pioneering works of Weatherson [2007] and Pryor [2007]. Although a full discussion of the proposals developed in those papers—and of their relationships with the proposal developed in this paper—will have to wait for another occasion, it'll be helpful to mention what in my view are the most important points of disagreement or, at least, of difference. *Generally*, both Weatherson [2007] and Pryor [2007] only focus on the problems for dogmatism arising from the *dynamics* of probabilities, without realising that, as I argued in section 9, there is a more fundamental conflict concerning the *structure* of probabilities between dogmatism and the classical theory of probabilities. *More specifically*, Weatherson [2007]'s theory still models learning from experience as consisting in a proposition about one's *experience* becoming *certain* and *indirectly* making *likely* a proposition about one's *surroundings*, and so is subject to a form of the third problem introduced in section 11, in particular the form discussed in this section (since the theory requires an experience as of a cube to privilege classical probability functions which exhibit a bias in favour of a link between experience and reality), as well as to the problem briefly mentioned in fn 36. Relatedly, while the relevant instances of LOWERING and CAPPING do fail on Weatherson [2007]'s theory, under extremely minimal assumptions the theory cannot allow for the probability of $\neg$*DECEIT* or of *CUBE* after an experience as of a cube to be *higher or equal* to the probability of $\neg$*DECEIT* before the experience. This feature of the theory is in grave tension with dogmatism$^{\natural\flat\flat}$. That being said, I should add that the theory shares with mine the aim of developing a framework hospitable to dogmatism$^{\natural}$ and also to the perhaps stronger view that the epistemic state before an experience may not provide *any* justification for believing in a link between experience and reality, *whether this link be a proposition or something else* (for example, think of a non-propositional link consisting in something along the lines of [*CUBE* given *EXPCUBE*], which perhaps may be justified even if corresponding propositions like ⟨Typically, if *EXPCUBE* is true, *CUBE* is true⟩ are not). (As I've mentioned above, the theory then diverges from mine in assuming that experience does provide a justification for believing in such a link.) This marks a crucial divergence from Pryor [2007]'s theory, one of whose distinctive features is instead that of building the justification for some such *non-propositional* link into the epistemic state *before an experience*. That theory too still models learning from experience as consisting in a proposition about one's experience becoming certain and indirectly making likely a proposition about one's surroundings, but avoids the letter of the third problem introduced in section 11 by postulating that the required justified link between experience and reality is only non-propositional (so that its being justified does not imply the existence of any justification for believing in any propositional link). However, since, contrary to what the theory seems to assume, I find it very plausible that, if one has a justification for [*CUBE* given *EXPCUBE*] (one's *conditional* probability for the former given the latter is high) in the way envisaged by the theory, one has a justification for believing at least some corresponding propositions like ⟨Typically, if *EXPCUBE* is true, *CUBE* is true⟩ (one's *unconditional* probability for at least some such propositions is high), I find it very plausible that the theory does ultimately contradict dogmatism$^{\natural}$ (whereas it may still not contradict (at least the letter of) dogmatism, for it may assume that the justification for believing in any propositional link is merely a *by-product* of the justification for the non-propositional link, and that it is at least partly thanks to the independent concourse *of the latter but not of the former* justification that, upon having the experience, a proposition like *EXPCUBE* being justified can determine that a proposition like *CUBE* is justified, which does not contradict (at least the letter of) dogmatism).

13 Coda

The dialectic of this paper has gone quite a long way. We started with an objection to dogmatism$^{\surd}$ (from Lowering) and one to dogmatism$^{\natural}$ (from Capping). We initially focussed on the former, showing how, against the background of a certain independently appealing assumption about the relationships between justification and probability, that objection fails. We also observed, however, that, at least if the further assumption (characteristic of dogmatism$^{\flat\flat}$) of a probabilistic threshold for justification is made, that line of reply to the objection from Lowering cannot also be used to rescue dogmatism$^{\surd\surd}$. And, in any event, the objection from Capping against dogmatism$^{\natural}$ was still being left unanswered. We then suspended temporarily consideration of the dynamics of probabilities to focus instead on consideration of their structure, discovering that, at least if the further assumption (characteristic of dogmatism$^{\flat}$) of a probabilistic sufficient condition for justification and of an inverse probabilistic sufficient condition for defeat are made, dogmatism$^{\natural}$ is incompatible with the classical theory of probabilities. Consequently, we briefly looked at an alternative theory, the Dempster-Shafer theory, more hospitable to dogmatism$^{\natural\flat}$. Coming back to the dynamics of probabilities, we then examined how learning should proceed in a dogmatist$^{\natural\flat\flat}$ framework adopting the Dempster-Shafer theory, and opted—at least for the the cases of learning from experience relevant for our context—in favour of **COMBINATION**. After some fine-tuning modelling choices with this rule, we eventually came full circle, and found a theory of the structure and dynamics of probabilities fully harmonious with both dogmatism$^{\natural\flat\flat}$ and dogmatism$^{\surd\surd}$.

References

Stewart Cohen. Why basic knowledge is easy knowledge. *Philosophy and Phenomenological Research*, 70:417–430, 2005.

Arthur Dempster. Upper and lower probabilities induced by a multivalued mapping. *Annals of Mathematical Statistics*, 38:325–339, 1967.

Didier Dubois and Henri Prade. A set-theoretic view on belief functions: Logical operations and approximations by fuzzy sets. *International Journal of General Systems*, 12:193–226, 1986.

Hartry Field. A note on Jeffrey conditionalization. *Philosophy of Science*, 45:361–367, 1978.

Roderick Firth. Are epistemic concepts reducible to moral concepts? In Alvin Goldman and Jaegwon Kim, editors, *Values and Morals*, pages 215–29. Reidel, Dordrecht, 1978.

John Hawthorne. *Knowledge and Lotteries*. Oxford University Press, Oxford, 2004.

Toshiyuki Inagaki. Interdependence between safety-control policy and multiple-sensor schemes via Dempster-Shafer theory. *IEEE Transactions on Reliability*, 40:182–188, 1991.

Richard Jeffrey. *The Logic of Decision*. The University of Chicago Press, Chicago, 2nd edition, 1983.

Peter Kung. On having no reason: dogmatism and Bayesian confirmation. *Synthese*, 177:1–17, 2010.

George Moore. Proof of an external world. *Proceedings of the British Academy*, 25:273–300, 1939.

Dana Nelkin. The lottery paradox, knowledge and rationality. *The Philosophical Review*, 109:373–409, 2000.

John Pollock. *Knowledge and Justification*. Princeton University Press, Princeton, 1974.

James Pryor. What's wrong with Moore's argument? *Philosophical Issues*, 14:349–378, 2004.

James Pryor. Uncertainty and undermining, 2007. URL `http://www.jimpryor.net/research/papers/Uncertainty.pdf`. ms.

Alfréd Rényi. On a new axiomatic theory of probability. *Acta Mathematica Academiae Scientiarum Hungaricae*, 6:286–335, 1955.

Stephen Schiffer. Skepticism and the vagaries of justified belief. *Philosophical Studies*, 119:161–184, 2004.

Glenn Shafer. *A Mathematical Theory of Evidence*. Princeton University Press, Princeton, 1976.

Nicholas Silins. Basic justification and the Moorean response to the skeptic. In Tamar Gendler and John Hawthorne, editors, *Oxford Studies in Epistemology*, volume 2, pages 108–140. Oxford University Press, Oxford, 2007.

Martin Smith. What else justification could be. *Nous*, 44:10–31, 2010.

Jonathan Vogel. Reliabilism leveled. *The Journal of Philosophy*, 97:602–623, 2000.

Jonathan Vogel. E & ~H. In this volume, 2013.

Brian Weatherson. The bayesian and the dogmatist. *Proceedings of the Aristotelian Society*, 107:169–185, 2007.

Brian Weatherson. Probability and scepticism. In this volume, 2013.

Roger White. Problems for dogmatism. *Philosophical Studies*, 131:525–557, 2006.

Crispin Wright. The perils of dogmatism. In Susana Nuccetelli and Gary Seay, editors, *Themes from G.E. Moore: New Themes in Epistemology and Ethics*, pages 25–48. Oxford University Press, Oxford, 2007.

Ronald Yager. On the Dempster-Shafer framework and new combination rules. *Information Sciences*, 41:93–137, 1987.

Lofti Zadeh. Review of books: A mathematical theory of evidence. *The AI Magazine*, 5: 81–83, 1984.

Lianwen Zhang. Representation, independence, and combination of evidence in the Dempster-Shafer theory. In Ronald Yager, Mario Fedrizzi, and Janusz Kacprzyk, editors, *Advances in the Dempster-Shafer Theory of Evidence*, pages 51–69. John Wiley, New York, 1994.