Luminosity and Determinacy*

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1 A Traditional Philosophical Idea

It is a traditional philosophical idea that certain domains of facts—such as, for example, facts concerning our sensations, our occurrent thoughts, basic conceptual truths etc.—are *fully open to our view* (*evident*, *manifest*, *transparent* etc.). In attempts at spelling out this idea, it is usual to replace what are for the most part merely suggestive *visual metaphors* with *epistemic notions* such as that of *knowledge*. However, one cannot say that the facts—whatever they are—belonging to those domains are *known*, for whether something is known depends on whether it is believed,[1] but whether something is fully open to our view does not so depend. It is natural to obviate to this problem by saying that the facts—whatever they are—belonging to those domains are *knowable*. Setting aside the many vagaries unearthed by philosophical reflection on the notion of knowability, the notion seems anyways ill-suited to capture the traditional philosophical idea: for example, Andrew Wiles' proof shows that Fermat's Last Theorem was knowable all along, but, before the

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[1]The question of what *confidence requirements* (if any) there are on knowledge is arguably of great momentum for some of the issues touched on by this paper (see Zardini [2012c] for an extensive discussion). However, as far as the specific arguments developed in the paper are concerned, we can for simplicity acquiesce in the traditional view that what is required for knowledge at the level of confidence is *belief*.

proof was offered, the truth of the theorem surely wasn't fully open to our view. To capture the traditional philosophical idea, we thus do need a *modalised* notion of knowledge, but a *stronger* one than mere knowability.

Say that one is *in a position to know* that P iff [P[2] and, in order for one to (come to) know that P, one only needs to (come to) believe that P on grounds one has *already* available].[3] For example, if one is looking at a chair under excellent perceptual conditions, is having the corresponding experience and meets whatever other epistemic (as opposed to psychological) conditions there are on one's knowing that there is a chair, then one is in a position to know that there is a chair, even if in fact, for some reason or other, one does not believe that there is a chair. One has already available certain grounds, at least partly constituted in some sense or other by one's perception that there is a chair, such that, if one were to come to believe that there is a chair on those grounds, one would also come to know that there is a chair. (By contrast, before the proof of Fermat's Last Theorem was offered, one did not (yet) have available any grounds such that, if one had come to believe Fermat's Last Theorem on those grounds, one would also have come to know Fermat's Last Theorem, and so one was not (yet) in a position to know Fermat's Last Theorem.) No doubt, the specific modality involved here would require a much more detailed explication and a thorough investigation, but for our purposes the explication just given will suffice (see e.g. Williamson [2000], p. 95 for a brief explication along the same lines).[4]

One can then spell out the traditional philosophical idea by saying that *one is always in a position to know* that the facts—whatever they are—belonging to the relevant domains obtain. Take for example the domain consisting of the facts concerning how hot one feels (e.g. the fact that one feels cold, or the fact that one feels warm, or the fact that one feels lukewarm). It is a

[2]This first conjunct of the definition is needed in order to guarantee the *factivity* of being in a position to know (i.e. if one is in a position to know that P, then P), which it is not clear is already enforced by the second conjunct. That depends on how the notion of a ground's being already available is further explicated, an issue on which I shall remain silent in this paper. To give a flavour of at least one worry here, suppose that one knows that beliefs whose content is expressed by a sentence of the form 'I am entertaining the thought that P' are self-verifying and typically knowledgeable. The notion of a ground's being already available may well be explicated in such a way that, although one is not actually entertaining the thought that snow is white, it is true that, in order for one to come to know that one is entertaining the thought that snow is white, one only needs to come to believe that one is entertaining the thought that snow is white on grounds one has already available. Under these assumptions, leaving out the first conjunct of the definition would result in a notion of being in a position to know such that, although one is not actually entertaining the thought that snow is white, one may nevertheless be in a position to know that one is entertaining the thought that snow is white.

[3]Throughout, I use square brackets to disambiguate constituent structure in English.

[4]What is the relation between *being in a position to be known* and *being knowable*? The modality involved in the notion of being knowable is of course in just as much need of explication and investigation as the modality involved in the notion of being in a position to be known. However, this much can be said: on at least one usual philosophical understanding of the notion (prominent for example in the tradition of *semantic anti-realism*, see e.g. Tennant [2000]), being knowable amounts in effect to some fairly strict possibility of being in a position to be known—a possibility that is at least no weaker than a nomological one relative to the state of the world at which the claim of knowability is to be evaluated. Hence, being in a position to be known implies being knowable but not *vice versa*. For example, I might not know whether there are dogs, the only dog might be next room, in excellent perceptual conditions for all and only those entering the room, and there might be no way of coming to know that there are dogs other than by perceiving the only existing dog. In such a situation, although I am not (yet) in a position to know that there are dogs, it is knowable for me that there are dogs, since I can very easily enter next room and perceive the dog, thereby coming to be in a position to know that there are dogs (thus, if I also form the belief that there are dogs on those grounds, I will know that there are dogs). Having drawn this distinction, I will move freely from being knowable to being in a position to be known when that is warranted by the context.

traditional philosophical idea that this domain of facts is fully open to our view, and one can spell out that idea by saying that one is always in a position to know that the facts—whatever they are—belonging to this domain obtain. Hence, if one in effect feels cold, one is in a position to know that one feels cold; if one in effect feels warm, one is in a position to know that one feels warm; if one in effect feels lukewarm, one is in a position to know that one feels lukewarm.[5]

Assuming the familiar apparatus of possible-world semantics, we can define a notion, *luminosity*, which will help to articulate further this traditional philosophical idea, while also providing a simple and neat framework in which the issues of concern for this paper can be sharply presented and investigated. We proceed stepwise by introducing the following notions:

Definition 1. A *case* is a centred world—that is, a possible world with a designated subject (referred to by the pronoun 'one') and time (referred to by the present tense).

Definition 2. A *condition* is a subject- and time-sensitive "proposition".[6]

Definition 3. A condition $\langle C \rangle$[7] is *luminous* iff, for every case α, if $\langle C \rangle$ is true at α, then in α one is in a position to know that C.

A set of luminous conditions thus *seems* to correspond fairly well to a domain of facts fully open to our view (at least *modulo* the remarks in fns 5 and 6).[8] The rest of this paper will be concerned with a very challenging argument (henceforth, 'the *AL-argument*'), first put forth by Tim Williamson (see Williamson [1995]—where however only an importantly different variant is developed—and then Williamson [1996]; Williamson [2000], pp. 93–113), to the effect that almost no interesting condition is luminous in this sense.

[5] I don't claim a perfect match between the visual metaphor of being fully open to our view and its proposed epistemic cash value in terms of being in a position to be known. An interesting case of potential divergence is familiar from the epistemological literature: in barn-façade country (see Goldman [1976]), the fact that that structure is a barn is intuitively fully open to Henry's view, although, at least according to many epistemologists, Henry is not in a position to know that that structure is a barn. This being noted, since I don't think that the potential divergence makes any difference to the *specific* arguments I develop in this paper, I'll henceforth assume that there is in this sense no gap between a fact's being fully open to *our view* and its being fully open to *our knowledge*. This is not to say that the potential divergence is in no way relevant to the *general* topic I'll be addressing: since the argument against the traditional philosophical idea which is the starting point of my discussion basically consists in the mere claim that certain beliefs are unreliable, one can reasonably wonder—on the strength of apparently similar examples such as Henry's—how that is supposed to show that every domain of facts is not even fully open to *our view* (over and above its merely not being fully open to *our knowledge*).

[6] For simplicity, I'm following very closely the set-up of Williamson [2000], pp. 93–113. Barring problems of apparent hyperintensionality (which we'll set aside here) and assuming (as we'll do) that there is a set of all cases and that all conditions are *bivalent* at all cases (i.e. at every case α, either true at α or false at α), a condition can be modelled by a *set* of cases (that is, most naturally, by the set of all and only those cases at which the condition is true). Arguably, the possibly most distorting feature of such a set-up is that it models the objects of knowledge, belief and confidence with entities whose truth and falsity are relative to subjects and times, while there are serious, if not necessarily compelling, reasons to think that the truth and falsity of what one knows, believes and is confident about are not relative to subjects or times (see e.g. Stalnaker [1981], pp. 145–147 and Richard [1981] respectively). Nevertheless, for uniformity, I'm henceforth going to assume conditions to be the objects of knowledge, belief and confidence, adapting traditional claims and arguments to this assumption. See fn 11 for a brief indication of how to recast the discussion to follow if this rather peculiar feature of the set-up is abandoned.

[7] Throughout, I'll use '$\langle \varphi \rangle$' as a singular term referring to the condition expressed by φ.

[8] Only 'seems'. A central point of this paper is that, on many theories of vagueness, a condition might be concerned with a domain of facts fully open to our view without being luminous in the sense of definition 3 (see section 3 and fn 32).

2 The Anti-Luminosity Argument

The argument takes as paradigmatic target the condition ⟨One feels cold⟩ and proceeds by considering the series of cases that correspond to the possible situation characterised by the following three features:

(POSNEG) One feels very cold at dawn and gradually warms up until one feels very warm at noon;

(FORM) Throughout the process, one is steadily attending to one's own thermal sensations and steadily considering, on this basis, whether one feels cold, forming, with regard to this matter and on this basis, whatever confidence one needs to form in order to achieve knowledge;

(CONF) While at dawn one is extremely confident that one feels cold and not confident at all that one does not feel cold, one gradually loses confidence that one feels cold and gradually acquires confidence that one does not feel cold, until at noon one is extremely confident that one does not feel cold and not confident at all that one feels cold.

Let t_1, t_2, $t_3 \ldots t_{1,000,000,000}$ be distinct equidistant times from dawn to noon, and, for every n,[9] let α_n be the case one is in at t_n.

The AL-argument starts with what I think is its crucial step, the derivation of a premise consisting in the *knowledge margin-for-error* principle:

(KMAR) For every n, if in α_n one knows that one feels cold, then in α_{n+1} one feels cold

(see Williamson [1992] for a seminal study of some philosophical motivations and consequences of margin-for-error principles). The derivation of this premise runs as follows. Assume that in α_n one knows that one feels cold. Then in α_n one is reasonably *confident* that one feels cold (otherwise, one would not know that one feels cold). Hence, by (CONF), in α_{n+1} one is at worst ever so slightly less confident that one feels cold. Now, does one feel cold in α_{n+1}? We argue by (classical) *reductio* that one does. For assume for *reductio* that in α_{n+1} one does not feel cold. Then one's still considerably high confidence in α_{n+1} that one feels cold is false and hence *mistaken*, and this in turn would seem to imply that one's confidence in α_n is not *reliable*.[10] This is so because a very similar confidence in a very similar condition[11] formed on a very similar basis in a very similar case—i.e. the confidence one has in α_{n+1} in ⟨One feels cold⟩—is mistaken. But if in α_n one's confidence that one feels

[9] Sometimes (as here) I will implicitly assume that the range of 'n' and of its like is restricted to the set $\{x : 1 \leq x \leq 1,000,000,000\}$ if that's clear enough from the context.

[10] In my view, it is precisely this inference in the derivation of (KMAR) that is fallacious and irremediably so, since, because of considerations I cannot go into here, one's confidence in α_n that one feels cold can arguably be reliable even if one's confidence in α_{n+1} that one feels cold is false. I develop and defend this view at length in Zardini [2012c].

[11] Indeed, in the very same condition (i.e. ⟨One feels cold⟩). In the text, I use a more general formulation since that is the formulation one would need to use if one wanted to run the AL-argument under the alternative assumption that the objects of knowledge, belief and confidence are insensitive to (subjects and) times, so that they are either (objectively and) eternally true or (objectively and) eternally false (see fn 6). On this assumption, the relevant object of a subject s's confidence in α_n is, roughly, the (subject- and) time-insensitive proposition that s feels cold at t_n, while the relevant object of s's confidence in α_{n+1} is, roughly, the *different* and yet *very similar* (subject- and) time-insensitive proposition that s feels cold at t_{n+1}.

cold is not reliable, in α_n one arguably does not know that one feels cold, contrary to our original hypothesis. This is so because of the general principle that knowledge requires reliability, which we interpret in our framework as requiring that, for every n, in α_n one knows that one feels cold only if in α_n one's confidence that one feels cold is reliable.[12] Therefore, by (classical) *reductio*, we'd better say that in α_{n+1} one does feel cold. But α_n was completely arbitrary, and so we can conclude to (KMAR).

Now, if any condition stands a chance of being luminous, it would seem that ⟨One feels cold⟩ does. As applied to our situation, the luminosity of that condition entails:

(LUM) For every n, if in α_n one feels cold, then in α_n one is in a position to know that one feels cold.

But at this point trouble arises. For the net effect of (FORM) is that, for every n, in α_n one knows that one feels cold iff in α_n one is in a position to know that one feels cold. Hence, (LUM) and (FORM) together entail:

(LUM$^+$) For every n, if in α_n one feels cold, then in α_n one knows that one feels cold.

This, together with (KMAR), yields the catastrophical *sorites* premise:

(SOR) For every n, if in α_n one feels cold, then in α_{n+1} one feels cold as well,

which in turn contradicts (POSNEG) by familiar reasoning.[13] Hence, (POSNEG), (FORM), (KMAR) and (LUM) are jointly inconsistent. Since (POSNEG) and (FORM) should be unproblematic, either (KMAR) or (LUM) has to give. But (KMAR) was supported by the appealing derivation given in the previous paragraph, derivation which relied only on (CONF) and on other seemingly plausible premises. There thus seems to be every reason to conclude against (LUM). This is the AL-argument.

3 Determinate Luminosity

There is no denying the fact that, when first confronted with the AL-argument, one is apt to feel that it is illicitly trading on the *vagueness* of the target conditions—in our presentation, on the

[12] In this paper, I will neither contest the general principle nor the interpretation offered in the text of what the principle requires in our framework. I think both issues are worthy of further investigation (Zardini [2012c] tries to make some progress on the latter issue).

[13] The reasoning is valid in most logics of some philosophical interest. However, it is not valid in the family of "*tolerant logics*" developed by Zardini [2008a]; Zardini [2008b], pp. 93–173; Zardini [2012b], logics which have been designed exactly for accommodating a view on vagueness which accepts the relevant sorites premises as correctly expressing what the vagueness of an expression consists in. On such a view, (SOR) is true (indeed, it is true for reasons independent of the AL-argument, as it correctly expresses what the vagueness of 'feels cold' consists in), but, because of the shift to a tolerant logic, it is now harmless and consistent with (POSNEG). As a consequence, on this view, the AL-argument has no bite, even if, up to (SOR), it may well be sound. To give the AL-argument its best chance, I'll henceforth ignore the view, and assume with most theorists that (SOR) does contradict (POSNEG).

vagueness of the conditions that one feels cold, that one is in a position to know that one feels cold and that one knows that one feels cold (and possibly other conditions as well that also, if in a more implicit fashion, play a crucial role in the argument). Vagueness arguably presents itself in a *variety* of phenomena: in a companion to this paper (Zardini [2012c]), I have looked at the AL-argument with a focus on that phenomenon of vagueness consisting in the *apparent absence of sharp boundaries* (see fn 32), while here I will focus instead on that phenomenon of vagueness consisting in the *presence of borderline cases*. The latter phenomenon is constituted by the fact that, on each episode of consideration, a vague expression presents *borderline cases*. For example, on each episode of consideration, some cases are borderline cases of feeling cold, in the sense that, on each episode of consideration, some cases are such that neither a positive nor a negative 'feels cold'-judgement about them can be warranted. We can say that in such cases neither does one *definitely* feel cold nor does one *definitely* not feel cold. More precisely and generally, I'm going to understand 'definitely' as defined in terms of 'borderline' as follows:

Definition 4. It is definitely the case that $\varphi := \varphi$ and it is not borderline whether φ.

Our official explanation of borderline cases is deliberately very weak and minimal (I defend and elaborate on such a minimal conception in Zardini [2008b], pp. 289–304). It thus bears emphasis that most contemporary theories of vagueness presuppose a stronger conception of borderline cases. For starters, on almost all such theories borderline cases are *absolutely* such and not simply relative to an episode of consideration: on these theories, for most F, the property of being a borderline F supervenes on features of an object that have nothing to do with episodes of consideration. Moreover, on almost all theories on which borderline cases are absolutely such, borderline cases are *unknowable* (see fn 4): if x is borderline F, one cannot know that x is F and one cannot know that x is not F (a notable exception to this presupposition is what may well be called 'the *New York tradition*' of Wright [2001]; Dorr [2003]; Barnett [2011]). Indeed, on many theories on which borderline cases are unknowable, this is so not because there is a fact of the matter concerning x's Fness about which we are *ignorant* (as is famously the case according to *epistemic* theories as variously defended by Sorensen [1988], pp. 199–252; Williamson [1994]), but simply because *there just is no fact of the matter* as to whether x is F or not (let's call theories on which in borderline cases there is no fact of the matter and such absence implies unknowability '*NFMU-theories*').

It is crucial to realise that the possibility of absence of a fact of the matter about a certain question brings in its wake *important qualifications to the inference from unknowability to ignorance*. Start by noticing that even in a fact-of-the-matter situation there should be no simple implication from unknowability to ignorance: for example, one cannot know that $2 + 2 = 5$, and so it is unknowable that $2 + 2 = 5$, but, plainly, this does not imply that there is a fact (i.e. the fact that $2 + 2 = 5$) about which we are ignorant (for the only fact there is to be had in the vicinity is that $2 + 2 = 4$, and we do know that!). Now, such a gap between unknowability and ignorance is not plausibly restricted to situations where what is unknowable is false (and so has a true negation which is such that, if one could not know it either, one would indeed be ignorant of some fact). For example, it is a plausible view on what Newton meant by 'mass' that there is no fact of the matter as to whether he meant rest mass or relativistic mass (see Field [1973], pp. 463–473), and it is plausibly a consequence of this view that one cannot know that Newtonian mass is rest mass, and so that it is unknowable that Newtonian mass is rest mass. Again, plainly, this does not imply that there is a fact (i.e. the fact that Newtonian mass is rest mass) about which we are ignorant (for the only fact there is to be had in the vicinity is that there is no fact of the matter as to whether

Newtonian mass is rest mass or not, and we do know that!). Indeed, the gap between unknowability and ignorance is nothing but *amplified* in no-fact-of-the-matter situations. For example, not only is it unknowable that Newtonian mass is rest mass, for the same reasons it is *also* unknowable that Newtonian mass is not rest mass, but, exactly because there is no fact of the matter as to whether Newtonian mass is rest mass or not, all this still does not imply that there is a fact about which we are ignorant (in contrast to fact-of-the-matter situations, in which the unknowability of a condition *and of its negation* does imply that there is a fact about which we are ignorant).

Yet, on any view on which absence of a fact of the matter is incompatible with knowledge, even though, as we've just seen, absence of a fact of the matter does not imply ignorance, it does imply that the relevant condition and its negation are not both luminous in the sense of definition 3, *for reasons completely independent of the AL-argument*. To see this, suppose that there is no fact of the matter as to whether C. Then, since absence of a fact of the matter is incompatible with knowledge, it is neither knowable that C nor knowable that it is not the case that C, and so *a fortiori* (see fn 4) one is neither in a position to know that C nor in a position to know that it is not the case that C. Contraposing on the luminosity of $\langle C\rangle$ and of its negation, we obtain the contradiction that it is not the case that C and that it is not the case that it is not the case that C. The lesson is that, on any view on which absence of a fact of the matter is incompatible with knowledge and on which such absence is possible (and note that, at least with respect to absence of a fact of the matter generated by borderline cases, any NFMU-theory is such), principles like (LUM) already have to be restricted for independent reasons that, as has just been brought out, *have nothing to do with our cognitive powers* (cf Hawthorne [2005a], pp. 3–9).[14] That is, principles like (LUM) already have to be restricted for reasons that do not at all imply that there is some fact about which we are ignorant, and so do not at all constitute a threat to the idea that some domains of facts are fully open to our view. A condition might be concerned with a domain of facts fully open to our view without being luminous in the sense of definition 3.[15]

Let's use a notion of *determinacy* defined as follows:

Definition 5. It is determinately the case that $\varphi := \varphi$ and there is a fact of the matter as to

[14] As has just been made explicit in the text, this specific result depends on the assumption that the negation of a condition concerned with a domain of facts fully open to our view is itself concerned with a domain of facts fully open to our view. I find the assumption very plausible for many conditions that are good candidates for being concerned with a domain of facts fully open to our view. In any event, even dropping that assumption we can still conclude that [it is not the case that C although there is no fact of the matter as to whether C] (and so although it is not a fact of the matter that it is not the case that C). In spite of its no longer being a truth-functional (or quantificational) *contradiction*, such a conclusion is typically considered *inconsistent* by NFMU-theorists (and absence-of-a-fact-of-the-matter theorists more generally), and rightly so in my view (notice that the conclusion is indeed inconsistent given the rule of inference (D-INTRO) introduced and defended in section 5). Hence, absence of a fact of the matter is still inconsistent with the luminosity of the relevant condition in the sense of definition 3, whether or not the condition's negation is also concerned with a domain of facts fully open to our view. Thanks to Paul Égré for raising this issue.

[15] Appealing as such a stance with respect to absence of a fact of the matter might seem, it must be mentioned that it sits very uncomfortably with *classical* patterns of reasoning. For take again a condition $\langle C\rangle$ such that there is no fact of the matter as to whether C. Then, since absence of a fact of the matter is incompatible with knowledge, it is neither knowable that C nor knowable that it is not the case that C. However, by the law of excluded middle, either C or it is not the case that C, wherefore, reasoning by cases with side premises, either C and it is unknowable that C or [it is not the case [that C] and it is unknowable that it is not the case that C]. Few would staunchly hold that such a result is unproblematically consistent with the spirit of the claim that there is no fact concerning $\langle C\rangle$ about which we are ignorant (see Field [2003], pp. 458–459 for a forceful presentation of this problem; Field's own conclusion is that we should give up the law of excluded middle for the relevant claims).

whether φ

and thus standing to the notion of there being no fact of the matter as the notion of definiteness stands to the notion of being borderline (see definition 4). It's easy to check that, on minimal assumptions, definition 5 validates the principle of *duality of absence of a fact of the matter and determinacy*:

(DUAL) There is no fact of the matter as to whether C iff it is neither determinately the case that C nor determinately the case that it is not the case that C

and the principle of *factivity of determinacy*:

(FACT) If it is determinately the case that C, then C

(see Zardini [2006], pp. 215–216 for a statement of the assumptions needed and a proof). Then, since the trouble cases for (LUM) exploited in the previous paragraph are cases in which there is no fact of the matter as to whether one feels cold, the obvious fix is to strengthen its antecedent (and hence weaken (LUM)) in such a way that it is only triggered when there is a fact of the matter as to [whether one feels cold] and one feels cold. Since, by definition 5, this conjunctive condition is equivalent with the condition ⟨It is determinately the case that one feels cold⟩, we obtain the principle of *determinate luminosity*:

(DLUM) For every n, if in α_n it is determinately the case that one feels cold, then in α_n one is in a position to know that one feels cold.[16]

What matters for our purposes is to observe that flat-footedly substituting (DLUM) for (LUM) in the AL-argument would make the argument conclude to the *determinacy margin-for-error* principle:

(DMAR) For every n, if in α_n it is determinately the case that one feels cold, then in α_{n+1} one feels cold

(see theorem 1 in section 5), which is entailed by (by (FACT)) but does not entail (SOR), and which, contrary to (SOR), does not seem to lead to any untoward consequence. It would thus seem that the friend of the idea that some domains of facts are fully open to our view has found a principled and stable response to the AL-argument: for reasons completely independent of the AL-argument, the idea had better be spelt out by (DLUM) rather than (LUM), and, contrary to the latter, the AL-argument does not seem to have any bite against the former (something like this end result, although not the specific train of thought that in this section has led to it, has been suggested by many commentators in either conversation or print as an apt response to the AL-argument, see e.g. Hawthorne [2005b], pp. 452–454).[17]

[16]Notice that (FACT) makes sure that (DLUM) respects the factivity of being in a position to know (see fn 2).

[17]In a *classical* setting, (DLUM) has the following consequence that may be worth mentioning. By familiar classical reasoning, there is a last n such that in α_n one determinately feels cold, and so, by (DMAR), in α_{n+1} [one

4 Determinate Luminosity and Definite Luminosity

Against this, I now wish to argue that (DLUM) is similarly affected by the spirit, if not the letter, of the AL-argument. First, notice that the strategy of endorsing (DLUM) rather than (LUM) is only going to be appealing to an NFMU-theory of vagueness, or, at least, to a theory that holds that borderline cases of feeling cold are cases in which there is no fact of the matter as to whether one feels cold (the latter theory being less committal than the former in that it does not require that absence of a fact of the matter is incompatible with knowledge).[18] For, if, on the contrary, borderline cases of feeling cold were cases in which there is nevertheless a fact of the matter as to whether one feels cold, then, by (DUAL), they would be cases in which either it is determinately the case that one feels cold or it is determinately the case that one does not feel cold, and (DLUM) would then quickly collapse onto (LUM).[19]

I now wish to make a general claim concerning the conditions under which weakenings of (LUM) along the lines of (DLUM) can be legitimately regarded as preserving the idea that some domains of facts are fully open to our view. In order to articulate such claim, we first need some additional background. As we've introduced it, the notion of determinacy is really a particular way of cashing out the notion expressed by 'definitely' as it occurs in definition 4. Let's consider then the more general principle obtained by substituting 'definitely' for 'determinately' in (DLUM):

(DFLUM) For every n, if in α_n it is definitely the case that one feels cold, then in α_n one is in a position to know that one feels cold.

The question arises if the strategy of endorsing (DFLUM) rather than (LUM) is going to be appealing to a theory of vagueness Θ different from an NFMU-theory once 'definitely' as it occurs in (DFLUM) is understood in the way favoured by Θ. Notice that, while the understanding of definiteness as determinacy is, roughly speaking, proper of what are usually called '*semantic*' and '*ontic*' theories of vagueness, other understandings are in effect possible: 'It is definitely the case that φ' could for example be understood along the lines of 'No obstacle of a certain kind prevents knowledge that φ' (as happens on *epistemic* theories of vagueness, see e.g. the references given in the previous section) or along the lines of 'It is correct to hold a standard (possibly partial) belief

still feels cold even if one does no longer determinately feel cold]. Thus, there is a case in which [one feels cold but one does not determinately feel cold]. That is certainly a problematic claim, but it is only very marginally more problematic than, for example, the claim that either there is case in which [one feels cold but it is not determinately the case that one feels cold] or there is a case in which [one does not feel cold but it is not determinately the case that one does not feel cold]. And, reasoning by cases with side premises, the latter claim follows *independently of* (DLUM) merely from the law of excluded middle together with the side premise that, in some case, it is neither determinately the case that one feels cold nor determinately the case that one does not feel cold. In fact, by familiar classical reasoning, the *former claim itself* follows independently of (DLUM) once it is further assumed (extremely plausibly on an NFMU-theory of vagueness) that the last case in which one feels cold is a case in which it is neither determinately the case that one feels cold nor determinately the case that one does not feel cold. Whatever problem emerges with these and similar claims should thus be imputed to classical logic rather than to (DLUM) (see also fn 15). Thanks to Aidan McGlynn for discussions of this issue.

[18] Having made this qualification, in the following I will ignore it, given that the latter kind of theory is in this dialectic considerably less interesting in view of its inability to provide an independent reason for preferring (DLUM) over (LUM).

[19] Here and in the following, I'm assuming that the situation described by the AL-argument is such that borderline cases are the only possible source of absence of a fact of the matter.

to the effect that φ' (as happens on *psychological* theories of vagueness, see e.g. Schiffer [2003], pp. 178–237).

In addressing this and related questions, it will prove useful to introduce the notion of *assertoric content* (see Dummett [1991], p. 48). For our purposes, we won't need a worked out theory of assertoric content and can leave matters at the observation that the assertoric content of φ[20] is determined by *the facts whose obtaining would make* φ *true*.[21] This presumably implies the constraint on *strength of assertoric contents*:[22]

(STR) The assertoric content of φ is at least as strong as the assertoric content of ψ iff, whatever fact whose obtaining would make φ true is also a fact whose obtaining would make ψ true.

Now, the "general claim" mentioned in the second last paragraph is that (DFLUM) can only be legitimately regarded as preserving the idea that some domains of facts are fully open to our view if 'definitely' is understood in such a way that the assertoric content of 'One feels cold' is *at least as strong as* the assertoric content of 'It is definitely the case that one feels cold' (as an interpretation of 'definitely' in terms of determinacy in effect does, since whatever fact whose obtaining would make 'One feels cold' true is also a fact whose obtaining would make 'It is determinately the case that one feels cold' true).[23] For reflect that, if this were not the case, there would be a fact whose

[20]I mean talk of assertoric content not to be restricted to sentences expressing ordinary, subject-insensitive propositions (see definition 2 and fn 6). In particular, I'm going to presuppose that it applies to (open) sentences expressing less ordinary, subject-sensitive conditions.

[21]Here and in the following, 'fact' is used to cover both actually obtaining and merely possibly obtaining facts. Moreover, first-order talk of actually obtaining and merely possibly obtaining facts is simply adopted for convenience and could in principle be eliminated in favour of higher-order locutions. What is not so eliminable is some version or other of talk of truth-making. This is not the place to elaborate on this issue: an intuitive understanding of truth-making will suffice for our purposes. Finally, as for truth itself, I'm assuming a notion of truth that validates the relevant instances of the schema ''P' is true iff P'.

[22]In the following, I will use 'strong' and its relatives to denote the ordering of strength of assertoric contents. It is crucial to see that equivalence in this ordering between the assertoric contents of φ and ψ does not guarantee that φ is *unrestrictedly inter-substitutable* with ψ even outside of (direct- or indirect-) quotation environments. For example, supposing that the reference of 'Antonius' is fixed as the inventor of the telephone, the assertoric content of 'Antonius liked spaghetti' is at least as strong and at least as weak as (and so equivalent with) the assertoric content of 'The inventor of the telephone liked spaghetti': in the intended sense, whatever fact whose obtaining would make 'Antonius liked spaghetti' true is also a fact whose obtaining would make 'The inventor of the telephone liked spaghetti' true and *vice versa*. (Indeed, given that any ordering $\leq$ requires the anti-symmetry property that, if $x \leq y$ and $y \leq x$, then $x = y$, *equivalence* in the ordering of strength of assertoric contents implies *identity*.) However, while 'It is possibly the case that [Antonius liked spaghetti and it is not the case that the inventor of the telephone liked spaghetti]' is true, the result of substituting 'Antonius liked spaghetti' for 'The inventor of the telephone liked spaghetti' would yield a falsehood (see fn 27 for another example of failure of inter-substitutability of sentences having equivalent assertoric contents). We thus need also another domain of entities ("*ingredient senses*", following the terminology of Dummett [1991], p. 48) and an ordering of strength on them such that equivalence in that ordering between the ingredient senses of φ and ψ does guarantee that φ is unrestrictedly inter-substitutable with ψ at least outside of (direct- or indirect-) quotation environments. I will use 'strong*' and its relatives to denote such an ordering of strength of ingredient senses. Notice that I will stick to ordinary English and use 'stronger' ('stronger*') and its relatives to mean what would be meant by 'strictly stronger' ('strictly stronger*') and its relatives in mathematical English, using 'at least as strong as' ('at least as strong* as') and its relatives to mean what would be meant by 'stronger' ('stronger*') in mathematical English.

[23]Here is the implicit reasoning a bit more in detail. If there obtains a fact f whose obtaining makes 'One feels cold' true, then, by our assumption about truth in fn 21, one feels cold and, by a natural assumption, there is a fact of the matter as to whether one feels cold. Hence, by definition 5, it is determinately the case that one feels cold.

obtaining would make 'One feels cold' true without making 'It is definitely the case that one feels cold' true, and hence every case in which that fact obtains would be a case of feeling cold, but would not be guaranteed also to be a case of definitely feeling cold. Thus, it would be a genuine open possibility that some cases of feeling cold are not cases of definitely feeling cold and hence not epistemically constrained by (DFLUM)—for all (DFLUM) would say, it would be genuinely possible that [some borderline cases of feeling cold are indeed cases of feeling cold and yet one is not in a position to know that they are such]. I take it that this would hardly square with the idea that the domain of facts concerning feeling cold is fully open to our view.

Granted, even if that idea is not captured by (DFLUM) as so understood, it may well be that (DFLUM) as so understood is, as far as it goes, *true*. It is not my intent here to argue against this (indeed, I think it's likely that there is a non-trivial strengthening of the antecedent of (LUM) that makes it true, like 'If in α_n one feels incredibly cold'). As a brief detour, I do however want to argue that it is unclear how much *interest* is retained by a principle which in effect amounts to the claim that the domain of facts concerning feeling cold is fully open to our view save for when these facts are borderline. And the first two points to be made in the argument will also cast some doubt as to how much interest is retained by (DLUM) itself.

Firstly, it does not seem that (DFLUM) or even (DLUM) could have all the philosophical impact that was hoped for from the idea that the domain of facts concerning feeling cold is fully open to our view. Think for example of the argument run by Wright [2001] for a broadly intuitionist revision of classical logic with respect to a certain vague discourse Δ: that argument assumes that, for some C belonging to Δ, there is no warrant for believing that either one is in a position to know that C or one is in a position to know that it is not the case that C (roughly, because it is borderline whether C), and concludes from this that there is no warrant for believing that either C or it is not the case that C (the instance of the law of excluded middle for $\langle C \rangle$).[24] Emphatically, that argument requires [that, [[if C, one is in a position to know that C] and, [if it is not the case that C, one is in a position to know that it is not the case that C]] *even if it is not definite or not even determinate whether* C]—indeed, that is in many respects the very crux of the argument. Although there is no space here to pursue this further, I believe that a similar situation obtains also for other arguments based on the idea that certain domains of facts are fully open to our view.

Secondly, it does not seem that typical arguments to the effect that certain domains of facts are fully open to our view actually have a *restriction* to definite facts built into them. Think for example of the argument that tries to establish that the domain of facts concerning having pain is fully open to our view on the grounds that:

(1) Having pain constitutively involves feeling one's pain (pain is something one feels);

(2) If one has the capacity for consciousness, and one feels one's pain, one can do so consciously (quite generally, what one feels one can bring to one's consciousness and consciously feel);

(3) If one has the concept of pain, of self and of possession, and one consciously feels one's pain, one can consciously feel one's pain as such (in consciously feeling one's pain, one's pain is presented

But, by our assumption about truth in fn 21, that suffices for 'It is determinately the case that one feels cold' to be true. Since, given natural assumptions, this conclusion has been secured by f's obtaining, f's obtaining makes 'It is determinately the case that one feels cold' true.

[24] I'm here adapting a bit Wright's argument to fit our setting in ways that seem to me innocuous.

to one in such a way that, if one exercises one's relevant conceptual capacities, one can feel one's pain as one's pain);

(4) Consciously feeling one's pain as such constitutively involves being aware of one's pain as such (quite generally, consciously feeling an F as F implies being aware of an F as F);

(5) Being aware of one's pain as such constitutively involves being aware that it is one's pain (quite generally, being aware of x as F implies being aware that x is F);

(6) Being aware that something is one's pain constitutively involves being aware that one has pain (quite generally, being aware that x is y's F implies being aware that y has an F);

(7) Being aware that one has pain constitutively involves knowing that one has pain (quite generally, being aware that C implies knowing that C).

(1)–(7) very plausibly entail that, if it is at least a fact of the matter that a suitably endowed subject has pain, it can be the case that it can be the case that she knows that she has pain (the double possibility deriving from the modalised consequents of (2) and (3)). Hence, assuming a modal logic strong enough as to yield collapse of possibilities (if it can be the case that it can be the case that C, it can be the case that C), (1)–(7) very plausibly entail that, if it is at least a fact of the matter that a suitably endowed subject has pain, it can be the case that she knows that she has pain. Moreover, (2) and (3) make clear that the modality expressed in this context by 'can' is strict enough as to guarantee that, if it can be the case that one knows that one has pain, one is in a position to know that one has pain. Therefore, concluding, if it is at least a fact of the matter that a suitably endowed subject has pain, she is in a position to know that she has pain.

This (sketch of an) argument relies on principles that, insofar as they can be assumed to enjoy an initial intuitive appeal, are most plausibly interpreted as being *perfectly general* principles about facts concerning having pain, feeling one's pain, being aware of one's pain etc., rather than being somehow tacitly restricted to definite cases of such facts. If something along the lines of this (sketch of an) argument is good (and I'm of course suspending judgement as to whether it is!), it thus gives reason to believe at least the relevant analogue of (DLUM) and not simply the relevant analogue of (DFLUM)—indeed, the seven connections between conditions stated in (1)–(7) may be so *tight* as to give reason to believe the relevant analogue of (LUM) and not simply the relevant analogue of (DLUM) (i.e. at each of (1)–(7), it may be that the first condition is, in the sense of fn 22, at least as *strong** as the second). Again, although there is no space here to pursue this further, I believe that a similar situation obtains also for other arguments in favour of the idea that certain domains of facts are fully open to our view.

Thirdly, recall that the understanding of 'definitely' under consideration is such that [it is a genuine open possibility [that C] even if it is borderline whether C]. Now, consider a possible situation where a human being, Sherry, with ordinary powers of sight, is looking at trees at a 100-yard distance such that, for each of the trees, it is borderline whether the tree is at least 50 ft tall[25] and such that some of the trees are in effect at least 50 ft tall and the others are not. Surely, this is not a situation where the domain of facts concerning a tree's being or not being at least 50

[25]True, 'at least 50 ft tall' is much less vague than 'tall', yet it also presents borderline cases. To get an idea of how this can be so, suppose that there is some wood on the top of a tree such that it would make a difference as to whether the tree is at least 50 ft tall or not, but also such that it is borderline whether it belongs to the tree or not.

ft tall is fully open to Sherry's view. Importantly, this is so not so much because of the vagueness of 'at least 50 ft tall'—the first and foremost source of Sherry's ignorance is rather the *limitation of her powers of sight*, which wouldn't allow her to know whether the trees are at least 50 ft tall even if they were not borderline 50 ft tall. Yet, an analogue for 'at least 50 ft tall' of (DFLUM) is vacuously satisfied in this situation. Thus, satisfaction of (analogues of) (DFLUM) doesn't even suffice to determine that any ignorance that there might be in a certain situation has its ultimate *source* in vagueness.

Finally, and relatedly, if satisfaction of (analogues of) (DFLUM) doesn't even suffice to determine that any ignorance that there might be in a certain situation has its ultimate source in vagueness, what philosophically *interesting* feature does it suffice to determine? More generally, a proponent of (DFLUM) is under the obligation of saying what philosophical interest there is in coming to know that a certain domain is "luminous" in the new, restricted sense of satisfying (analogues of) (DFLUM). Especially in view of some of the points just made, it is not at all clear that there is a particularly satisfactory answer to this question.

Coming back from this brief detour, I think that we are thus warranted to assume not only that (DLUM) itself, taken as a response to the original AL-argument, is only going to be appealing to an NFMU-theory of vagueness, but also that (DFLUM) can only be legitimately regarded as preserving the idea that some domains of facts are fully open to our view if 'definitely' is understood in such a way that the assertoric content of 'One feels cold' is at least as strong as the assertoric content of 'It is definitely the case that one feels cold'. Now, many notions would obviously vindicate this claim about the assertoric content of 'It is definitely the case that one feels cold': for example, the notion of possibility, since it is the case that the assertoric content of 'One feels cold' is at least as strong as the assertoric content of 'It is possibly the case that one feels cold' (of course, (DFLUM) with this reading would be blatantly false). However, not only does the notion of determinacy also vindicate that claim (see fn 23), but it also seems to be the *strongest** notion that does so.[26] Therefore, (DLUM) itself will be entailed by any version of (DFLUM) that preserves the idea that some domains of facts are fully open to our view. Thus, I think that in the following we can without loss of generality focus on how (DLUM) fares within the framework of an NFMU-theory of vagueness.

5 Running down D-Luminous Slopes

Having thus become clearer on the relationships between luminosity and determinacy, consider now the rule of inference (very much discussed in the contemporary vagueness debate):

(D-INTRO) From φ, infer 'It is determinately the case that φ'

(singled out for the first time for careful study of his logical behaviour by, I believe, Heck [1993],[27] although earlier mentions can be found e.g. in Fine [1975], pp. 287–300). Given that, on an NFMU-

[26]For example, not only is the ingredient sense of 'It is determinately the case that φ' stronger* than the ingredient sense of 'It is possibly the case that φ', it is even stronger* than the ingredient sense of φ itself (see fn 27).

[27]Let's briefly and informally mention the most characteristic feature of such a behaviour. This consists basically in the fact that (D-INTRO) *cannot be unrestrictedly employed in an argument some of whose premises are assumptions to be discharged* (hence, (D-INTRO) cannot be unrestrictedly employed in a sub-argument for conditional proof, *reductio*, reasoning by cases etc.). To give an example, while in many standard logics (like classical, intuitionist,

theory of vagueness, the assertoric content of φ is at least as strong as the assertoric content of 'It is determinately the case that φ', (D-INTRO) is presumably valid on such a theory: whenever its premise is made true by the obtaining of a fact, so is its conclusion (see fn 23), and hence the rule will always lead from obtaining facts to obtaining facts.

Of course, especially in view of the feature mentioned in fn 27, one may object to the validity *stricto sensu* of (D-INTRO): if a rule is "really" valid, it should be permissible to employ it even under an assumption to be discharged (see McGee and McLaughlin [2004], pp. 132–136 for some relevant discussion). Fortunately, we don't need to get bogged down in a debate about what validity "really" amounts to: even if, on account of this or other feaures, (D-INTRO) were not strictly speaking *valid*, it would certainly still be *something one can reason by* if one reasons from premises that are made true by the obtaining of a fact, for, as we've just seen, (D-INTRO) *preserves the property of being made true by the obtaining of a fact*: if the premise of an application of (D-INTRO) is made true by the obtaining of a fact, so is its conclusion. Hence, (D-INTRO) is at least reliable in the sense that, if one is reasoning only from premises that are made true by the obtaining of a fact and, trusting them, one is not going to discharge them, one is guaranteed to arrive only at conclusions that are still made true by the obtaining of a fact. The argument against (DLUM) that I'm developing will only rely on this property of (D-INTRO), independently of the issue of whether or not such property suffices to make (D-INTRO) "really" valid.

The stage is finally completed for the first argument against (DLUM). Crucially, one retreats to (DLUM) because one does find the original AL-argument compelling. In particular, it is dialectically warranted in arguing against (DLUM) to assume that (KMAR) has indeed been established by the considerations appealed to in the original AL-argument. Unfortunately, the following two theorems are readily available:

Theorem 1. (DLUM)*,* (FORM) *and* (KMAR) *jointly entail* (DMAR)*.*

Proof. Assume that in α_n one determinately feels cold. Then, by (DLUM), in α_n one is in a position to know that one feels cold, and so, by (FORM), one knows that one feels cold. Therefore, by (KMAR), in α_{n+1} one feels cold.

□

Theorem 2. *In the presence of* (D-INTRO)*,* (DMAR) *is inconsistent with* (POSNEG)*.*

or minimal) whenever φ entails ψ the conditional 'If φ, then ψ' is a logical truth, this is not so once a rule like (D-INTRO) is added to one's logic (more generally, letting Γ be an arbitrary set of premises, what fails is the principle of conditional proof that, whenever Γ, φ entails ψ, Γ entails 'If φ, then ψ'). For, while (D-INTRO) makes φ entail 'It is determinately the case that φ', the conditional 'If φ, then it is determinately the case that φ' is not a logical truth—indeed, some of its instances are arguably untrue. (To see this, let φ be 'One feels cold' and suppose that there is no fact of the matter as to whether one feels cold. Then, by contraposition, 'If one feels cold, then it is determinately the case that one feels cold' yields 'If it is not the case that it is determinately the case that one feels cold, then it is not the case that one feels cold', which is arguably untrue in this situation, as it has a true (indeed, determinate) antecedent and an indeterminate consequent. Notice that, since the untrue (in the situation just described) 'If one feels cold, then it is determinately the case that one feels cold' can be obtained from the true 'If one feels cold, then one feels cold' by substituting sentences that have equivalent assertoric contents, we have here another example of failure of inter-substitutability of sentences having equivalent assertoric contents (see also fn 22): indeed, quite generally, the ingredient sense of 'It is determinately the case that φ' is stronger* than the ingredient sense of φ.)

Proof. By (POSNEG), in α_1 one feels cold, and so, by (D-INTRO), in α_1 one determinately feels cold. Hence, by (DMAR), in α_2 one feels cold, and so, by (D-INTRO), in α_2 one determinately feels cold. Hence, by (DMAR), in α_3 one feels cold. Apply analogous arguments another 999,999,997 times to reach the conclusion that in $\alpha_{1,000,000,000}$ one feels cold, which contradicts (POSNEG).

□

Interestingly, an analogous inconsistency result could have been reached even without (D-INTRO). To this end, we first need the eminently plausible principle of *closure of determinacy under logical consequence*:

(CD) If $\langle C_1\rangle$, $\langle C_2\rangle$, $\langle C_3\rangle\ldots$ entail $\langle D\rangle$, then $\langle$It is determinately the case that $C_1\rangle$, $\langle$It is determinately the case that $C_2\rangle$, $\langle$It is determinately the case that $C_3\rangle\ldots$ entail $\langle$It is determinately the case that $D\rangle$.

Theorem 3. *Let 'determinately^n' be short for the result of concatenating n occurrences of 'determinately'). Then* (CD) *entails:*

(CD*) *For every n, if $\langle C_1\rangle$, $\langle C_2\rangle$, $\langle C_3\rangle\ldots$ entail $\langle D\rangle$, then $\langle$It is determinately^n the case that $C_1\rangle$, $\langle$It is determinately^n the case that $C_2\rangle$, $\langle$It is determinately^n the case that $C_3\rangle\ldots$ entail $\langle$It is determinately^n the case that $D\rangle$.*

Proof. By induction. The induction base is trivial since 'determinately^0 φ' is definitionally equivalent with φ. For the induction step, assume that (CD*) holds at least up to n and that $\langle C_1\rangle$, $\langle C_2\rangle$, $\langle C_3\rangle\ldots$ entail $\langle D\rangle$. Then, since (CD*) holds at least up to n, $\langle$It is determinately^n the case that $C_1\rangle$, $\langle$It is determinately^n the case that $C_2\rangle$, $\langle$It is determinately^n the case that $C_3\rangle\ldots$ entail $\langle$It is determinately^n the case that $D\rangle$, from which it follows by (CD) that $\langle$It is determinately^{n+1} the case that $C_1\rangle$, $\langle$It is determinately^{n+1} the case that $C_2\rangle$, $\langle$It is determinately^{n+1} the case that $C_3\rangle\ldots$ entail $\langle$It is determinately^{n+1} the case that $D\rangle$.

□

We also need the following "determinisations" of (DMAR) and (POSNEG):

($\mathrm{D}^{999,999,998}$DMAR) It is determinately^{999,999,998} the case that, for every n, if in α_n one determinately feels cold, then in α_{n+1} one feels cold;

($\mathrm{D}^{999,999,999}$POSNEG) It is determinately^{999,999,999} the case that [one feels very cold at dawn and gradually warms up until one feels very warm at noon].

Given (CD*), ($\mathrm{D}^{999,999,998}$DMAR) has to be granted as soon as it is granted, very plausibly, that the premises of the argument establishing (DMAR) in theorem 1 are determinately^{999,999,998} true (which is in turn hard to avoid once it is granted that they are true, as we've seen it is granted by a proponent of (DLUM)). ($\mathrm{D}^{999,999,999}$POSNEG) clearly enjoys independent high plausibility.

Given these minimal assumptions, the following inconsistency result is easily established:

Theorem 4. *In the presence of* (CD), ($\mathrm{D}^{999,999,998}$DMAR) *is inconsistent with* ($\mathrm{D}^{999,999,999}$POSNEG).

Proof. ⟨In α_1 one determinately feels cold⟩ and ⟨If in α_1 one determinately feels cold, then in α_2 one feels cold⟩ entail ⟨In α_2 one feels cold⟩, and so, by (CD*), ⟨It is determinately^{999,999,998} the case that in α_1 one determinately feels cold⟩ (given by (D^{999,999,999}POSNEG)) and ⟨It is determinately^{999,999,998} the case that, if in α_1 one determinately feels cold, then in α_2 one feels cold⟩ (given by (D^{999,999,998}DMAR)) entail ⟨It is determinately^{999,999,998} the case that in α_2 one feels cold⟩. ⟨In α_2 one determinately feels cold⟩ and ⟨If in α_2 one determinately feels cold, then in α_3 one feels cold⟩ entail ⟨In α_3 one feels cold⟩, and so, by (CD*), ⟨It is determinately^{999,999,997} the case that in α_2 one determinately feels cold⟩ (given by the lemma just established) and ⟨It is determinately^{999,999,997} the case that, if in α_2 one determinately feels cold, then in α_3 one feels cold⟩ (given by (D^{999,999,998}DMAR)) entail ⟨It is determinately^{999,999,997} the case that in α_3 one feels cold⟩. Apply analogous arguments another 999,999,996 times to reach the conclusion that in $\alpha_{999,999,999}$ one determinately feels cold, which, together with (D^{999,999,998}DMAR), yields that in $\alpha_{1,000,000,000}$ one feels cold, thereby contradicting (D^{999,999,999}POSNEG).[28]

□

To sum up, our first argument against (DLUM) is this. On the one hand, (DLUM), taken as a response to the original AL-argument, is only going to be appealing to an NFMU-theory of vagueness, but any such theory will need to understand determinacy (and, in particular, the strength of the assertoric content of claims of the form 'It is determinately the case that φ') in such a way as to make (D-INTRO) unobjectionable. Moreover, any theory of determinacy whatsoever will be hard-pressed to find fault with (CD) or (D^{999,999,999}POSNEG). On the other hand, a proponent of (DLUM) as a response to the AL-argument will have to accept (KMAR), and hence she will also have to accept (DMAR) and will be hard-pressed to find fault with (D^{999,999,998}DMAR). Unfortunately, as shown by theorem 2, in the presence of (D-INTRO), (DMAR) is inconsistent with the uncontroversial (POSNEG), and, as shown by theorem 4, in the presence of (CD), (D^{999,999,998}DMAR) is inconsistent with the fairly uncontroversial (D^{999,999,999}POSNEG).

[28] Theorem 2 is a close kin of a result offered by Fara [2003], pp. 196–205. The important difference consists in the fact that Fara's result uses the principle obtained from (DMAR) by substituting 'in α_{n+1} one does *not determinately not* feel cold' for 'in α_{n+1} feels cold'. The result is more significant than theorem 2 in that the new principle is weaker than (DMAR); it is less significant in that it also requires higher-order versions of the new principle, whereas theorem 2 does not require higher-order versions of either (DMAR) or the new principle (although, of course, (CD) and (D^{999,999,998}DMAR) jointly entail the relevant higher-order versions of both (DMAR) and the new principle). Theorem 4 is a close kin of a result offered by Gómez-Torrente [1997], pp. 243–245. The important difference consists in the fact that Gómez-Torrente's result uses *higher-order versions* of (DMAR). The result is more significant than theorem 4 in that it requires neither (CD) nor (D^{999,999,998}DMAR); it is less significant in that it also needs to appeal to higher-order versions of (DMAR), whereas theorem 4 does not need to appeal to them (although, of course, (CD) and (D^{999,999,998}DMAR) jointly entail the relevant higher-order versions of (DMAR), and hence the sense in which Gómez-Torrente's result is less significant than theorem 4 is again only the sense in which theorem 4 shows how what in that result is merely a not very compelling assumption actually follows from jointly stronger premises each of which is very compelling). (I note in passing that the (CD)-based reasoning of theorem 4 takes its inspiration from a related argument developed in Zardini [2012a].) Neither Fara nor Gómez-Torrente relate their discussion to the AL-argument, let alone to (DLUM). Thanks to Paul Égré for raising the issue of the relationships between theorems 2 and 4 and these two important results in the literature.

6 Mistake and Absence of a Fact of the Matter

I want to close by offering a second argument against (DLUM), argument which manages to do away with many of the assumptions of the first argument (i.e. (D-INTRO), (CD), ($D^{999,999,999}$POSNEG) and ($D^{999,999,998}$DMAR), assumptions which I nevertheless stress are hardly objectionable on an NFMU-theory of vagueness which finds the original AL-argument compelling). This second argument in effect establishes a sorites premise by mimicking closely the way in which the original AL-argument establishes (KMAR) and by employing a very plausible principle about *mistake*:

(MIS) One's confidence that C is mistaken if there is no fact of the matter as to whether C.

To appreciate the plausibility of (MIS), let's briefly go back to the example of section 3. If one were confident that Newtonian mass is rest mass, we would judge such confidence to be mistaken, not because it is *false* that the condition one is confident in holds, but because there is *no fact of the matter* as to whether that condition holds. Or suppose that a view to the effect that reference failure of complex demonstratives generates absence of a fact of the matter (rather than falsity) were correct, and that, hallucinating a dagger on the table, one were confident that that dagger (pointing towards the table) is covered with blood. Again, we would judge such confidence to be mistaken. Or suppose that there is no fact of the matter as to whether in $\alpha_{500,000,000}$ one feels cold and that in $\alpha_{500,000,000}$ one were confident that one feels cold. Again, we would judge such confidence to be mistaken.[29] Quite generally then, absence of a fact of the matter would seem to be incompatible with the correctness of one's confidence no less than falsity is (and so would seem to imply mistake in one's confidence no less than falsity does).[30] Of course, assuming that

[29]I wish to emphasise that, in this discussion, the operative sense of 'mistake' is that of *objective* mistake, a failure located in the brute relation between a subject's belief (and confidence) and reality rather than in the formation of the belief (and confidence) in the light of the subject's epistemic position. I don't contest that there is a sense of 'mistake' and its like which targets the *epistemic pedigree* of a subject's belief (and confidence), and which is thus such that e.g. a belief formed on the basis of tossing a coin to the effect that the number of stars in the universe is even would count as mistaken even if the number of stars in the universe were in fact even. This epistemically loaded sense of 'mistake' and its like would be of dubious relevance for the discussion connecting mistake and reliability. For example, it might be that a (epistemic) norm of belief (and confidence) is knowledge: in such case, the general principle that knowledge requires reliability would have extremely problematic consequences. For recall that, in our framework, we interpret the principle as requiring that, for every n, in α_n one knows that one feels cold only if in α_n one's confidence that one feels cold is reliable, and that we take this in turn to require that a very similar confidence in a very similar condition formed on a very similar basis in a very similar case—e.g. the confidence one has in α_{n+1} in ⟨One feels cold⟩—is not mistaken. The epistemically loaded sense of 'mistake' would then require that the latter confidence is also knowledgeable, thus generating a sorites premise which, in conjunction with (POSNEG) and the factivity of knowledge, would entail the sceptical conclusion that in α_1 one does not know (and, by (FORM), is not in a position to know) that one feels cold.

[30]Somehow related to the point made in fn 29, I wish to emphasise that, in this discussion, the operative sense of 'mistake' is that of *representational* mistake, failure to represent reality (the totality of obtaining facts) as it is. It may be thought that there is a sense of 'mistake' and its like which, while still purely objective, permits as not mistaken a positive degree of confidence (or even belief) in at least some conditions representing no fact of the matter, and which is thus such that e.g. today a belief (or at least a confidence) that there will be a sea-battle tomorrow would not count as mistaken even if today there were no fact of the matter as to whether there will be a sea-battle tomorrow. Such view has a dubious plausibility even with respect to the domains for which it is not clearly a non-starter (possibly among others, the future, vagueness and conditionals): the very real intuitions on which it relies concerning the warrantedness of belief (or at least confidence) are more plausibly taken to tell against the assumption that the relevant condition does not represent any fact of the matter. Be that as it may, belief (and

truth is a (objective, representational) norm of belief (and confidence), this incompatibility could be explained rather smoothly if absence of a fact of the matter unavoidably induced lack of truth (just as falsity does), but the incompatibility is independently plausible even on a view (such as the one championed e.g. in Field [1994]) that divorces absence of a fact of the matter from lack of truth and falsity, in such a way that absence of a fact of the matter does not require lack of truth (thus treating factuality as being stronger* than truth). Even if stronger* than truth, factuality seems to be a (objective, representational) norm of belief (and confidence) no less than truth is (for one, truth is at least as strong as factuality).

With (MIS) in place, we can now use (DLUM) to establish a sorites premise by mimicking closely the way in which the original AL-argument establishes (KMAR). Assume that in α_n one determinately feels cold. Then, by (DLUM) and (FORM), in α_n one knows that one feels cold, and so in α_n one is reasonably *confident* that one feels cold (otherwise, one would not know that one feels cold). Hence, by (CONF), in α_{n+1} one is at worst ever so slightly less confident that one feels cold. Now, does one *determinately* feel cold in α_{n+1}? We argue by (classical) *reductio* that one does.[31] For assume for *reductio* that in α_{n+1} one does not determinately feel cold. Then, by (MIS), one's still considerably high confidence in α_{n+1} that one feels cold is *mistaken*, and this in turn implies that one's confidence in α_n is not *reliable*. This is so because a very similar confidence in a very similar condition formed on a very similar basis in a very similar case—i.e. the confidence one has in α_{n+1} in ⟨One feels cold⟩—is mistaken. But if in α_n one's confidence that one feels cold is not reliable, in α_n one arguably does not know that one feels cold, contrary to what we have just established on the basis of our original hypothesis. Therefore, by (classical) *reductio*, we'd better say that in α_{n+1} one does determinately feel cold. But α_n was completely arbitrary, and so we are still stuck with a catastrophical sorites premise:

(DSOR) For every n, if in α_n one determinately feels cold, then in α_{n+1} one determinately feels cold as well,

which, given (CD), contradicts the following extremely plausible strengthening of (POSNEG):

(DPOSNEG) It is determinately the case that [one feels very cold at dawn and gradually warms up until one feels very warm at noon]

by familiar reasoning (see fn 13).

To sum up, our second argument against (DLUM) is this. Granted that absence of a fact of the matter no less than falsity implies mistake in confidence, (DLUM) can be used as a crucial

even confidence) that is in this sense objectively not mistaken albeit representationally mistaken is very plausibly all the same sufficient for giving rise to a charge of unreliability against a very similar confidence in a very similar condition formed on a very similar basis in a very similar case. For example, suppose that a respected general said that there was a sea-battle yesterday and also that there will be a sea-battle tomorrow. Suppose also that, while it is a fact of the matter that there was a sea-battle yesterday, there is no fact of the matter as to whether there will be a sea-battle tomorrow. Suppose finally that, purely on the basis of this testimony and without knowing that there is no fact of the matter as to whether there will be a sea-battle tomorrow, one becomes confident both that there was a sea-battle yesterday and that there will be a sea-battle tomorrow. Then, very plausibly, the fact that the latter confidence is placed in a condition representing no fact of the matter (and so is representationally mistaken) does give rise to a charge of unreliability against the former confidence.

[31] I won't go into the details of this, but I note that the particular application of *reductio* to follow is unobjectionable even in view of the remarks in fn 27.

premise in an argument that mimics closely the way in which the original AL-argument establishes (KMAR) and that concludes to an unacceptable sorites premise.

7 Conclusion

The upshot of our two arguments against (DLUM) is that, granting the compellingness of the original AL-argument, even with respect to the most cognitively accessible conditions (such as ⟨One feels cold⟩) *not even determinacy can guarantee knowability*. Some may be ready to take this as an important philosophical discovery. Others (including myself) may want to take pause at this point, and go back to re-examine the original AL-argument, sensing that the worry from vagueness as presence of borderline cases has misidentified the illicit role played by vagueness in the argument.[32]

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[32]Having argued this much against (DLUM), I hasten to add that in Zardini [2012c] I try to rescue (LUM) by focussing instead on that phenomenon of vagueness consisting in the apparent absence of sharp boundaries, and that the considerations offered in that paper can easily be converted into a rescue of (DLUM) itself from the two arguments against it that I have put forth in this paper (given that these arguments ultimately rely on a crucial kind of inference at work both in the derivation of (KMAR) and in the derivation of (DSOR), see fn 10). Such a rescue is particularly important if an NFMU-theory of vagueness is correct. For in that case, because of the reasons pointed out in section 3, (LUM) has anyways to be abandoned in favour of the more guarded (DLUM), without this constituting in any way a retreat from the idea that some domains of facts are fully open to our view.

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