Naive *Modus Ponens*[*]

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1 Deflationism about Logical Consequence

In his [2011], Lionel Shapiro proposes a very interesting *deflationist* conception of *logical consequence*, according to which the logical-consequence predicate applying to *sentences* serves as a *merely expressive device* allowing us to talk in a *generalising* way about typically *non-linguistic* entailment facts (just as, according to a deflationist conception of truth, the truth predicate applying to sentences serves as a merely expressive device allowing us to talk in a generalising way about typically non-linguistic facts). Entailment facts are in turn facts expressed using a certain suitably strong conditional (henceforth written as $\Rightarrow$), just as conjunctive facts are facts expressed using a conjunction operator. (I'll assume with Shapiro that there are such facts and that we have a workable conception of them; when talking in English, I'll express them using the canonical construction

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'Its being the case that φ entails that ψ', which will thus serve as official paraphrase of $\varphi \Rightarrow \psi$.)[1]

Although I find it very interesting, I don't share this view. In this paper, I'll be concerned with a logical difficulty, represented by *Curry's paradox* (see Curry [1942]), which the view gives rise to—a difficulty that also arises on my favoured, non-deflationist view of logical consequence, and that I'll propose to treat by applying the general solution to the semantic paradoxes that I've developed more fully in other work (see Zardini [2011]; [2012c]). More in detail, the rest of this paper is organised as follows. In section 2, after presenting the difficulty, I'll introduce the solution offered by Shapiro consisting in the rejection of the structural metarule of *contraction* and I'll briefly criticise an alternative solution consisting in the rejection of the structural metarule of *transitivity*, on the grounds that such alternative solution cannot but fail to validate a very important version of *modus ponens* for the entailment conditional. I'll also note how the difficulty which Shapiro's view gives rise to also arises on my favoured, non-deflationist view of logical consequence. In section 3, I'll proceed to argue that Shapiro's approach to solving the difficulty, while having the great virtue of validating the *deduction theorem* and at least some versions of *modus ponens* for the entailment conditional, is extremely problematic in that it invalidates certain other principles that would equally well seem to express what *modus ponens* is all about. In section 4, I'll close by offering a theory which, while sharing with Shapiro's the very important feature of rejecting contraction, also consequently rejects, contrary to Shapiro's, the *equivalence between* φ *and* $\varphi \wedge \varphi$. I'll show that this difference is crucial in allowing my theory to avoid the problem identified for Shapiro's and hence to validate all the principles that are naively associated with *modus ponens*.

2 Entailment, Curry's Paradox and Structural Properties

For it to serve its expressive purpose, and as restricted to one-premise arguments, the logical-consequence predicate should satisfy the two rules:

(C-IN^1) $\varphi \Rightarrow \psi \vdash C(\ulcorner\varphi\urcorner, \ulcorner\psi\urcorner)$[2]

(C-OUT^1) $C(\ulcorner\varphi\urcorner, \ulcorner\psi\urcorner) \vdash \varphi \Rightarrow \psi$

[1] Throughout, 'fact'-talk *may* (although it *need not*) be understood as a mere *façon de parler* carrying no commitment to a separate category of entities: talk of the fact that snow is white may be understood as talk of snow's being white, talk of the fact that its being the case that snow is white entails that either snow is white or grass is green may be understood as talk of [[its being the case that snow is white] entailing that [either snow is white or grass is green]]. (Here as elsewhere, I'll use square brackets to make constituent structure more perspicuous.)

[2] Throughout, under an assignment of an object-language sentence as a value to 'φ', '$\ulcorner\varphi\urcorner$' refers to a designated singular term in the object language referring (by some means or other) to the same object-language sentence referred to by 'φ' under the assignment.

(with $\vdash$ being the *metalinguistic* logical-consequence predicate—understood as expressing a logic encompassing the logic of validity and truth—and with C being the *object-linguistic* logical-consequence predicate). (C-IN^1) in particular seems to require $\Rightarrow$ to obey the *two-premise* version of the *rule* of *modus ponens*:

(RMP^2) $\varphi \Rightarrow \psi, \varphi \vdash \psi$

For we should expect that $C(\ulcorner\varphi\urcorner, \ulcorner\psi\urcorner), \varphi \vdash \psi$ holds, which, together with (C-IN^1) and some natural assumptions, forces (RMP^2) to hold (in any event, Shapiro and I agree that $\Rightarrow$ should obey (RMP^2)). And (C-OUT^1) in particular seems to require $\Rightarrow$ to obey the *one-premise* version of the *deduction theorem*:

(DT^1) If $\varphi \vdash \psi$ holds, then $\langle . \rangle \vdash \varphi \Rightarrow \psi$ holds[3]

For we should expect that, if $\varphi \vdash \psi$ holds, so does $\langle . \rangle \vdash C(\ulcorner\varphi\urcorner, \ulcorner\psi\urcorner)$, which, together with ($C$-OUT^1) and some natural assumptions, forces (DT^1) to hold (in any event, Shapiro and I agree that $\Rightarrow$ should obey (DT^1)).

Let us also assume that T is an object-language *truth* predicate and that there is a sentence κ identical to $T(\ulcorner\kappa\urcorner) \Rightarrow \bot$ (with $\bot$ being your favourite nightmare sentence, like 'Real Madrid will win the next Champions League').[4] Let us finally assume that our theory of truth is *transparent*—that is, such as to treat $T(\ulcorner\varphi\urcorner)$ as *fully intersubstitutable* with φ (given the semantic paradoxes, this forces our logic to be non-classical).[5]

Under these assumptions, a version of Curry's paradox seems to preclude any operator obeying both (RMP^2) and (DT^1):

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{T(\ulcorner\kappa\urcorner) \Rightarrow \bot, T(\ulcorner\kappa\urcorner) \vdash \bot}\,(\mathrm{RMP}^2)}{T(\ulcorner\kappa\urcorner), T(\ulcorner\kappa\urcorner) \vdash \bot}\,\text{transparency}}{T(\ulcorner\kappa\urcorner) \vdash \bot}\,\text{contraction}}{\langle . \rangle \vdash T(\ulcorner\kappa\urcorner) \Rightarrow \bot}\,(\mathrm{DT}^1)}{\langle . \rangle \vdash T(\ulcorner\kappa\urcorner)}\,\text{transparency} \qquad \dfrac{\dfrac{\dfrac{}{T(\ulcorner\kappa\urcorner) \Rightarrow \bot, T(\ulcorner\kappa\urcorner) \vdash \bot}\,(\mathrm{RMP}^2)}{T(\ulcorner\kappa\urcorner), T(\ulcorner\kappa\urcorner) \vdash \bot}\,\text{transparency}}{T(\ulcorner\kappa\urcorner) \vdash \bot}\,\text{contraction}}{\langle . \rangle \vdash \bot}\,\text{transitivity}$$

[3]For reasons that will shortly become apparent, I assume in this paper that the collections mentioned on the left and on the right of $\vdash$ are *sequences*. $\langle . \rangle$ is the *empty sequence*.

[4]I'll sometimes lapse into grandiose wishful thinking and assume that $\bot$ is in fact untrue and even logically absurd.

[5]I mention now once and for all that, with the appropriate modifications, all the arguments to follow could be recast directly in terms of C in a $\Rightarrow$- and T-free theory that includes the natural C-analogues of (RMP^2) and (DT^1) (see Beall and Murzi [2012]; Zardini [2012c] for the details). In this paper, I'll follow Shapiro in presenting the arguments in terms of $\Rightarrow$ and T.

The paradox shows that, on pain of the catastrophical conclusion $\langle . \rangle \vdash \bot$, no operator (whether an entailment conditional or not) can obey the analogues of (RMP^2) and (DT^1) if all of transparency, contraction and transitivity hold. Since most non-classical logics proposed to accommodate for transparency validate contraction and transitivity, the lesson drawn from the paradox is that either (RMP^2) or (DT^1) has to give (and typically, since (RMP^2) is considered sacrosanct, the choice falls on (DT^1)).[6] However, as Shapiro correctly remarks, one can give up *contraction* instead,[7] and thus preserve both (RMP^2) and (DT^1).

As made evident by the above version of Curry's paradox, another way of preserving both (RMP^2) and (DT^1) in a transparent theory of truth may be afforded by giving up *transitivity*. Matters prove however not that simple. The first non-transitivist approach to the semantic paradoxes, developed by Alan Weir (see Weir [2005] for a recent presentation), actually validates the particular instance of transitivity used in the above version of Curry's paradox (since it does not involve side premises), and would rather block the paradox at the transparency steps, by rejecting the full intersubstitutability of $T(\ulcorner \kappa \urcorner)$ with κ. Thus, (RMP^2) and (DT^1) are preserved in Weir's theory only at the cost of transparency.[8]

More promisingly, at least for friends of transparency, it has always seemed quite likely that the family of non-transitive logics developed in Zardini [2008a]; [2008b], pp. 93–173; [2012a] for dealing with *vagueness* (including the subfamily of very strong logics subsequently studied by Cobreros et al. [2012]) could be used to obtain a non-transitive transparent theory of truth preserving both (RMP^2) and (DT^1), and that this is in fact so has recently been established by Ripley [2012]. Contrary to Weir's, Ripley's theory validates transparency and blocks the above version of Curry's paradox only at the final transitivity step. However, by then, the damage to *modus ponens* has already been done. For, even without the final transitivity step, the above version of Curry's paradox still derives both $\langle . \rangle \vdash T(\ulcorner \kappa \urcorner) \Rightarrow \bot$ and $\langle . \rangle \vdash T(\ulcorner \kappa \urcorner)$; since $\langle . \rangle \vdash \bot$ does not hold, this requires

[6] I emphasise once and for all that the conditionals employed in non-classical logics proposed to deal with the semantic paradoxes are usually *not* designed to be *entailment* conditionals. For ease of comparison, I'll however assume, plausibly enough, that these logics' account of a more ordinary conditional extends in the relevant respects to an account of an entailment conditional. In any event, the critical points I'll be making against these logics in terms of an entailment conditional could be made just as well in terms of the logics' more ordinary conditional. Thus, to go back to the point just made in the text, the analogues of (RMP^2) and (DT^1) for a more ordinary conditional also have a strong *prima facie* plausibility, and Curry's paradox teaches us that transparent theories of truth validating contraction and transitivity cannot have them both. In my view, this is a substantial disadvantage of these theories (I elaborate on this problem in Zardini [2011], pp. 516–517, 522–524; Zardini [2012b]).

[7] Although Shapiro does not give many details on his envisioned non-contractive logic, there are in effect many well-behaved and well-understood such logics (see Girard [1987] for a seminal paper).

[8] Weir's theory validates the rules $T(\ulcorner \varphi \urcorner) \vdash \varphi$ and $\varphi \vdash T(\ulcorner \varphi \urcorner)$, and so, in the theory, $T(\ulcorner \varphi \urcorner)$ and φ *logically follow from one another* even if they *are not fully intersubstitutable*. Importantly, such a failure of transparency should not be taken as a symptom of an *idiosyncratic* weakness of the rules $T(\ulcorner \varphi \urcorner) \vdash \varphi$ and $\varphi \vdash T(\ulcorner \varphi \urcorner)$ in the theory: it is a *completely general* feature of the theory—and something indeed to be naturally expected in a non-transitive logic—that sentences that logically follow from one another (even those that "truth-functionally" follow from one another) are not fully intersubstitutable. Thanks to Alan Weir for very illuminating and enjoyable discussions on these issues.

that the *law* version of the *metarule* of *modus ponens*:

(MMP$^{\mathrm{L}}$) If $\langle . \rangle \vdash \varphi \Rightarrow \psi$ and $\langle . \rangle \vdash \varphi$ hold, then $\langle . \rangle \vdash \psi$ holds

fail. And failure of (MMP$^{\mathrm{L}}$) in turn casts serious doubts on the alleged significance of the fact that (RMP^2) nevertheless holds: given the concession—made in rejecting (MMP$^{\mathrm{L}}$)—that the laws of logic themselves do not obey *modus ponens*, what substance is left to the claim—made in accepting (RMP^2)—that *modus ponens* is nevertheless a valid rule?

In fact, the situation is even worse. For the *logical* idea of *modus ponens* can be seen as intimately connected with and arguably grounded in the *semantic* idea that *a true conditional cannot have a true antecedent and an untrue consequent*;[9] yet, on the strategy under consideration, since $\langle . \rangle \vdash T(\ulcorner \kappa \urcorner) \Rightarrow \bot$ and $\langle . \rangle \vdash T(\ulcorner \kappa \urcorner)$ hold, we're presumably asked to accept that they are true (and, given other details of the theory, false), while $\bot$ is untrue. So we're in effect asked to reject the idea that a true conditional cannot have a true antecedent and an untrue consequent. This is not only in itself rebarbative; as I've just indicated, it also means to reject the very semantic idea with which *modus ponens* is intimately connected and in which it is arguably grounded. And that in turn casts even more serious doubts on the alleged significance of the fact that (RMP^2) nevertheless holds, since, on the strategy under consideration, its instance with $T(\ulcorner \kappa \urcorner)$ for φ and $\bot$ for ψ has true premises and an untrue conclusion: given the concession just made that (RMP^2) can have true premises and an untrue conclusion, what substance is left to the claim that it is nevertheless a valid rule?[10] (Notice that the problems discussed in this paragraph are even further exacerbated by the fact that, in this dialectic, 'true' and 'untrue' can be strengthened to 'logically true' and 'logically absurd' respectively.)[11] I conclude that, at least for friends of transparency and *modus ponens*, the difficulties faced by non-transitivist

[9]For some sophisticated readers, it may not be trivial to specify that 'untrue' is understood throughout in a sense that *rules out* truth (if such readers know what I mean).

[10](MMP$^{\mathrm{L}}$) is a special case of the more general *rule* version of the *metarule* of *modus ponens*:

(MMP$^{\mathrm{R}}$) If $\Gamma \vdash \varphi \Rightarrow \psi$ and $\Delta \vdash \varphi$ hold, then $\Gamma, \Delta \vdash \psi$ holds

(contrary to (MMP$^{\mathrm{L}}$), (MMP$^{\mathrm{R}}$) also fails in Weir's theory of truth). If (MMP$^{\mathrm{L}}$) can be glossed as the claim that *logic* is closed under *modus ponens*, (MMP$^{\mathrm{R}}$) can be glossed as the more general claim that *theory combination* is closed under *modus ponens*. All the problems discussed so far in the last two paragraphs in the text find natural analogues cast in terms of (MMP$^{\mathrm{R}}$) rather than (MMP$^{\mathrm{L}}$). Given the concession—made in rejecting (MMP$^{\mathrm{R}}$)—that theory combination itself does not obey *modus ponens*, what substance is left to the claim—made in accepting (RMP^2)—that *modus ponens* is nevertheless a valid rule? Moreover, in this context it is natural to assume that, in some of the cases in which (MMP$^{\mathrm{R}}$) fails, we may still jointly accept Γ and Δ even if ψ is absurd. Then, since $\Gamma \vdash \varphi \Rightarrow \psi$ and $\Delta \vdash \varphi$ hold, we would presumably again be committed to accepting that $\varphi \Rightarrow \psi$ and φ are true, while ψ is untrue. So we would in effect again be asked to reject the idea that a true conditional cannot have a true antecedent and an untrue consequent, and the problems discussed so far in the paragraph in the text would again ensue.

[11]It may be worth noting that, for reasons I can't go into in this paper, I don't think that any of the problems discussed in the last two paragraphs applies in the case of the use of non-transitive logics for dealing with vagueness made in my work referenced above in the text.

approaches give ample reason to focus instead on non-contractive approaches in the search for a satisfactory solution to Curry's paradox (if (DT^1) is also to be kept fixed).

Before proceeding further in the investigation of non-contractive approaches, it should be noted that the difficulty created by Curry's paradox does not arise only for a *deflationist* theory of logical consequence. For example, on the *non-deflationist* theory I favour, logical consequence is a substantive relation among sentences consisting in a certain form of *truth preservation* from the premises to the conclusions: *if* all premises are true, *then* some conclusions are true (where the implication[12] in question is entailment). Moreover, as for truth, I favour a (in turn non-deflationist) transparent theory. These two views jointly require that the conditional $\Rightarrow$ expressing entailment obey (DT^1). (Suppose that $\varphi \vdash \psi$ holds. Then the truth-preservation theory of logical consequence requires that $\langle . \rangle \vdash T(\ulcorner\varphi\urcorner) \Rightarrow T(\ulcorner\psi\urcorner)$ hold, given which the transparent theory of truth requires that $\langle . \rangle \vdash \varphi \Rightarrow \psi$ hold.) And (RMP^2) is as sacrosanct as ever. Thus, with (RMP^2) and (DT^1) in place, Curry's paradox threatens also on my non-deflationist theory of logical consequence, and I agree with Shapiro that this difficulty should be solved by giving up contraction rather than either (RMP^2) or (DT^1).

3 *Modus Ponens* and Paradox

End of story? Not so fast. A serious problem for Shapiro's approach arises when we consider the *one-premise* version of the *rule* of *modus ponens*:

(RMP^1) $(\varphi \Rightarrow \psi) \wedge \varphi \vdash \psi$

(RMP^1) and (DT^1) jointly imply the *law* of *modus ponens*:

(LMP) $\langle . \rangle \vdash ((\varphi \Rightarrow \psi) \wedge \varphi) \Rightarrow \psi$

Instantiated with $T(\ulcorner\kappa\urcorner)$ for φ and $\bot$ for ψ, by transparency (LMP) implies that $\langle . \rangle \vdash (T(\ulcorner\kappa\urcorner) \wedge T(\ulcorner\kappa\urcorner)) \Rightarrow \bot$ holds, and, *by the equivalence between* φ *and* $\varphi \wedge \varphi$, that in turn implies that $\langle . \rangle \vdash T(\ulcorner\kappa\urcorner) \Rightarrow \bot$ holds, which I'll assume (*pace* the second non-transitivist theory of truth discussed in section 2) to imply the catastrophical conclusion that $\langle . \rangle \vdash \bot$ holds.

Since Shapiro's approach accepts all the other principles involved in this argument, it has to reject (RMP^1). It is this rejection, and the ensuing unnatural severing of (RMP^1) and (RMP^2), that I find extremely problematic. I'll make three points to substantiate my misgivings (in what I deem to be an order of increasing strength).

[12] Throughout, *implication* is understood to be the *genus* comprising all operations expressed by some *conditional* or other. Material implication and entailment are two different *species* of implication lying at the opposite ends of its spectrum.

Firstly, rejection of (RMP^1) simply defies credibility (although, in the context of the semantic paradoxes, this is admittedly a less serious flaw than elsewhere).[13]

Secondly, (RMP^1) is forced if one accepts (RMP^2) (as Shapiro does) and extends the deflationist theory of logical consequence so as to cover *multi-premise* arguments in the obvious (and eminently plausible) way by using *conjunction*:

(C-IN$^i_{\mathrm{C}}$) $(\varphi_1 \wedge \varphi_2 \wedge \varphi_3 \ldots \wedge \varphi_i) \Rightarrow \psi \vdash C(\langle \ulcorner\varphi_1\urcorner, \ulcorner\varphi_2\urcorner, \ulcorner\varphi_3\urcorner \ldots, \ulcorner\varphi_i\urcorner\rangle, \ulcorner\psi\urcorner)$

(C-OUT$^i_{\mathrm{C}}$) $C(\langle \ulcorner\varphi_1\urcorner, \ulcorner\varphi_2\urcorner, \ulcorner\varphi_3\urcorner \ldots, \ulcorner\varphi_i\urcorner\rangle, \ulcorner\psi\urcorner) \vdash (\varphi_1 \wedge \varphi_2 \wedge \varphi_3 \ldots \wedge \varphi_i) \Rightarrow \psi$

Shapiro shows awareness of the glitch, saying that in this and similar cases "the two-premise entailment locution 'That p_1 and that p_2 together entail that q' may need to be interpreted otherwise than as 'That (p_1 and p_2) entails that q'" (Shapiro [2011], p. 326, fn 10). Still, it is very unclear what alternative to the obvious (and eminently plausible) interpretation in terms of conjunction could be offered.[14]

One possible proposal, at least suggested by one of Shapiro's comments (see Shapiro [2011], p. 340), is to replace conjunction with *entailment* (associating to the right), so as to obtain:

(C-IN$^i_{\mathrm{E}}$) $\varphi_1 \Rightarrow (\varphi_2 \Rightarrow (\varphi_3 \ldots \Rightarrow (\varphi_i \Rightarrow \psi)) \ldots) \vdash C(\langle \ulcorner\varphi_1\urcorner, \ulcorner\varphi_2\urcorner, \ulcorner\varphi_3\urcorner \ldots, \ulcorner\varphi_i\urcorner\rangle, \ulcorner\psi\urcorner)$

(C-OUT$^i_{\mathrm{E}}$) $C(\langle \ulcorner\varphi_1\urcorner, \ulcorner\varphi_2\urcorner, \ulcorner\varphi_3\urcorner \ldots, \ulcorner\varphi_i\urcorner\rangle, \ulcorner\psi\urcorner) \vdash \varphi_1 \Rightarrow (\varphi_2 \Rightarrow (\varphi_3 \ldots \Rightarrow (\varphi_i \Rightarrow \psi)) \ldots)$

However, if the aim here is to find an acceptable *proposition* to associate with the accepted *rule of inference* (RMP^2) (a proposition which, as it were, expresses "the thought behind" the rule of inference),[15] the proposal would seem utterly misguided. For (RMP^2) would be associated with the trivial reflexive proposition $(\varphi \Rightarrow \psi) \Rightarrow (\varphi \Rightarrow \psi)$, which does not seem to have much to do with *modus ponens*.[16]

[13] I should note that at least one author has indeed defied credibility, and has done so even without the justification of being grappling with paradox. I'm thinking of course of McGee [1985]'s much discussed "counterexamples" to *modus ponens*. Setting aside the moot question of what is to be made of those examples, it'll suffice here to observe that they are doubly irrelevant in this dialectic. Firstly, the examples crucially involve consequents that are themselves *conditionals*, while $\perp$ (which is the consequent in the problematic instance of (RMP^1)) clearly need not be a conditional. Secondly, the examples target (RMP^2) just as well as (RMP^1), and so do not give any support to the unnatural *severing* of (RMP^1) and (RMP^2) required by Shapiro's approach. Thanks to Paul Égré for discussion of this issue.

[14] In the quoted passage, Shapiro seems to talk as though only a "*local*" reinterpretation were needed, targeted at the specific instances of (RMP^2) that cause trouble (or targeted specifically at (RMP^2) in its entirety as against other rules). I would think that such treatments would be objectionably *ad hoc* and disjunctive, and that a "*global*" reinterpretation of how premises are combined in *any* argument is much preferable. This being noted, the point to follow in the text is mostly neutral with respect to the extent of the proposed reinterpretation.

[15] Since I'm treating rules of inference as *schematic*, to be precise I should here really be talking of a "proposition schema" rather than of a (full-blooded) proposition. In the text, I'll however sacrifice precision for conciseness.

[16] Priest [1980], pp. 432–433, who airs what is basically this proposal for (RMP^2), claims oddly enough that "[t]his [i.e. the trivial reflexive proposition, my insertion] is the genuine *modus ponens* axiom; the other [i.e. (LMP), my insertion] is a counterfeit".

It might be thought that things look rosier if we swap the order of the premises in (RMP^2), so that the proposal in question yields $\varphi \Rightarrow ((\varphi \Rightarrow \psi) \Rightarrow \psi)$ instead. I would agree that this second proposition is not as trivial as the first one. But I would add that it is so far from being trivially true as to be clearly false. Instantiate φ with 'Snow is white' and ψ with 'Snow is white or grass is green'. Since the main antecedent of the resulting instance of $\varphi \Rightarrow ((\varphi \Rightarrow \psi) \Rightarrow \psi)$ is, as a matter of fact, true (for, as a matter of fact, snow is white), by (RMP^2) we could detach that instance's main consequent—that is, we could unconditionally infer that [its being the case that [[its being the case that snow is white] entails that [either snow is white or grass is green]]] entails that [either snow is white or grass is green]. I take it that the positive entailment claim we could thus unconditionally infer to is, in spite of snow's whiteness, clearly false: its antecedent is most plausibly logically true while its consequent is most definitely not, the former might easily have been true with the latter false and, more generally, the validity of the rule of addition has nothing to do with the colour of either snow or grass.[17]

Moreover, even if, contrary to what I've just argued, $\varphi \Rightarrow ((\varphi \Rightarrow \psi) \Rightarrow \psi)$ is after all an acceptable proposition to associate with the swapped version of (RMP^2), we would still have done nothing to find an acceptable proposition to associate with the original version of (RMP^2). (We might of course try to take again $\varphi \Rightarrow ((\varphi \Rightarrow \psi) \Rightarrow \psi)$, but it's hard to think of a general theory associating propositions with rules of inference that would have this as a non-*ad hoc* consequence.)

An alternative proposal is to replace conjunction not with entailment (associating to the right), but with the operation (henceforth written as $\circ$ and known as '*fusion*' in the literature on relevant logics) of which entailment is the "*residual*" (associating to the right), so as to obtain:

$$(C\text{-IN}^i_{\mathrm{F}})\ \ (\varphi_1 \circ (\varphi_2 \circ (\varphi_3 \ldots \circ \varphi_i)) \ldots) \Rightarrow \psi \vdash C(\langle \ulcorner\varphi_1\urcorner, \ulcorner\varphi_2\urcorner, \ulcorner\varphi_3\urcorner \ldots, \ulcorner\varphi_i\urcorner \rangle, \ulcorner\psi\urcorner)$$

$$(C\text{-OUT}^i_{\mathrm{F}})\ \ C(\langle \ulcorner\varphi_1\urcorner, \ulcorner\varphi_2\urcorner, \ulcorner\varphi_3\urcorner \ldots, \ulcorner\varphi_i\urcorner \rangle, \ulcorner\psi\urcorner) \vdash (\varphi_1 \circ (\varphi_2 \circ (\varphi_3 \ldots \circ \varphi_i)) \ldots) \Rightarrow \psi$$

Without loss of generality, for our purposes we can simply understand fusion so that $\varphi \circ \psi$ is fully intersubstitutable with $\neg(\varphi \Rightarrow \neg\psi)$. (RMP^2) would now be associated with the proposition $((\varphi \Rightarrow \psi) \circ \varphi) \Rightarrow \psi$. Again, I would agree that this third proposition is not as trivial as the first one. But, similarly, I would add that it is so far from being trivially true as to be clearly false. Instantiate φ with 'Snow is blue' and ψ with 'Snow is blue or grass is red'. Then the main antecedent of the resulting instance of $((\varphi \Rightarrow \psi) \circ \varphi) \Rightarrow \psi$ is equivalent with the claim that [its being the case that [[its being the case that snow is blue] entails that [either snow is blue or grass is red]]] does not entail that snow is not blue. I take it that that negative entailment claim is, in spite of snow's whiteness, clearly

[17] This point about the illegitimacy of replacing conjunction with entailment (associating to the right) is related to the general fact, well-known in the literature on relevant logics (see e.g. Brady [2003], p. 4), that the rule of *exportation* $(\varphi \wedge \psi) \Rightarrow \chi \vdash \varphi \Rightarrow (\psi \Rightarrow \chi)$ breeds irrelevance. (The rule is so called because it allows in effect to "extract" ψ from its combination with φ and stand alone as the antecedent of the conditional having χ as consequent. The rule is most clearly objectionable for conditionals that are suitably strong and that obey (RMP^2).)

true: its antecedent is most plausibly logically true while its consequent is most definitely not, the former might easily have been true with the latter false and, more generally, the validity of the rule of addition has nothing to do with the colour of snow. Thus, since the main antecedent of the resulting instance of $((\varphi \Rightarrow \psi) \circ \varphi) \Rightarrow \psi$ is true, by (RMP^2) we could detach that instance's main consequent—that is, we could unconditionally infer to the conclusion that either snow is blue or grass is red, which is, as a matter of fact, false (since, as a matter of fact, snow is white and grass is green).

Moreover, not only, as I've in effect just argued, is the main antecedent of $((\varphi \Rightarrow \psi) \circ \varphi) \Rightarrow \psi$ *in certain respects too weak* to capture the intended circumstances in which (RMP^2) is triggered; it is also *in other respects too strong* to capture such circumstances. Instantiate φ with 'Snow is white' and ψ with 'Snow is white and snow is not white'. Then the main antecedent of the resulting instance of $((\varphi \Rightarrow \psi) \circ \varphi) \Rightarrow \psi$ is false (for, [its being the case that [its being the case that snow is white entails that [snow is white and snow is not white]]] does entail that snow is not white—let's indeed assume such a plausible instance of *reductio*), and it should arguably be regarded as false even in a circumstance in which both 'Its being the case that snow is white entails that [snow is white and snow is not white]' and 'Snow is white' are assumed (say, for *reductio*). In such a circumstance, the resulting instance of $((\varphi \Rightarrow \psi) \circ \varphi) \Rightarrow \psi$ remains objectionably silent as to whether 'Snow is white and snow is not white' follows (since we cannot rely on its main antecedent), even if that sentence does follow by *modus ponens* (since we can rely on our assumption of 'Snow is white' to detach the sentence from our other assumption of 'Its being the case that snow is white entails that [snow is white and snow is not white]' by (RMP^2)).[18,19]

Leaving behind us the proposals just discussed, the more general point emerging here is that the unnatural severing of (RMP^1) and (RMP^2) required by Shapiro's approach does violence to what seems to be our notion of *how premises are combined* in a multi-premise argument (i.e. conjunctively). For an argument with premises 'A' and 'B' and conclusion 'C' seems to be an argument saying that 'C' logically follows from 'A' *and* 'B', which in turn seems equivalent with saying that 'C' logically follows from 'A *and* B'.

Thirdly, (RMP^1) is again forced if one accepts (RMP^2) (as Shapiro does) and the extremely compelling metarule:

(CONJ$^{\leq}$) If $\varphi, \psi \vdash \chi$ holds, then $\varphi \wedge \psi \vdash \chi$ holds

[18] Thanks to Aaron Cotnoir for urging me to consider this alternative proposal.

[19] Having thus criticised this alternative proposal, I should add that the use of conjunction to combine premises in a multi-premise argument can in effect be seen as a variant of the proposal, for *material implication* can be seen as the residual of conjunction. In fact, in section 4, I'll offer a non-contractive theory of entailment and transparent truth which uses conjunction to combine premises and has a material implication that is the residual of conjunction. However, the distinctive features of material implication *vis-à-vis* entailment will be crucial for such a theory not to suffer from the problems identified for the proposal in the text. These problems do not stem from the mere fact that premises are combined using an operation that has some implicative residual or other; they rather stem from the more specific fact that the operation has as implicative residual *entailment*.

That a conjunction has as logical consequence everything that is a logical consequence of its two conjuncts jointly is necessary (at least *modulo* transitivity) to guarantee the logical strength in which the whole point of conjunction seems to reside: that of having as logical consequence *both* of its conjuncts.[20]

These three points are not meant to provide a conclusive refutation of Shapiro's approach—on the contrary, I think that it would be worthwhile to investigate the prospects for a non-standard conception of conjunction which can accommodate for the approach's anomalies that have just been brought up.[21] Still, for those of us loyal to a more standard conception of conjunction, the points raised do provide reasons for deep dissatisfaction with the approach. It is thus even more important to realise that its problematic specifics are driven by certain assumptions which, although apparently taken for granted by Shapiro (and many other authors, even if they do envisage failure of contraction, see e.g. Restall [1994], p. viii), are actually not forced on a broadly non-contractive approach to entailment and transparent truth.

4 A Non-Contractive Theory of Naive *Modus Ponens*

The central assumption in question is the *equivalence between* φ *and* $\varphi \wedge \varphi$, and in particular the assumption that $\varphi \vdash \varphi \wedge \varphi$ holds (which in most systems suffices for the argument against (LMP) of section 3 to go through).[22] Given failure of contraction and the points made in section 3 about certain desirable properties for conjunction, that assumption is easily seen to be untenable. For example, suppose that $\varphi, \varphi \vdash \psi$ holds. Then, by (CONJ$^{\leq}$), $\varphi \wedge \varphi \vdash \psi$ holds, and hence, by $\varphi \vdash \varphi \wedge \varphi$ (and transitivity), $\varphi \vdash \psi$ holds, which precludes the possibility of failure of contraction for $\varphi, \varphi \vdash \psi$. Contraposing, failure of contraction for $\varphi, \varphi \vdash \psi$ together with (CONJ$^{\leq}$) forces failure of $\varphi \vdash \varphi \wedge \varphi$, and hence failure of the argument against (LMP) of section 3.

It is however one thing for that *specific* argument to fail, and quite another thing to have a theory guaranteed to be proof against *any* kind of paradoxical argument. Let me thus briefly offer some details of a provably consistent non-contractive theory of entailment

[20] Unsurprisingly, it is typically on the strength of these or similar considerations that many systems include something along the lines of $\wedge$-L of section 4, a metarule from which (CONJ$^{\leq}$) falls out as a special case.

[21] One obvious place to start here would be constituted by the so-called '*additive* operators' studied in the literature on linear logics (see again Girard [1987], p. 5), which, as far as I can tell, would comply well with the non-standard requirements on conjunction made by Shapiro's approach. An "additive conjunction" $\overset{A}{\wedge}$ exhibits however yet another non-standard—and philosophically hugely controversial—feature: namely, that $\langle . \rangle \vdash \neg(\varphi \overset{A}{\wedge} \psi)$ holds only if either $\langle . \rangle \vdash \neg\varphi$ holds or $\langle . \rangle \vdash \neg\psi$ holds (the dual "additive disjunction" would exhibit the usual constructivist disjunction property).

[22] I hasten to add that the converse direction of the equivalence (i.e. $\varphi \wedge \varphi \vdash \varphi$) is completely unproblematic in this dialectic, and in fact holds in virtually every system of interest here, including the one which I'm about to offer.

and transparent truth on which (CONJ$^{\leq}$) holds while $\varphi \vdash \varphi \wedge \varphi$ fails, and then examine how the theory deals with the serious problem faced by Shapiro's approach discussed in section 3. For our purposes, it will suffice to focus on the quantifier-free fragment of the theory (which, for reasons that will shortly become obvious, I'll call '$\mathbf{ICKT}_{\Rightarrow}$'). I stress that such focus is compatible with discerning at least the subsentential structure of *predication* (which is of course desirable in a discussion involving a truth predicate!).

$\mathbf{ICKT}_{\Rightarrow}$ can very naturally be presented in sequent-calculus style. The *background logic* is defined as the smallest logic containing as axiom the structural rule:

$$\frac{}{\varphi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \varphi}\ \mathrm{I}$$

and closed under the structural metarules:

$$\frac{\varphi_1, \varphi_2, \varphi_3 \ldots, \varphi_i, \varphi_{i+1} \ldots, \varphi_j \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta}{\varphi_1, \varphi_2, \varphi_3 \ldots, \varphi_{i+1}, \varphi_i \ldots, \varphi_j \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta}\ \text{C-L} \qquad \frac{\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} \varphi_1, \varphi_2, \varphi_3 \ldots, \varphi_i, \varphi_{i+1} \ldots, \varphi_j}{\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} \varphi_1, \varphi_2, \varphi_3 \ldots, \varphi_{i+1}, \varphi_i \ldots, \varphi_j}\ \text{C-R}$$

$$\frac{\Gamma_1 \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta}{\Gamma_1, \Gamma_2 \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta}\ \text{K-L} \qquad \frac{\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_1}{\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_1, \Delta_2}\ \text{K-R}$$

$$\frac{\Gamma_1 \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_1, \varphi \qquad \Gamma_2, \varphi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_2}{\Gamma_1, \Gamma_2 \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_1, \Delta_2}\ \mathrm{S}$$

and under the operational metarules:

$$\frac{\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta, \varphi}{\Gamma, \neg\varphi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta}\ \neg\text{-L} \qquad \frac{\Gamma, \varphi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta}{\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta, \neg\varphi}\ \neg\text{-R}$$

$$\frac{\Gamma, \varphi, \psi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta}{\Gamma, \varphi \wedge \psi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta}\ \wedge\text{-L} \qquad \frac{\Gamma_1 \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_1, \varphi \qquad \Gamma_2 \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_2, \psi}{\Gamma_1, \Gamma_2 \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_1, \Delta_2, \varphi \wedge \psi}\ \wedge\text{-R}$$

$$\frac{\Gamma_1, \varphi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_1 \qquad \Gamma_2, \psi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_2}{\Gamma_1, \Gamma_2, \varphi \vee \psi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_1, \Delta_2}\ \vee\text{-L} \qquad \frac{\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta, \varphi, \psi}{\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta, \varphi \vee \psi}\ \vee\text{-R}$$

$$\frac{\Gamma_1 \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_1, \varphi \qquad \Gamma_2, \psi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_2}{\Gamma_1, \Gamma_2, \varphi \rightarrow \psi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta_1, \Delta_2}\ \rightarrow\text{-L} \qquad \frac{\Gamma, \varphi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta, \psi}{\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} \Delta, \varphi \rightarrow \psi}\ \rightarrow\text{-R}$$

$$\frac{\Gamma_1 \vdash_{\mathbf{ICKT}_\Rightarrow} \Delta_1, \varphi \qquad \Gamma_2, \psi \vdash_{\mathbf{ICKT}_\Rightarrow} \Delta_2}{\Gamma_1, \Gamma_2, \varphi \Rightarrow \psi \vdash_{\mathbf{ICKT}_\Rightarrow} \Delta_1, \Delta_2} \Rightarrow\text{-L} \qquad \frac{\Gamma, \varphi \vdash_{\mathbf{ICKT}_\Rightarrow} \Delta, \psi}{\Gamma \vdash_{\mathbf{ICKT}_\Rightarrow} \Delta, \varphi \Rightarrow \psi} \Rightarrow\text{-R}$$

where Γ and its likes are finite and with the restriction that in $\Rightarrow$-R all the members of Γ and Δ be *logical*$^{\Rightarrow}$ (i.e. be either a sentence of the form $\varphi \Rightarrow \psi$ or be the negation, conjunction, disjunction or $\rightarrow$-implication of logical$^{\Rightarrow}$ sentences).

$\mathbf{ICKT}_\Rightarrow$ can be extended to a *theory of truth* by adding the following metarules for the truth predicate:

$$\frac{\Gamma, \varphi \vdash_{\mathbf{ICKT}_\Rightarrow} \Delta}{\Gamma, T(\ulcorner \varphi \urcorner) \vdash_{\mathbf{ICKT}_\Rightarrow} \Delta} T\text{-L} \qquad \frac{\Gamma \vdash_{\mathbf{ICKT}_\Rightarrow} \Delta, \varphi}{\Gamma \vdash_{\mathbf{ICKT}_\Rightarrow} \Delta, T(\ulcorner \varphi \urcorner)} T\text{-R}$$

In Zardini [2011], I investigate in some detail the background-logical and truth-theoretic strength of what is essentially $\mathbf{ICKT}_\Rightarrow$ (while proving its consistency), especially in the surprisingly many respects of philosophically interesting strength in which it outperforms other non-classical solutions to the semantic paradoxes (indeed, I do all this for my favoured $\mathbf{ICKT}_\Rightarrow$'s extension to the first order).[23] I refer the reader to that paper for a sustained philosophical and technical study of $\mathbf{ICKT}_\Rightarrow$, and limit myself here to three remarks that are germane to the issues at hand.

Firstly, while $\rightarrow$ is meant to express ordinary *material implication*, $\Rightarrow$ is meant to express *entailment*. This difference between the two operators is determined by the restriction stated above on $\Rightarrow$-R, which blocks the derivation in $\mathbf{ICKT}_\Rightarrow$ of the "paradoxes" of material implication for $\Rightarrow$ (while the absence of an analogous restriction on $\rightarrow$-R allows the derivation in $\mathbf{ICKT}_\Rightarrow$ of those "paradoxes" for $\rightarrow$, so that both $\varphi \vdash_{\mathbf{ICKT}_\Rightarrow} \psi \rightarrow \varphi$ and $\neg\varphi \vdash_{\mathbf{ICKT}_\Rightarrow} \varphi \rightarrow \psi$ hold).[24]

Secondly, technically speaking, the $\Rightarrow$-free fragment of $\mathbf{ICKT}_\Rightarrow$ corresponds to the multiplicative fragment of sentential *affine* logic (i.e. linear logic plus K-L and K-R). The choice of *multiplicative* operators is crucial for $\mathbf{ICKT}_\Rightarrow$ to have the advantages over Shapiro's "additively inclined" theory of truth (see fn 21) which I'll discuss shortly, and it is arguable that, with the presence of *monotonicity* (i.e. K-L and K-R) as a structural

[23] While transparent theories of truth typically do accept the *structural metarule* of contraction (i.e. what I'm calling throughout simply 'contraction'), such theories almost invariably have to reject what may be called 'the *law* of contraction' for their conditional $\rightarrow$ (i.e. the statement $(\varphi \rightarrow (\varphi \rightarrow \psi)) \rightarrow (\varphi \rightarrow \psi)$), whereas $\mathbf{ICKT}_\Rightarrow$ rejects both the structural metarule of contraction and the law of contraction (the latter both for $\rightarrow$ and for $\Rightarrow$). Recent examples of transparent theories that accept the structural metarule of contraction but reject the law of contraction for their conditional $\rightarrow$ include Field [2008]; Beall [2009]. These theories accept the $\rightarrow$-analogues of (RMP^2) and (RMP^1), but reject the $\rightarrow$-analogues of (DT^1) and (LMP), whereas, as I'll explain in the text, $\mathbf{ICKT}_\Rightarrow$ accepts all these principles (both for $\rightarrow$ and for $\Rightarrow$).

[24] To avoid a possible misunderstanding, I should add that, although $\Rightarrow$ is meant to express a reasonable notion of *entailment*, it is not meant to express a notion of *relevant entailment*. For, by I, $\varphi \Rightarrow \varphi \vdash_{\mathbf{ICKT}_\Rightarrow} \varphi \Rightarrow \varphi$ holds, and hence, by K-L, $\varphi \Rightarrow \varphi, \psi \vdash_{\mathbf{ICKT}_\Rightarrow} \varphi \Rightarrow \varphi$ holds, and so, by $\Rightarrow$-R, the relevant anathema $\varphi \Rightarrow \varphi \vdash_{\mathbf{ICKT}_\Rightarrow} \psi \Rightarrow (\varphi \Rightarrow \varphi)$ holds.

metarule in affine logics, multiplicative operators can adequately represent our informal, *extensional* notions of conjunction and disjunction.

This last claim is however open to a natural worry: for $\mathbf{ICKT}_{\Rightarrow}$ is designed for $\varphi \vdash \varphi \wedge \varphi$ to fail, and that might seem like a big cost of the theory—in fact, one that is paid exactly in terms of failure of extensionality. In reply, I'll make three points. For starters, given the extreme compellingness of (CONJ$^{\leq}$) and the argument offered at the beginning of this section to the effect that (CONJ$^{\leq}$) and $\varphi \vdash \varphi \wedge \varphi$ jointly imply contraction, it is arguable that failure of $\varphi \vdash \varphi \wedge \varphi$ is *no additional cost* over and above failure of contraction. In fact, by I and $\wedge$-R, contraction conversely implies $\varphi \vdash \varphi \wedge \varphi$, so that, under extremely compelling assumptions, $\varphi \vdash \varphi \wedge \varphi$ *just is* contraction in "conjunctive clothing". These considerations are enough to clear $\mathbf{ICKT}_{\Rightarrow}$—with its characteristic failure of $\varphi \vdash \varphi \wedge \varphi$—of any alleged disadvantage *vis-à-vis* alternative theories of truth (such as Shapiro's) *on which too contraction fails.*[25] Moreover, to speak directly to the alleged failure of extensionality, failure of $\varphi \vdash \varphi \wedge \varphi$ is perfectly compatible with the principle that *the truth of both conjuncts is sufficient for the truth of a conjunction* (formalised as $T(\ulcorner\varphi\urcorner) \wedge T(\ulcorner\psi\urcorner) \vdash T(\ulcorner\varphi \wedge \psi\urcorner)$), which encodes well the relevant aspect of extensionality allegedly threatened by failure of $\varphi \vdash \varphi \wedge \varphi$ and which is in fact provable in $\mathbf{ICKT}_{\Rightarrow}$ (see Zardini [2011], pp. 521–522 for further discussion). In fact, the *full truth-functionality* of conjunction (formalised as $\langle . \rangle \vdash ((T(\ulcorner\varphi\urcorner) \wedge T(\ulcorner\psi\urcorner)) \rightarrow T(\ulcorner\varphi \wedge \psi\urcorner)) \wedge (T(\ulcorner\varphi \wedge \psi\urcorner) \rightarrow (T(\ulcorner\varphi\urcorner) \wedge T(\ulcorner\psi\urcorner))))$ is provable in $\mathbf{ICKT}_{\Rightarrow}$ (see Zardini [2011], p. 521), so that $\mathbf{ICKT}_{\Rightarrow}$ upholds the principle that *the truth of a conjunction wholly consists in the truth of both conjuncts*, which in turn is all what extensionality can reasonably be taken to require. Finally, if one so wishes, one can *consistently add* to $\mathbf{ICKT}_{\Rightarrow}$ the "additive conjunction" $\overset{A}{\wedge}$ of fn 21, which does validate $\varphi \vdash \varphi \overset{A}{\wedge} \varphi$. The problem with $\overset{A}{\wedge}$ is not at all that it expresses a notion that, on pain of some sort of "revenge", cannot be expressed in $\mathbf{ICKT}_{\Rightarrow}$—it is simply that it expresses a notion which, as I'm variously arguing in this paper, does not square well with our informal notion of conjunction.[26]

Thirdly, it should be recorded that an apparently minor variation on (MMP$^{\mathrm{R}}$) (see fn 10):

(MMP$^{\mathrm{R}}_{\mathrm{W}}$) If $\Gamma \vdash \varphi \Rightarrow \psi$ and $\Gamma \vdash \varphi$ hold, then $\Gamma \vdash \psi$ holds

fails in $\mathbf{ICKT}_{\Rightarrow}$. If (MMP$^{\mathrm{R}}$) can be glossed as the claim that *theory combination* is closed under *modus ponens*, it may be thought that (MMP$^{\mathrm{R}}_{\mathrm{W}}$) can be glossed as the claim that *theories* are closed under *modus ponens*.

[25] In turn, I believe that failure of contraction can be well motivated on the basis of a metaphysical picture quite congenial to certain intuitions of "instability" which we seem to have about the behaviour of paradoxical sentences, but that is a story better left for another occasion (see Zardini [2011], pp. 503–506 for a start).

[26] A completely dual dialectic could of course be had about *disjunction* in $\mathbf{ICKT}_{\Rightarrow}$. Thanks to an anonymous referee for comments that led to the clarifications in this paragraph.

However, I don't think that failure of $(\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}})$ is particularly problematic for $\mathbf{ICKT}_{\Rightarrow}$. To begin with, notice that the $\rightarrow$-analogue of $(\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}})$ is bound to fail *on just about every non-contractive theory of anything*, as long as the theory satisfies I and $\rightarrow$-R. For example, suppose that $\varphi, \varphi \vdash \psi$ holds. Then, by $\rightarrow$-R, $\varphi \vdash \varphi \rightarrow \psi$ holds. Since, by I, $\varphi \vdash \varphi$ also holds, it follows by the $\rightarrow$-analogue of $(\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}})$ that $\varphi \vdash \psi$ holds, which precludes the possibility of failure of contraction for $\varphi, \varphi \vdash \psi$. Contraposing, failure of contraction for $\varphi, \varphi \vdash \psi$ together with I and $\rightarrow$-R forces failure of the $\rightarrow$-analogue of $(\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}})$.

In fact, restoring our focus on transparent theories of truth, the very general result is available that *no transparent theory validating* (DT^1) *validates* $(\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}})$. For notice that $(\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}})$ affords the following derivation of κ:

$$\dfrac{\dfrac{\dfrac{}{T(\ulcorner\kappa\urcorner) \ \vdash \ T(\ulcorner\kappa\urcorner)}\,\text{reflexivity}}{T(\ulcorner\kappa\urcorner) \ \vdash \ T(\ulcorner\kappa\urcorner) \Rightarrow \bot}\,\text{transparency} \qquad \dfrac{}{T(\ulcorner\kappa\urcorner) \ \vdash \ T(\ulcorner\kappa\urcorner)}\,\text{reflexivity}}{\dfrac{T(\ulcorner\kappa\urcorner) \ \vdash \ \bot}{\langle . \rangle \ \vdash \ T(\ulcorner\kappa\urcorner) \Rightarrow \bot}\,(\mathrm{DT}^1)}\,(\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}})$$

(call this derivation 'M'). With M in place, we would then get the following version of Curry's paradox:

$$\dfrac{\dfrac{}{\langle . \rangle \ \vdash \ T(\ulcorner\kappa\urcorner) \Rightarrow \bot}\,\mathrm{M} \qquad \dfrac{\dfrac{}{\langle . \rangle \ \vdash \ T(\ulcorner\kappa\urcorner) \Rightarrow \bot}\,\mathrm{M}}{\langle . \rangle \ \vdash \ T(\ulcorner\kappa\urcorner)}\,\text{transparency}}{\langle . \rangle \ \vdash \ \bot}\,(\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}})$$

Thus, $(\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}})$ has just got to fail in every transparent theory that validates (DT^1).

Interestingly, while such failure might be problematic to explain for certain transparent theories of truth, it is particularly unproblematic for $\mathbf{ICKT}_{\Rightarrow}$, from whose perspective $(\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}})$ can easily be recognised as a spurious principle of *modus ponens*. For *modus ponens*, most paradigmatically in its version as (RMP^2), requires for its being triggered the *joint availability* of a conditional and its antecedent. However, contrary to what happens in contractive frameworks, in a non-contractive framework the mere facts that $\Gamma \vdash \varphi \Rightarrow \psi$ holds and that $\Gamma \vdash \varphi$ holds do not ensure that Γ suffices for the joint availability of $\varphi \Rightarrow \psi$ and φ (in the terminology of fn 25, Γ may contain sentences that behave "unstably", and so, if it does yield $\varphi \Rightarrow \psi$, it cannot be assumed to remain available to yield φ as well). And precisely because facts of that type do not ensure that a theory suffices for the joint availability of the relevant conditional and its antecedent, any gloss of $(\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}})$ in terms of closure under *modus ponens* is actually misconceived: there can only be an issue of something being closed under *modus ponens* if that thing suffices for the joint availability of a conditional and its antecedent.

The point really has nothing in particular to do with *modus ponens*. In a non-contractive framework, *absolutely every two-premise rule* $\chi_1, \chi_2 \vdash \chi_3$ (with χ_1, χ_2 and χ_3 being logically contingent) will have an analogue of ($\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}}$) that may fail, and *precisely for the same reason*: the mere facts that $\Gamma \vdash \chi_1$ holds and that $\Gamma \vdash \chi_2$ holds do not ensure that Γ suffices for the joint availability of χ_1 and χ_2. (The point extends in the obvious way to multi-premise rules more generally.) Those facts do ensure that Γ, Γ suffices for the joint availability of χ_1 and χ_2, but that just indicates that, in a non-contractive framework, the notion of *closure under an i-premise rule* only sensibly applies to the *i-fold combination of theories*[27] (with the theories jointly supplying all the premises of the rule) rather than to a *theory in itself* (where the limit case of a single theory supplying all the premises of the rule is recovered by taking the i-fold combination of a theory with itself), with this reducing to *closure of theories in themselves* in the special case of *one*-premise rules and in the special case in which the single theory supplying all the premises of the rule is *either logically true or logically absurd*. And it is indeed the case that, for every i-premise rule valid in $\mathbf{ICKT}_{\Rightarrow}$, i-fold theory combination is closed under that rule in $\mathbf{ICKT}_{\Rightarrow}$. In particular, that this is so for (RMP^2) results in the fact that ($\mathrm{MMP}^{\mathrm{R}}$) holds in $\mathbf{ICKT}_{\Rightarrow}$ (where the case of a single theory supplying both a conditional and its antecedent—the case at issue with ($\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}}$)—is recovered by taking the two-fold combination of a theory with itself), and that this is so for (RMP^1) results in the fact that theories in themselves are closed under (RMP^1) in $\mathbf{ICKT}_{\Rightarrow}$ (that is, if $\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} (\varphi \Rightarrow \psi) \wedge \varphi$ holds, then $\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} \psi$ holds—which incidentally makes it particularly vivid where the spurious plausibility of ($\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}}$) comes from, since the mere facts that $\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} \varphi \Rightarrow \psi$ holds and that $\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} \varphi$ holds only ensure that $\Gamma, \Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} (\varphi \Rightarrow \psi) \wedge \varphi$ holds, but do not ensure that $\Gamma \vdash_{\mathbf{ICKT}_{\Rightarrow}} (\varphi \Rightarrow \psi) \wedge \varphi$ holds).[28]

Let's now move on to the examination of how $\mathbf{ICKT}_{\Rightarrow}$ deals with the serious problem faced by Shapiro's approach. For starters, recall that I agree with Shapiro that $\Rightarrow$ should obey (RMP^2). And (RMP^2) in fact holds in $\mathbf{ICKT}_{\Rightarrow}$. (By I, $\varphi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \varphi$ and $\psi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \psi$ hold, and hence, by $\Rightarrow$-L, $\varphi, \varphi \Rightarrow \psi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \psi$ holds, and so, by C-L, $\varphi \Rightarrow \psi, \varphi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \psi$ holds.)

Moreover, recall that, *contra* Shapiro, I endorse ($\mathrm{CONJ}^{\leq}$). And ($\mathrm{CONJ}^{\leq}$) in fact holds in $\mathbf{ICKT}_{\Rightarrow}$. (It's just a restricted version of $\wedge$-L, see also fn 20.) Pleasingly enough, (RMP^2) and ($\mathrm{CONJ}^{\leq}$) jointly imply that (RMP^1) also holds, thus avoiding the unnatural severing of (RMP^1) and (RMP^2) which constituted the serious problem faced by Shapiro's approach.

Finally, recall that I agree with Shapiro that $\Rightarrow$ should obey (DT^1). And (DT^1) in fact holds in $\mathbf{ICKT}_{\Rightarrow}$. (It's just a restricted version of $\Rightarrow$-R.) Pleasingly enough, (RMP^1) and

[27] The i-fold combination of theory Γ_1, theory Γ_2, theory $\Gamma_3 \ldots$ and theory Γ_i is the sequence $\Gamma_1, \Gamma_2, \Gamma_3 \ldots \Gamma_i$.

[28] The considerations in the text should have made it abundantly clear why the first objection raised in fn 10 against failure of ($\mathrm{MMP}^{\mathrm{R}}$) does not apply to failure of ($\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}}$) in a non-contractive framework. As for the second objection, it would apply to failure of ($\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}}$) in $\mathbf{ICKT}_{\Rightarrow}$ only if, in this new context, it were also natural to assume that, in some of the cases in which ($\mathrm{MMP}^{\mathrm{R}}_{\mathrm{W}}$) fails, we may still accept Γ even if ψ is absurd. But while that assumption was natural in the context of the kind of theory of truth discussed in fn 10, it is no longer so in the context of $\mathbf{ICKT}_{\Rightarrow}$.

(DT[1]) jointly imply that (LMP) also holds, thus providing the proposition that is most obviously and plausibly associated with the rules of inference (RMP[2]) and (RMP[1]). And since $\mathbf{ICKT}_{\Rightarrow}$ is provably consistent, the validity of (LMP) does not subject $\mathbf{ICKT}_{\Rightarrow}$ to any paradoxical reasoning (for example, the argument against (LMP) of section 3 fails because in $\mathbf{ICKT}_{\Rightarrow}$ φ is provably not equivalent with $\varphi \wedge \varphi$, and in particular $\varphi \vdash_{\mathbf{ICKT}_{\Rightarrow}} \varphi \wedge \varphi$ provably does not hold).[29]

Summing up these findings, it looks like $\mathbf{ICKT}_{\Rightarrow}$ can have its cake and eat it, validating all of (RMP[2]) (as just about every theory of truth does for its own implication, including Shapiro's), (RMP[1]) (as just about every theory does for its own implication and conjunction, save for Shapiro's), (DT[1]) (as just about no transparent theory does for its own implication, save for Shapiro's) and (LMP) (as just about no transparent theory does for its own implication and conjunction, including Shapiro's).[30] It is worth pointing out that these pleasing results about *modus ponens* in $\mathbf{ICKT}_{\Rightarrow}$ follow from a more general fact about the connection in $\mathbf{ICKT}_{\Rightarrow}$ between logical consequence on the one hand and entailment and conjunction (and disjunction) on the other hand: if $\varphi_1, \varphi_2, \varphi_3 \ldots, \varphi_i \vdash_{\mathbf{ICKT}_{\Rightarrow}} \psi_1, \psi_2, \psi_3 \ldots, \psi_j$ holds, then, by $\wedge$-L, $\vee$-R and $\Rightarrow$-R, $\langle . \rangle \vdash_{\mathbf{ICKT}_{\Rightarrow}} (\varphi_1 \wedge \varphi_2 \wedge \varphi_3 \ldots \wedge \varphi_i) \Rightarrow (\psi_1 \vee \psi_2 \vee \psi_3 \ldots \vee \psi_j)$ holds. And, by transparency, that in turn yields that, if $\varphi_1, \varphi_2, \varphi_3 \ldots, \varphi_i \vdash_{\mathbf{ICKT}_{\Rightarrow}} \psi_1, \psi_2, \psi_3 \ldots, \psi_j$ holds, then $\langle . \rangle \vdash_{\mathbf{ICKT}_{\Rightarrow}} (T\ulcorner\varphi_1\urcorner \wedge T\ulcorner\varphi_2\urcorner \wedge T\ulcorner\varphi_3\urcorner \ldots \wedge T\ulcorner\varphi_i\urcorner) \Rightarrow (T\ulcorner\psi_1\urcorner \vee T\ulcorner\psi_2\urcorner \vee T\ulcorner\psi_3\urcorner \ldots \vee T\ulcorner\psi_j\urcorner)$ holds as well. $\mathbf{ICKT}_{\Rightarrow}$ is thus revealed as a particularly suitable theory of entailment and transparent truth for the truth-preservation theory of logical consequence mentioned at the end of section 1, since the last result amounts to the fact that $\mathbf{ICKT}_{\Rightarrow}$ implies of each of its valid arguments that it preserves truth (a result that is unavailable on just about every other theory, see Field [2006] for an insightful discussion which however does not encompass a theory like $\mathbf{ICKT}_{\Rightarrow}$).

[29](LMP) is sometimes very tendentiously attached pejoratives like 'pseudo *modus ponens*', but, after our discussion, that should be more apt to seem the invidious reaction of someone who's shot oneself in the foot by endorsing a theory of truth inconsistent with it.

[30]I should note that (DT[1]) and (LMP) are available in a transparent theory based on the paraconsistent logic **LP** (for which see e.g. Priest [1979]) *if* one interprets implication as *material implication*. Setting aside the obvious point that material implication is a completely inadequate representation of entailment by everyone's lights, that is not usually taken to be a viable option even for a more ordinary implication as, on that understanding of implication, both (RMP[2]) and (RMP[1]) fail in **LP**. I'm of course sympathetic to the idea of validating (LMP), but I also think that achieving that at the expenses of the validity of (RMP[2]) and (RMP[1]) deprives it of much of its value. And whatever value is left is spoiled by the consideration that, in **LP**, the failure of (RMP[2]) and (RMP[1]) implies that there are cases where a true conditional has a true (and, given other details of the theory, false) antecedent and an untrue consequent (recall fn 9). Similarly to my discussion of the second kind of non-transitivist theory in section 2, that is not only in itself rebarbative; it also means to reject the very semantic idea with which *modus ponens* is intimately connected and in which it is arguably grounded. And that in turn casts even more serious doubts on the alleged significance of the fact that (LMP) nevertheless holds, since, on the strategy under consideration, its instance with $T(\ulcorner\kappa\urcorner)$ for φ and $\perp$ for ψ has a true antecedent and an untrue consequent: given the concession just made that (LMP) can have a true antecedent and an untrue consequent, what substance is left to the claim—made in accepting it—that it is nevertheless a true implication? (Notice that the problems discussed in this fn are even further exacerbated by the fact that, in this dialectic, 'true' and 'untrue' can be strengthened to 'logically true' and 'logically absurd' respectively.) Thanks to Paul Égré for pressing me on this point.

I like to think that (RMP^2), (RMP^1) and (LMP) form the golden triangle of naive *modus ponens*: the route from the first to the third can be seen as a progressive naive *descent* from the metalanguage to the object language, and the converse route from the third to the first can be seen as a progressive naive *ascent* from the object language to the metalanguage. To wit, (RMP^2) says that ψ logically follows from $\varphi \Rightarrow \psi$ *and* φ. Using conjunction, (RMP^1) then interiorises into the object language the metalinguistic premise-combining particle, thus saying that ψ *logically follows from* $(\varphi \Rightarrow \psi) \wedge \varphi$. Using entailment, (LMP) finally interiorises into the object language the metalinguistic residuum constituted by the logical-consequence predicate, thus providing the fully non-metalinguistic proposition that is most obviously and plausibly associated with the metalinguistic rules of inference (RMP^2) and (RMP^1)—that is, the proposition expressed by $((\varphi \Rightarrow \psi) \wedge \varphi) \Rightarrow \psi$. It is the topic for another occasion to investigate the relationships among the sides of this golden triangle; the point of this paper was to show that, contrary to what Shapiro (alongside many other authors) seems to assume, once a broadly non-contractive approach to entailment and transparent truth is adopted the triangle comes at last within reach.

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