No-no. Paradox and consistency

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1. A Critique of the Sorensenian Solution to the No-No Paradox

Classical logic and *naive truth* – roughly, the unrestricted equivalence between '*P*' and ''*P*' is true' – are a cocktail for disaster. Sometimes the disaster consists in the derivation of a straightforward *contradiction* (as happens in a standard Liar paradox). Some other times, however, it consists instead in the derivation of a conclusion which, *while formally consistent, is clearly unwarranted*, lying beyond the bounds of what one can rationally infer when one reasons about truth (as happens in a version of Curry's paradox with the conditional:

(C) If (C) is true, Italy will win the next World Cup,

using which one can derive, by classical logic and naive truth alone, that Italy will win the next World Cup!).[1] It is in this vein that we wrote that a *paradox* is any situation where 'despite the apparent validity of the argument, the premisses do not appear rationally to support the conclusion' (López de Sa & Zardini 2007: 246), never mind whether the premisses are apparently true and the conclusion apparently false (which is only one specific way in which the former may not appear rationally to support the latter). Ideally, one would avoid all these dangers in a principled and systematic way by deploying a theory of truth which is provably consistent and conservative over the relevant background theories (a theory that will perforce involve a weakening of either classical logic or naive truth). In the absence of such a theory, however, when one reasons about truth one should be *extremely wary* of any substantial conclusion reached *via* the joint deployment of classical logic and naive truth, even if the black mark of a contradiction has not appeared in one's reasoning. For, as we've just seen in the case of (C), the mere preservation of formal consistency is no guarantee at all that

[1] Throughout, we understand the conditional to be material. Armour-Garb & Woodbridge 2010: 13, a paper which we'll discuss in greater detail in sections 2 and 3, put forth the principle:

(DT) If the only consistent truth-value assignment for a given sentence token, S, assigns it truth (falsity), then that will be the correct truth-value assignment for S.

A discussion of the (dubious) philosophical merits of (DT) is rendered unnecessary by its disastrous consequences as applied to (C) and its like. For notice that the assignment of falsity to (C) is inconsistent, as, by classical logic and naive truth, the falsity of (C) entails the falsity of its antecedent and so (C)'s truth. The assignment of truth to (C) is, on the contrary, consistent, even given the full joint power of classical logic and naive truth. Hence, (DT) disastrously implies that (C) is true, and so, by classical logic and naive truth, that Italy will win the next World Cup. (Armour-Garb & Woodbridge 2010: 13–4 further claim that (DT) has the consequence that the Truth Teller is either true or false. This is certainly mistaken as the antecedent of (DT) is only triggered by the fact that *only one* truth-value assignment is consistent, which is manifestly not the case for the Truth Teller. For the same reason, (DT) and its relatives are equally idle with respect to the no-no paradox to be introduced shortly.)

one is not spinning into paradox.

Such wariness is called for, for example, when one considers the pair of sentences known as 'the *no-no paradox*':

(1) (2) is false

(2) (1) is false.

Roy Sorensen (2001: 165–70), who has the great merit of having brought the paradox to contemporary attention, maintains that one sentence is true and the other one is false, since, *by classical logic and naive truth*, only these two truth-value assignments are consistent.

Pursuing a programme initiated in López de Sa & Zardini 2006, we argued in López de Sa & Zardini 2007 that Sorensen's argument can be rightly regarded as a paradox in the sense specified above—in particular, that it is a case where the joint deployment of classical logic and naive truth leads to an unwarranted conclusion. We argued for this conclusion by claiming that Sorensen's argument cannot be any better than the following foolish argument based on the pair of sentences:

(1′) If (2′) is true, then [(1′) is false and it is not the case that [(1′) is short and (2′) is long]][2]

(2′) If (1′) is true, then [(2′) is false and it is not the case that [(2′) is short and (1′) is long]].

By classical logic and naive truth, the only two consistent truth-value assignments are those that assign truth to one sentence and falsity to the other one, but both such assignments entail, again by classical logic and naive truth, that the true sentence is long while the false one is short! As we wrote:

> The situation with (1′) and (2′) is [...] structurally identical to the situation with (1) and (2): both pairs of sentences are perfectly symmetrical in form; the reasoning is for all intents and purposes the same in both cases (and classically valid); only the instances of [principles of naive truth] for the relevant sentences are used as assumptions in both cases; both conclusions are perfectly consistent – even though startling, offending as they do in one case against our intuition of uniformity in truth-value and in the other case against our perception of uniformity in length. Whatever may be specifically wrong about it, the argument to the effect that either (1′) is true (and long) and (2′) is false (and short) or *vice versa* clearly fails rationally to support its conclusion, and hence so does the argument to the effect that either (1) is true and

[2] Throughout, we use square brackets to disambiguate constituent structure.

(2) is false or *vice versa*, whose soundness wholly relies on the very same abstract formal features. (López de Sa & Zardini 2007: 245–46)

2. Tokenism and the Meaninglessness Strategy

In their recent reply in this journal, Bradley Armour-Garb and James Woodbridge (2010) contend that we've failed to establish our conclusion. To begin with, they notice that a proponent of the Sorensenian solution might be sympathetic to the following doctrines:

- *tokenism* about semantic features, according to which these features attach to tokens rather than types and can differ for different co-typical tokens

- the *meaninglessness strategy* for solving the semantic paradoxes, according to which paradoxical tokens are, in some sense, 'meaningless'

and suggest that endorsement of these doctrines would allow one to resist our argument (Armour-Garb and Woodbridge 2010: 13, 16; we'll adopt their labels for the two doctrines and we'll moreover use the acronym 'TMS' to denote their fashionable combination).[3]

There should be initial reasons for puzzlement about the suggestion that our argument fails to take into account TMS. As we emphasised in López de Sa & Zardini 2007: 249, we wanted to remain neutral as to what exactly goes wrong in the various kinds of paradoxical reasoning—and this neutrality was of course supposed to include TMS, which is one of the main competitors in the debate on the semantic paradoxes. Moreover, as Armour-Garb & Woodbridge 2010: 11, n. 1 themselves note, one of the most sophisticated defenders of TMS, Laurence Goldstein, has recently endorsed our argument (2009: 378–79), which would be hard to explain if the argument failed to take into account TMS.

Be that as it may, we find Armour-Garb & Woodbridge's appeal to TMS a red herring. Notice that, in our argument, we did make use of a natural if informal notion of the situation with (1)–(2) being '*structurally identical*' to the situation with (1′)–(2′) (see the quote at the end of section 1). Now, Armour-Garb & Woodbridge seem to think that this is exactly the *same kind* of structural identity as that within which TMS typically draws crucial distinctions (so that, against our argument, a distinction could be drawn between (1)–(2) and (1′)–(2′)). Is it really? A look at the simplest example, mentioned by Armour-Garb & Woodbridge 2010: 13 themselves, will suffice to show that it isn't. Consider the two tokens:

(A) (A) is not true

[3] Fashionable, but by no means uncontroversial (for example, one of us has mounted a recent attack against it in Zardini 2008: 561–66, 2010).

(B) (A) is not true.

TMS typically holds that while (A) is meaningless and hence not true, (B) is true. The sense in which this differential assignment of truth values goes against a 'structural identity' between (A) and (B) is the plain one in which (A) and (B) are *tokens of the same sentence*. But that is clearly *not* the sense of 'structural identity' appealed to in our argument. On the one hand, that sense is in certain respects *looser* than co-typicality, since (1′)–(2′) are obviously not tokens of the same sentences as (1)–(2). On the other hand, that sense is in certain respects *stricter* than co-typicality, since co-typicality does not preserve referential structure (for example, (A) is self-referential while (B) is not), while it was clearly important for our sense of 'structural identity' that, on the contrary, (1′)–(2′) have the same 'cross-referential' structure as (1)–(2).[4] These are decisive differences in the notions of 'structural identity' employed respectively in discussions of TMS and in our argument. Only a confusion between the two can encourage Armour-Garb & Woodbridge's thought that TMS's differential treatment of (A) over (B) can support a differential treatment of (1)–(2) over (1′)–(2′).

3. Consistency, Meaninglessness and Arbitrariness

Having thus clarified the relationships between TMS and our argument, we proceed to discuss the main reply that Armour-Garb & Woodbridge offer on behalf of a proponent of the Sorensenian solution:

> while we can grant that the reasoning that [López de Sa and Zardini] employ regarding the semantic statuses of [(1′)–(2′)] is structurally identical to the reasoning Sorensen employs regarding the semantic statuses of [(1)–(2)], this by no means requires Sorensen to conclude that [(1′)–(2′)] have the same semantic statuses as those of [(1)–(2)]. In fact, since (1′) and (2′) seem unable to tolerate any (consistent) truth-value assignments, Sorensen would declare them *meaningless* [...] In contrast, since [(1)–(2)] appear to tolerate consistent truth-value assignments, Sorensen would take them to be, while epistemically indeterminate in truth-value, both meaningful and (consistently) truth-valued. (Armour-Garb and Woodbridge 2010: 16–7)

The central claim of this passage to the effect that there are no consistent truth-value assignments for (1′)–(2′) is however clearly mistaken. As we explicitly said in López de Sa & Zardini 2007 (see again the quote at the end of section 1), an essential part of the structure of our analogy is precisely that, just as the conclusion of the reasoning involving (1)–(2)—namely, that one sentence is true

[4] Thus, while Armour-Garb & Woodbridge 2010: 16, n. 4 are quite correct in assuming that co-typicality is not *necessary* for 'structural identity' in our sense, they are quite mistaken in assuming that it is nevertheless *sufficient*, as when they write: 'Co-typicality is just one, rather exacting, way by which different tokens (of sentence types or forms of reasoning) can be said to be structurally identical in [López de Sa and Zardini]'s sense'.

and the other one is false – the conclusion of the reasoning involving (1′)–(2′) – namely, that the true sentence is long while the false one is short – *is perfectly consistent*! True, as we wrote, both conclusions are *startling*, offending as they do in one case against our intuition of uniformity in truth value and in the other case against our perception of uniformity in length, but we should all agree in distinguishing *incompatibility with some firmly held belief* from *inconsistency*, lest we start counting 'Snow is black' as inconsistent and thereby miss the whole point of that notion.

(1′)–(2′) cannot thus be dismissed as meaningless because of inconsistency, for the simple reason that they are, on the contrary, perfectly consistent. But maybe (1′)–(2′) could be dismissed as meaningless because, roughly, they are incompatible with the firmly held belief that they are uniform in length? It is not entirely clear to us what, according to TMS, is supposed to licence a claim of meaninglessness, but let us grant for the sake of argument that, in this case, one can let one's grounds for that firmly held belief override the joint deliverances of classical logic and naive truth and thus licence a claim of meaninglessness for (1′)–(2′). However, we believe that, given our analogy, such a move would be damning for the dialectical force of the Sorensenian solution to the no-no paradox: for many of us think to have very good grounds for an equally firmly held belief concerning the uniformity in truth value of (1)–(2),[5] which, by parity of reasoning, should then be let override the joint deliverances of classical logic and naive truth – on which the Sorensenian solution relies – and thus licence a claim of meaninglessness for (1)–(2). To do otherwise, and follow the perilous combination of classical logic and naive truth wherever it leads in the case of (1)–(2) while repudiating it in the case of (1′)–(2′), would smack of intellectual arbitrariness.

We conclude that we still haven't seen convincing reasons for teasing apart (1)–(2) from (1′)–(2′), and hence that Sorensen's argument still appears to be no better than the foolish argument concluding that one of either (1′) or (2′) is long and the other one short.[6]

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[5] For example, Priest 2005: 690 writes that the non-uniformity in truth value of (1)–(2) is 'a manifest *a priori* repugnance'.

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