Truthmakers, knowledge, and paradox

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According to the *truthmaker principle*, every truth – at least, every contingent truth – has a truthmaker, something in virtue of which it is true.

This is perhaps too strong. True negative existentials – such as the truth that there are no unicorns – might motivate a retreat to the weaker principle that truth requires at most a truthmaker *or* lack of a falsemaker (in our example, lack of a unicorn). Such a principle would still vindicate the idea that truth supervenes on being – on what there is. However, some true (non-essential) predications – such as the truth that the rose is red – might motivate a further retreat to an even weaker principle, if one were to hold the view that it is not required that there be some *thing* (say, a state of affairs) in virtue of which it is true that the rose is red. On this view, truth would no longer supervene on *what* there is, but only on what there is *and how* it is. Notice though that both weakenings still respect what seems to be the essential core of the thought that truths require truthmakers: that truths are about things, that they 'do not float in a void', in Lewis's (1992: 218) apt phrase.

According to Roy Sorensen (2001: 165–84), however, that very same core is jeopardized by what has come to be known as the '*no-no paradox*'. As the reconstruction to be presently offered will make clear, Sorensen's argument wholly relies on an (alleged) abstract formal feature of the predicate 'is true'.[1] The cogency of such an argument can thus fruitfully be tested by looking at how structurally identical arguments fare. In this paper, we provide parallel arguments which, though relying on the very same feature, clearly fail to carry any conviction – even if their conclusions are in themselves consistent, or epistemically open, or indeed evident. Hence, Sorensen's own argument should likewise be rejected. Since the no-no paradox is assuming increasing weight in discussions on indeterminacy (ranging from the semantic paradoxes to vagueness), we think this result already has considerable intrinsic interest. We close by generalizing it to what we believe are further instances of the same phenomenon in the recent literature on truthmaking and knowledge.

1. The no-no paradox and 'truthmaker gaps'

Consider the following pair of sentences:

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[1]This being its satisfaction of the schema:

(T) '*P*' is true iff *P*,

with any restriction that may be required in order to avoid the usual semantic paradoxes. For future reference, we call the left-to-right and right-to-left directions 'disquotation' and 'enquotation' respectively.

(1) (2) is false

(2) (1) is false.

Sorensen (2001: 165–70) maintains that one is true and the other is false. His grounds for this can be reconstructed as follows. (1) and (2) cannot both be true, for, by disquotation, they would then both be false. But they cannot both be false either, for, by enquotation, they would then both be true. It only remains that one is true and the other false. But which is which?[2]

Sorensen's own proposal consists in positing '*truthmaker gaps*' in order to account for the apparent absolute unknowability of the answer to this question. The move needs some explaining. As we have seen, some philosophers think of true negative existentials and of some true (non-essential) predications as lacking a truthmaker, but this hardly gives them any reason to believe that knowledge of them is not readily available, let alone absolutely impossible. Rather, Sorensen's key thought is that '[a] contingent statement that does not owe its truth-value to anything else is epistemically isolated. When the truth of a statement rests on further facts, then I can gain evidence by examining those further facts. But when the truth-value is possessed autonomously, then there is no trail of truthmakers' (Sorensen 2001: 177).

This thought might be tentatively expanded as follows. Even if (possibly) having no truthmaker, the truth of 'There are no unicorns' or of 'The rose is red' does rest on what there is and how it is – namely on there being no unicorns and on the rose's being red – so that knowledge of these mundane facts can be seen as grounding knowledge that the corresponding sentences are true. But such *epistemic mediation* would seem to be unavailable in the no-no paradox: whichever sentence turns out to be the true one, its truth would seem to fail to be appropriately related to what there is and how it is. For, one might think, assuming without loss of generality that (1) is true and (2) is false, (1)'s truth would have to rest on (2)'s falsity, but (2)'s falsity would in turn have to rest on (1)'s truth – and how could one break into this circle and gain knowledge of (1) and (2)'s respective truth-values?[3]

Be that as it may, Sorensen's talk of 'truthmaker gaps' should be taken with a grain of salt, as what he really seems to mean are truths that not only lack a truthmaker, but are in addition 'free-floating' in the peculiar way supposed to be exemplified by (1) and (2) (however this notion might be best understood).

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[2]As the reader will have noticed, we are availing ourselves of very strong logical resources (ie full propositional classical logic). While one might find this dubious in the present context, the relevant patterns of inference are simply taken for granted in the debates we are interested in, and so can be safely used for our dialectic purposes. We also assume the definition of falsity of a sentence as its untruth – given classical logic and the relevant instances of (T), this is equivalent to the definition of falsity of a sentence as truth of its negation.

[3]The picture briefly sketched here is of course inspired by Kripke's (1975: 701–2) insightful informal explanation of the notion of semantic grounding.

2. 'Proving' surprising asymmetries

We don't wish to rule out that there might be other reasons for the postulation of 'truthmaker gaps' in Sorensen's sense and that the notion, once properly understood and refined, might indeed play some role in dealing with (at least some of) the semantic paradoxes and the paradoxes of vagueness. We do claim, however, that the argument offered for the conclusion that either (1) is true and (2) is false or *vice versa* fails to carry any conviction. This is so given that a similar pair of sentences can be constructed such that they exhibit both the same formal symmetry and – assuming the relevant instances of (T) – the same property that all and only asymmetric assignments of truth-values are admissible, but also such that – assuming again the relevant instances of (T) – any such assignment entails that one of them is long while the other is short!

Consider the following pair of sentences:

(1′) If (2′) is true, then [(1′) is false and it is not the case that [(1′) is short and (2′) is long]]

(2′) If (1′) is true, then [(2′) is false and it is not the case that [(2′) is short and (1′) is long]],[4]

where 'if' expresses the usual truth function.

Exactly as with the no-no paradox, it would seem that we can reason as follows. (1′) and (2′) cannot both be true, for, by disquotation, they would then both be false. But they cannot both be false either, for, by enquotation, they would then both be true.[5] It only remains that one is true and the other false, which entails, again by enquotation, that the true one is long while the false one is short.

The situation with (1′) and (2′) is, as we have already indicated, structurally identical to the situation with (1) and (2): both pairs of sentences are perfectly symmetrical in form; the reasoning is for all intents and purposes the same in both cases (and classically valid); only the instances of (T) for the relevant sentences are used as assumptions in both cases; both conclusions are perfectly consistent – even though startling, offending as they do in one case against our intuition of uniformity in truth-value and in the other case against our perception of uniformity in length.[6] Whatever may be specifically

[4]Throughout, we use square brackets to disambiguate scope in English.

[5]Given our definition of falsity as untruth, (the contrapositive of) enquotation yields the negations of (1′) and (2′). The rest is routine.

[6]For dramatic purposes, we have chosen an example yielding an apparently *false* asymmetry. But, of course, nothing essential depends on this. Consider the question about which sentences will officially be pronounced at some time by the next Spanish King – something which is presumably *epistemically open* – and consider the following pair of sentences:

(1″) If (2″) is true, then [(1″) is false and it is not the case that [(1″) will officially be pronounced at some time by the next Spanish King and (2″) will not]]

wrong about it, the argument to the effect that either (1′) is true (and long) and (2′) is false (and short) or *vice versa* clearly fails rationally to support its conclusion, and hence so does the argument to the effect that either (1) is true and (2) is false or *vice versa*, whose soundness wholly relies on the very same abstract formal features.

A traditional wisdom has it that a *paradox* consists in, roughly, a case where apparently true premises apparently entail an apparently false conclusion (see for instance Sainsbury 1995: 1). As a characterization, this is at best *too narrow*. A paradox need not exhibit all these elements, as witnessed by the familiar argument using:

(3) If (3) is true, then Italy will win the next World Cup

to 'prove' that Italy will win the next World Cup. There can thus be paradox even if the conclusion, far from being apparently false, is epistemically open, or indeed evident (just consider the corresponding argument 'proving' that Italy won the last World Cup). What really seems to be of the essence is that, despite the apparent validity of the argument, the premises do not rationally support the conclusion. In this sense, Sorensen's argument and the others considered below are in our view rightly regarded as paradoxes.

3. Further paradoxes

Sorensen's argument has recently been taken up by Bradley Armour-Garb and James Woodbridge (2006). Unlike us, they are willing to grant its soundness and merely aim to show that Sorensen's own explanation of the ensuing ignorance lacks the required generality. Apparently not finding any fault in the kind of argument Sorensen's is an instance of, it is quite natural for them to devise a similar pair of sentences exhibiting both the same formal symmetry[7] and – assuming the relevant instances of (T) – the same property that all and only asymmetric assignments *of truth-values* are admissible, but also such that – assuming again the relevant instances of (T) – any such assignment entails a suitable asymmetric assignment *of truthmakers* as well. The possibility of applying Sorensen's original explanation to this new case of ignorance would thereby be foreclosed.

(2″) If (1″) is true, then [(2″) is false and it is not the case that [(2″) will officially be pronounced at some time by the next Spanish King and (1″) will not]].

A structurally identical argument would establish that the true one will not be officially pronounced at some time by the next Spanish King while the false one will. We hope that at this point the reader won't jump on her seat, rub her eyes and go to a betting shop to invest her lifetime savings in a bizarre bet on the official utterances of the next Spanish King (and hereby disclaim any responsibility if she will!).

[7] Actually, Armour-Garb and Woodbridge think that symmetry is not an essential feature of the indeterminacy exhibited by the no-no and similar paradoxes. We won't discuss this aspect of their view here; we only note that a point analogous to the one about to be made will apply to the asymmetric cases they consider.

Consider the following pair of sentences:

(4) If (5) is true, then (4) is false and (5) has no truthmaker

(5) If (4) is true, then (5) is false and (4) has no truthmaker.

Armour-Garb and Woodbridge reason as follows:

> Working through the possible truth-values for (4) and (5), one can see that matching ascriptions yield inconsistency, while divergent ascriptions are consistent. … In order to see why, ascribe truth to (4) and falsity to (5). In that case, (4) is true, in virtue of the falsity of its antecedent. In order for (5) to be false, the antecedent of this (material) conditional must be true (which, *ex hypothesi*, it is), with the consequent false. Since the first conjunct of the consequent is true, the second must be false. That is, it must be false that (4) has no truthmaker, from which (via obvious fiddling) we conclude that (4) *has* a truthmaker. Thus, we can consistently maintain that (4) is true and (5) is false, provided we also maintain that (4) has a truthmaker. A parallel argument shows that we can maintain that (4) is false and (5) is true, provided we also maintain that (5) has a truthmaker. (Armour-Garb and Woodbridge 2006: 405, numbering altered)

In our view, to grant the soundness of the argument to the conclusion that either (1) is true and (2) false or *vice versa* is a *premier pas fatal*. This is not remedied by putting forward (4) and (5), since the argument leading to the conclusion that either (4) is true (and so has a truthmaker) and (5) is false or *vice versa* is just as effective as the argument below, which relies on exactly the same abstract formal features.

Consider the following pair of sentences:

(4′) If (5′) is true, then (4′) is false and (5′) is not short

(5′) If (4′) is true, then (5′) is false and (4′) is not short.

We reason as follows:

> Working through the possible truth-values for (4′) and (5′), one can see that matching ascriptions yield inconsistency, while divergent ascriptions are consistent. In order to see why, ascribe truth to (4′) and falsity to (5′). In that case, (4′) is true, in virtue of the falsity of its antecedent. In order for (5′) to be false, the antecedent of this (material) conditional must be true (which, *ex*

> *hypothesi*, it is), with the consequent false. Since the first conjunct of the consequent is true, the second must be false. That is, it must be false that (4′) is not short, from which (via obvious fiddling) we conclude that (4′) *is* short. Thus, we can consistently maintain that (4′) is true and (5′) is false, provided we also maintain (against the deliverances of our senses) that (4′) is short. A parallel argument shows that we can maintain that (4′) is false and (5′) is true, provided we also maintain (again, against the deliverances of our senses) that (5′) is short.

The phenomenon we have brought out here is quite general. There are a number of recent arguments that purport to prove substantial philosophical claims but merely rely on (alleged) abstract formal features of the relevant predicates in a way which makes them similarly problematic.

Thus, Peter Milne (2005) aims to refute the truthmaker principle, contending that:

(6) (6) has no truthmaker

is a truth without a truthmaker. In so arguing, Milne only uses enquotation and the factivity of the truthmaking predicate.[8] Unfortunately, a structurally identical argument, using only enquotation and the factivity of the predicate 'is both true and short', would also establish that:

(6′) (6′) is not both true and short

is (true but) not short (see López de Sa and Zardini 2006 for details).[9]

Roy Cook (2006) aims to refute the view that all truths are knowable, contending that one of:

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[8]A predicate Φ is *factive* iff every instance of the schema 'If '*P*' is Φ, then *P*' holds.

[9]In that paper, we didn't argue that the truthmaker principle is consistent (let alone true), only that Milne's argument gives no warrant to believe that (6) is a truth without a truthmaker. We conjectured there that the various ways of resolving the usual semantic paradoxes would provide the defender of the truthmaker principle with corresponding strategies for blocking Milne's attack. Gonzalo Rodriguez-Pereyra (2006)'s recent criticism of Milne's argument can be seen as an implementation of this suggestion. He contends that the argument begs the question against the defender of the truthmaker principle, since the principle entails (together with factivity of truthmaking) that [(6) has a truthmaker iff (6) has no truthmaker], which – so he says – makes (6) 'paradoxical' (Rodriguez-Pereyra 2006: 261). He then proceeds to sketch a 'no-proposition-expressed' strategy for solving the problem. We do have doubts about the tenability of this particular strategy. However, here we should only like to stress that, as argued in our López de Sa and Zardini 2006, far from simply begging the question *against the defender of the truthmaker principle*, Milne's argument should not be taken *by anyone's lights* to demonstrate that (6) is a truth without a truthmaker.

(7) (8) cannot be known to be true

(8) (9) cannot be known to be true

(9) (7) cannot be known to be true

is true but unknowable. In so arguing, Cook only uses enquotation and the factivity of 'can be known to be true'. Again, a structurally identical argument, using only enquotation and the factivity of the predicate 'is both true and short', would also establish that one of:

(7′) (8′) is not both true and short

(8′) (9′) is not both true and short

(9′) (7′) is not both true and short

is (true but) not short (see Duke-Yonge 2006 for details).

Patrick Grim (2000) aims to refute the existence of an omniscient (and infallible) being *g*, contending that:

(10) (10) is not believed by *g*

is true but not believed by *g*. In so arguing, Grim only uses enquotation and the factivity of 'is believed by *g*' (remember that *g* is supposed to be infallible as well). Once again, a structurally identical argument, using only enquotation and the factivity of the predicate 'is truly officially pronounced at some time by the next Spanish King', would also establish that:

(10′) (10′) is not truly officially pronounced at some time by the next Spanish King

is (true but) not officially pronounced at some time by the next Spanish King (details left to the reader).

We believe there are other examples in the recent literature, but must draw the present note to an end. Examination of *what exactly* goes wrong in the arguments

considered must also wait for another occasion. Our aim here has only been to show that, despite their appeal and ingenuity, *something* does indeed go wrong in them.[10]

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[10]We would like to thank Bradley Armour-Garb, Sebastiano Moruzzi, Stephen Read, Sven Rosenkranz, Roy Sorensen, James Woodbridge and the members of *Arché* for very stimulating comments and discussion. In writing this paper, the first author has benefitted from a GenCat-Fulbright Postdoctoral Fellowship and Project HUM2004-05609-C02-01 (MEC), the second author from an AHRC Doctoral Award.

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